A Topological Model of Particle Physics

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June 2018†

Abstract

A mathematical model for interpreting Newtonian gravity by means of elastic deformation of space is given. Based on this model, a possible theory of particle physics, interpreting particles as compact manifold attached to space by means of a connected sum is given. This theory is based on little mathematical ground and should be seen more as a description of the world as we would like to be (being enthusiast of low dimensional manifold topology) then a proper theory. However, we think that this theory is fun and it was worth writing about it!

Key Words: Particle Physics, Topology.

1 Introduction

In this paper we show as Newtonian gravity may be interprets in a framework of equilibrium in elastic material. Based on this fact, we propose a theory of particles made of compact manifold direct summed to space. This topological theory of particles is based on very little mathematical ground but we believe it is interesting and it deserves to be further analysed.

String theory, although with little success so far, has shown that geometry and topology are somehow connected to theoretical physics and can be used as a powerful language to describe it.

We believe that, if for decades thousands of physicists have devoted their efforts to prove that space has several compactified dimensions, it would not be more crazy to believe that particles are compact manifolds and try to prove it. If a couple of physicists would investigates this hypothesis, although they may most likely disprove it, in the worst case they would prove some interesting theorems in low dimensional topology, a thing physicist with a background in string theory seems very good at.

2 Elastostatic for Shearless Material

We want to study the equilibrium in an elastic material in presence of a solution with vanishing Shear (i.e. strain due to layers laterally shifted in relation to each other). The vanishing shear may be due to the particular geometry. For example

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†Posted at: www.vixra.org (Classical Physics) - Current version: v1 - 16^{th} of June 2018

it is possible to show that spherical symmetric solution have always vanishing shear. However, in the general case, we have to assume that the vanishing shear is due to the characteristic of the material (i.e. vanishing non diagonal coefficients of the constituent equation tensor).

To find our solution we start from the Navier-Lame equation for the equilibrium in elastic materials:

$$p + \mu\nabla^2 u + (\lambda + \mu)\nabla\nabla \cdot u = 0 \tag{1}$$

where p is the distributed force in the material, μ and λ are Lame's parameters end u is a field of displacements that solve our problem. In absence of distributed force ($p = 0$) and using the following identity:

$$\nabla^2 u = \nabla(\nabla \cdot u) - \nabla \times (\nabla \times u) \tag{2}$$

we can write (3) as folows:

$$\mu\nabla^2 u + (\lambda + \mu)[\nabla^2 u + \nabla \times (\nabla \times u)] = 0 \tag{3}$$

since vanishing shear implies that $\nabla \times u = 0$, we have immediately that the above equation simplifies as follows:

$$\nabla^2 u = 0 \tag{4}$$

The above equation together with the boundary conditions will allow us to find the field of displacement in a shearless material.

3 Solution for the Removed Ball Problem

Let us consider a three-dimensional space composed of an uniform, isotropic and elastic material. We want to evaluate the elastostatic solution (i.e. the displacement u) when we remove a ball of material of radius R and we identify the boundary of the ball to a point.

The stress in the material is proportional to the deformation of the material with respect to the rest position that we will suppose to be the one where all points of the material are at no distance each other. This means that if we take any finite chunk of our material and we let it shrink, it will shrink to a point. We need this assumption so that, when we identify the boundary of the ball to a point, we have no wrinkles in the material. This is quite a remarkable propriety of our the material that we require in addition to the property of the material not to take shear as discussed in the previous paragraph.

The general spherical symmetric solution of the (4) can be expressed as:

$$u(r, \theta, \phi) = f(r)\hat{i}_r \tag{5}$$

where $f(r)$ is the displacement along the radius r of the elastic material and $\hat{i}_r$ is the versor of the radial axis.

Moreover, for our problem, f is defined on $[R, \infty]$ and has to satisfy boundary condition $f(R) = -R$ (point at position R is displaced to the centre of the spherical coordinates), $f(\infty) = 0$ (no displacement at infinity).

Let u be a vector field defined as follows:

$$u(r,\theta,\phi) = -\frac{R^3}{r^2}\hat{i}_r \quad \text{with } r > R \tag{6}$$

we want to show that (6) is the solution we are looking for.

To show that, we note (6) verifies the boundary condition. Moreover, since (6) is a radial vector fields that goes like r^2, for the Gauss theorem we have that $\nabla \cdot u = 0$. For the above reason, the fact that $\nabla \times u = 0$ and given the identity (2), solution (6) satisfy $\nabla^2 u = 0$ and is therefore the solution to our problem.

