Charge Stability Approach to Finite Quantum Field Theory

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This paper analyses elementary particle stability and applies the resulting stability principle to resolve divergence issues in quantum field theory without renormalization. For quantum electrodynamics (QED), stability is enforced for an electron by requiring that the positive electromagnetic field energy $\mathcal{E}_{em}^{+}$ be balanced by a negative interaction energy $\mathcal{E}_{em}^{-}$ between the observed charge and a local vacuum potential. Then in addition to the observed core mechanical mass m, an electron system consists of two electromagnetic mass components $m_{em}^{\pm}$ of equal magnitude $M \equiv \eta m$ but opposite sign; consequently, the net electromagnetic mass is zero. Two virtual, electromagnetically dressed mass levels $m \pm \eta m$ are constructed to form a complete set of mass levels and isolate the electron-vacuum interaction; in general, the vacuum current associated with transient dressed core mass (DCM) states for a fermion opposes that of the core. Similarly, electroweak theory is used to define a stability condition for bosons and determine dressed boson mass states. For quantum chromodynamics, the stabilized amplitude takes into account confinement. Total scattering amplitudes for radiative corrections, including core and DCM states, are shown to be convergent in the limit $\eta \to \infty$ and equal to renormalized amplitudes when Feynman diagrams for all mass levels are included. In each case, the infinity in the core mass amplitude is canceled by the average amplitude for DCM levels, which become separated in intermediate states and account for the stabilizing interaction energy between a particle and its surrounding vacuum. In this manner, S-matrix corrections are shown to be finite for all particles of the Standard Model, all the while maintaining their mass and charge at physically observed values. The method is verified for radiative corrections in QED and non-Abelian gauge theories. The results demonstrate that quantum field theory is fundamentally scale invariant.

I. INTRODUCTION

A long-standing enigma in particle physics is how an elementary charged particle such as an electron can be stable in the presence of its own electromagnetic field [1, 2]. Critical accounting for charge stability is essential since radiative corrections in quantum field theory (QFT) involve self-interactions that appear to change the mass and charge of a particle. This analysis identifies and accounts for the hidden interaction that energetically stabilizes a particle such that its mass and charge assume their physically observed values.

The agreement between renormalization theory and experiment confirms the effect of vacuum fluctuations on the dynamics of elementary particles to astounding accuracy. For example, electron anomalous magnetic moment calculations currently agree with experiment to about 1 part in a trillion [3, 4]. This achievement is the result of seven decades of effort since the relativistically invariant form of the theory took shape in the works of Feynman, Schwinger, and Tomonaga (see Dyson's unified account [5]). The agreement leaves little doubt that QFT predictions are correct; however, the renormalization technique [6, 7] used to overcome divergence issues in radiative corrections offers little insight into the underlying physics behind charge stability in the high-energy regime. Recall that divergent integrals occur in scattering amplitudes for self-energy processes and arise in sums over intermediate states of arbitrarily high-energy virtual particles. This stymied progress until theoretical improvements were melded with renormalization to isolate the physically significant parts of radiative corrections by absorbing the infinities into the electron mass and charge. Although the renormalization method used to eliminate ultraviolet divergences results in numerical predictions in remarkable agreement with experiments, redefinition of fundamental physical constants remains an undesirable feature of the current theory in this author's opinion.

Our main purpose is to develop an alternative to mass and charge renormalization in QFT. A minimal requirement for this proposal is that it reproduce the successes of the accepted theory: these include the successful higher-order multiloop calculations of QED, the modern understanding of QED as a part of a non-Abelian electroweak theory, and asymptotic freedom in quantum chromodynamics (QCD). Starting with the classical self-energy problem in Sec. III, we define an energetically stable electrical charge. We

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then generalize the stability principle to apply to all interacting particles of the Standard Model. Scattering matrix corrections for stability are simply constructed using core amplitudes from the literature, involve two additional Feynman diagrams associated with dressed core mass (DCM) states, and account for the action of the vacuum back on the charge via an opposing current. After defining divergent integrals for DCM diagrams, we verify that net S-matrix corrections in QED for vacuum polarization, fermion self-energy, and vertex processes are finite to all orders in perturbation theory. Finally, we apply the method to one-loop diagrams in non-Abelian Yang-Mills [8] and electroweak theories [9–11].