A vector field u, defined in an open set, which has the properties $\nabla \cdot u = 0$ and $\nabla \times u = 0$, admits a scalar potential ϕ such that:

$$u = -\nabla\phi \tag{7}$$

In our case we have:

$$\phi = -\frac{R^3}{r}, \;\Rightarrow\; u = -\frac{R^3}{r^2}\hat{i}_r = -\nabla\left(-\frac{R^3}{r}\right) \tag{8}$$

4 Analogy with Gravity

There is a perfect analogy between the solution (6) for the displacement field of a removed ball of material and the Newtonian Gravitational field generated by a ball of radius R and with a total mass proportional to its volume. In this analogy we interpret displacement field to be equivalent to gravitational field and the function ϕ defined in (7) to be equivalent of the gravitational potential.

Given the above analogy, we can evaluate the displacement field of two separate removed ball configurations of radius R_1 and R_2 to be equal, apart from multiplicative constant, to the the gravitational field generated by two balls of radius R_1 and R_2 of total mass proportional to their volumes.

Moreover, the energy density stored in the displacement field is proportional, as for the gravitational field, to the square module of the field itself in a point.

Now, if we suppose that we can freely move the centre of the two balls of removed material, in the material itself, in order to make them closer by a distance δL, the energy stored in the displacement field will decrease by a quantity δE. The two balls will experience a force F attracting each other that, apart from a multiplicative constant, is equal two $F = -\frac{\delta E}{\delta L}$. From the analogy with the gravitational field we know already that this force is equal to:

$$F = -\frac{\delta E}{\delta L} = \frac{(R_1)^3\,(R_2)^3}{d^2} \tag{9}$$

where d is the distance between the centre of the two balls.

From the above discussion, we may interpret the elastic material as space itself, the mass as space deficiency (with mass proportional to the space removed) and displacement as gravitational field. By doing this, we have a perfectly working theory of Newtonian gravitation. We may say somehow that gravity is generated by deficiency of space.

We note that the theory can explain only what happen in the space external to the radius R. However, if we take the radius R to be the Schwarzschild radius, the theory can explain everything, an observer external to the horizon event, can see.

5 A Particle Physics Theory

So far we have shown a simple, although interesting mathematical fact from which it is possible to see that removed material can be used to define a Newtonian theory of gravity. From now on, we let our fantasy free and we describe how this fact can be use to define a topological theory of particle physics which has noting to do with real physics but that it is nice to be described.

Let us suppose space itself is modelled as an the elastic material with all the properties discussed in the paragraphs above and let us suppose we confine a hight energy in a limited region of space. Energy can be interpreted as oscillations of the material. In this situation the material, for some sort of "tunnelling" effect, may get twisted with itself in such a way it cannot be untwisted. This tunnelling effect would imply that the fabric, space is made of, would open somewhere to recompose in a different way. This is a standard procedure in topology (cutting and glueing) to get a manifold from another. For example, in 2D, we can cut a sphere on a line and glue the two edges in opposite directions to get a real projective plane.

The final configuration would then be equal to a compact 3D manifold attached to the space by means of a connected sum. At that point the material would pull the manifold due to its elastic property and the manifold would shrink to a point where the energy due to the bending of the space would be in equilibrium with the external space which is pulling. Let us say the equilibrium is given when the radius of curvature of the manifold reach a minimum, for example the Planck length.

In this theory an elementary particle would therefore be a tiny compact 3D manifold attached to the "space" by means of a connected sum and free to move around. Since space can be twisted but cannot disappear, a manifold attached to the space would lead to deficiency of space in that point and a generation of an Newtonian equivalent gravitational field around it as described in the paragraph above. In this situation every possible particle would just be a separate prime[1] compact manifold.

We believe such a theory may help to explain many features of particle physics by means of the analogous topological properties of the associated compact manifold:

Mass=Energy. When a particle (i.e. a compact manifold attached to space) shrinks pulled by the elastic forces of space, at the equilibrium, we can assume that it will reach a configuration where the curvature of the manifold is constant (let say with radius of curvature equal to the Planck length) throughout it. Space in the manifold will then store a density of energy proportional to the curvature and equal to the energy required to bend the space. Since both mass and energy are proportional to the deficiency of space (volume of the manifold) then they will be proportional each other as state by theory of relativity.

Massless particles cannot rest. For what we have said before, massless particle cannot be associated to a manifold since deficiency of space would imply mass. For this reason they have to be interpreted as locally confined perturbations in space (transversal bumps in the space). Given the theory of elastic

[1] A prime manifold is an n-manifold that cannot be expressed as a non-trivial connected sum of two n-manifolds. Non-trivial means that neither of the two is an n-sphere

waves in elastic material this perturbation cannot be at rest but have to move at the speed of waves in that material, speed that depends on the characteristics of the material and that we can assume to be equal to speed of light.

Energy gravitates. General relativity tells us that energy as well matter curves space and generates gravity. As seen before, massless particles (i.e. pure energy) are not responsible for space deficiency but the wrinkles associated with them will pull and deform space generating gravity also in our theory.

Singularity of black holes. This theory would remove the singularity in black holes which is so difficult to be treated in gravity and substitute it with a bunch on manifold grouped together. This situation may be not easier to dealt with but, at least, there is not theoretical issues relate to it.

Black holes evaporates. Black holes have a temperature and emits photons. In our model the centre of a black hole is a bunch of manifolds grouped together. In such a situation, the whole manifold we get, is quite big and therefore unstable. It is likely that it will untwist itself for some kind of tunnelling effect and became a bit simpler. The excess of energy, due to the fact that some of the manifolds the black hole is composed of untwists, may then be released as a massless particle.

Unification of the 4 forces. Unification of the 4 forces is one of the big challenges of modern physics. In our theory gravity is already part of the way particles are created. This may hep in the task of unifying gravity with other forces.

Hierarchy of particles mass. Standard model does not explain why matter particles came in family of three with increasing mass. This may be explained using topology and finding manifolds that have similar characteristics, and therefore being associated to the same particles, but being of bigger size so that they have an higher mass.

Standard model constants. Standard mode has many physical constant which are measured experimentally and that have no theoretical explanation. This may be explained by means of topological characteristics of the associated manifold.

Antimatter. Explain antimatter with our model is not easy. We may say that particles and antiparticles are associated to manifolds somehow symmetric and stable. When they come together they form a manifold that for same geometrical reason reaches an equilibrium configuration that is very unstable and can untwist itself for some tunnelling effect and became a photon. This may be due, for example, to the fact that the part that can untwist themselves in the equilibrium configuration are very close each other. The inverse process would also be possible (i.e. a a photon to became a pair of matter-antimatter particles) and this would explain why, among the large number of compact manifolds only some would be suitable to became the stable particles we know.

Matter-antimatter unbalance. Matter antimatter unbalance may be explained by the fact the homologous matter-antimatter particles may be associated to manifolds that, although symmetric, would have characteristics such that one of them had an higher likelihood to untwist itself and became a photon. This would explain why matter is far more present in the universe than antimatter.

Dark matter. Given the large number of 3D compact manifold, there are same that may be stable and not interact with regular matter and light for topological reasons. This would be good candidates for antimatter particles.

Al the discussions above should be made more quantitative. A task that for some of them is not easy.

6 Relativity

Being a Newtonian theory of gravity, our model does not agree with relativity. However, we can do something about it:

Speed of light. We have already discussed in the previous paragraph that massless particles travels in our space a the speed of elastic waves in the material the space is made of. If we assume this speed to be speed of light we find that classical physics tells us no particles (i.e. the relevant manifold attached to the space) can travel at speed higher to speed of light (elastic waves) in agreement with relativity.

Privileged frame of reference. We need to assume that there is no privileged frame of reference. To do that we have to assume that the elastic material, the space is made of, is more to be seen as a field that can stretch and bend like an elastic material. However, every particles at rest in any frame of reference will see this field at rest with respect to itself. This is a bit strange but it is not much different from the situation we have with fields of the standard model. In this framework, inertial mass is just the statement that there is no privileged frame of reference. When a particles is in motion, it feels just at rest with respect of its frame of reference and it keeps going in absence of external forces.

Increasing mass of moving particles. When a particles is moving with respect to an observer, if we accept the principles of special relativity, the observer will see lengths to contract in detection of motion. This will apply also to the manifold the particle is made of. The manifold that at the beginning will look like a 3D sphere (i.e. constant curvature), will them became more like an ellipsoid. Given the principle that maximum curvature cannot be less then the Planck length, The manifold will keep its minimum curvature constant by growing in the other direction and pull external space inside. The curvature will not be uniform any more and the total curvature of the manifold, as well as its mass and the space deficiency, will grow as required by special relativity.

General relativity. This is a theory of curved space and not a theory of curved space-time. However, we are sure that by accepting the equivalence principle of general relativity it is possible to modify the theory and write equations that agree with the Einstein field equations. Not an easy task though.

Once again the above discussions should be made more quantitative.

7 Particles Interaction

In particle physics, particles interact and decay in other particles. Particle numbers and types are not in general conserved in the process. We want to show a possible way topology may be used to describe particle interaction in our framework where particles are interpreted as compact manifold direct summed to space. For simplicity we will do that for a 2D case although the idea can be easily extended to the 3D case.

There are many ways to decompose compact n-dimensional manifolds in simpler elements. A possible way is to decompose it as a connected sum of prime compact manifolds. For example, for 2-dimensional manifolds, any compact manifold can be obtained by connected sum of 2-real projective planes (cross-caps) and 2-tori (handles) to a 2-sphere.

However, it is possible to obtain any compact 2-dimensional manifold by adding and removing projective planes from a 2-sphere without using handles. For example you can obtain a tours by the following procedure:

- Add three cross-caps to a sphere by means of connected sums.
- Combine two of the three cross-caps on the sphere in order to obtain a Klein Bottle attached to the sphere.
- Readjust the Klein bottle so that it can be seen as an handle with one end attached to one side of the sphere and the other end attached to the opposite side of the sphere (see Fig. 1a).
- Drag one end of the handle (Klein Bottle) inside to the third cross-cap (see Fig 1a), and move it in order to make it emerge from the cross-cap attached to the opposite side of the sphere (see Fig. 1b). Now you have a proper handle (2-torus).
- Remove the third cross-cap. Now you have eventually a 2-torus.

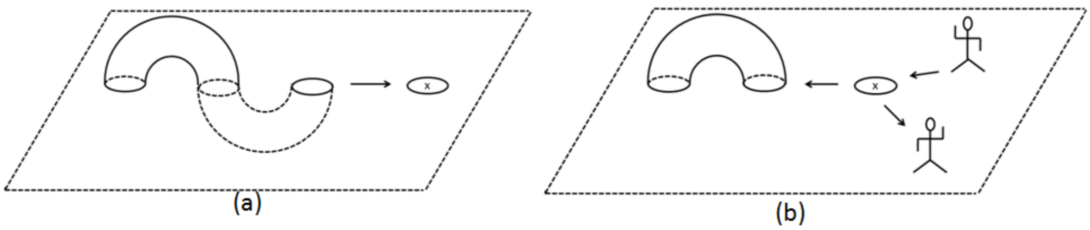

Figure 1: From Klein Bottle to Torus

From the example above we see that projective planes are somehow more fundamental then tori because from them you can get any compact manifold. In this paper, we will call such kind of fundamental spaces "Essential Spaces".

We can use the concept sketched above to describe a particle interaction process in a 2D space where a prime but not essential manifold (i.e. a particle) is hit by another prime manifold (i.e. another particle). The process would transform the prime manifold in a connected sum of separate prime essential manifold as illustrate in Fig. 2).

Of course in 2D there are only two prime compact manifold (torus and cross-cap) only one of which (the cross-cap) is essential. In 3D there is a huge quantity of prime compact manifolds leading to much more possibilities for particles and interactions.

Particles may interact also in other ways. For example two manifolds may interact and became too big to be stable. In this case for some tunnelling effect

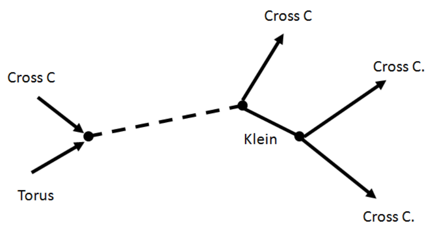

Figure 2: Interaction between a Torus and a Cross-Cap

the manifold may twist itself and became the connected sum of two prime manifolds, different to the ones interacted in the first place, and then decay. Another possibility is a manifold that is hit by a massless particle (i.e. a travelling wave but not a manifold) and have too much energy (i.e. stationary waves in the manifold) to be stable. Once again the manifold may twist itself and decay as in the previous examples.