II. LAGRANGIAN AND NOMENCLATURE

In this section, we summarize the required machinery of the Standard Model utilizing references [12, 13]. Natural units are assumed; that is, $\hbar = c = 1$.

A. Electroweak

The electroweak Lagrangian for the physical particles

$$\mathcal{L}_{EW} = \mathcal{L}_{\mathrm{G}} + \mathcal{L}_{\mathrm{H}} + \mathcal{L}_{\mathrm{F}} \tag{1}$$

includes gauge, Higgs, and fermion parts. Gauge fixing and ghost terms are omitted in (1) since it is only necessary to consider physical particles for this development. The gauge part, based on a Yang-Mills prototype (29), is given by

$$\mathcal{L}_{\mathrm{G}} = -\frac{1}{4} W^{a}_{\mu\nu} W^{a,\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} \, , \tag{2}$$

where the field strength tensors

$$\begin{aligned} W^{a}_{\mu\nu} &= \partial_\mu W^{a}_{\nu} - \partial_\nu W^{a}_{\mu} + g_W \varepsilon_{abc} W^{b}_{\mu} W^{c}_{\nu} \, , \text{ and} \\ B_{\mu\nu} &= \partial_\mu B_\nu - \partial_\nu B_\mu \end{aligned} \tag{3}$$

are expressed in terms of derivatives of the gauge fields: a triplet W^{a}_{μ}, $a = 1, 2, 3$ of vector bosons and a singlet B_μ which transform according to $SU(2)$ and $U(1)$ symmetry groups [9], respectively. In (3), g_W is the non-Abelian SU(2) gauge coupling constant, and ε_{abc} is the Levi-Civita tensor representing the structure constants of $SU(2)$.

The Higgs part is given by

$$\mathcal{L}_{\mathrm{H}} = \left(D_\mu \Phi\right)^{\dagger} \left(D^\mu \Phi\right) - V(\Phi) \, , \tag{4}$$

where Φ is an isospin doublet coupled to the gauge fields via the covariant derivative

$$D_\mu = \partial_\mu - i g_W T_a W^{a}_{\mu} - i g_B \frac{Y}{2} B_\mu \, , \tag{5}$$

$\overrightarrow{T} = \overrightarrow{\sigma}/2$ are weak isospin generators, $\overrightarrow{\sigma}$ are Pauli matrices satisfying the SU(2) algebra $[\sigma_i, \sigma_j] = 2i\varepsilon_{ijk}\sigma_k$, and g_B is the Abelian coupling constant. Φ carries hypercharge $Y = Y_\Phi \equiv 1$ and a third component of isospin $T_3\Phi = -\frac{1}{2}\Phi$. Minimizing the Higgs potential

$$V(\Phi) = \mu_\Phi^2 \Phi^\dagger \Phi + \lambda_\Phi \left(\Phi^\dagger \Phi\right)^2 \tag{6}$$

with $\lambda_\Phi > 0$ and $\mu_\Phi^2 < 0$ for symmetry breaking leads to a stable ground state

$$\begin{aligned} \Phi(x) &= \left(\begin{array}{c} \phi^+ \\ \left(v + h(x) + i\chi\right)/\sqrt{2} \end{array} \right)_{\phi^+ = \chi = 0} \\ &= \frac{1}{\sqrt{2}} \left(\begin{array}{c} 0 \\ v + h(x) \end{array} \right) \, , \end{aligned} \tag{7}$$

where in a unitary gauge, $\phi^+ = \chi = 0$, and the real Higgs field $h(x)$ fluctuates about a vacuum

$$v = \sqrt{-\frac{\mu_\Phi^2}{2\lambda_\Phi}} \, .$$

Physical fields for charged W-bosons, neutral Z, and photon are

$$W_\mu^\pm = \frac{1}{\sqrt{2}} \left(W_\mu^1 \mp W_\mu^2 \right) \text{ , and} \tag{8}$$

$$\begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_W & \sin\theta_W \\ -\sin\theta_W & \cos\theta_W \end{pmatrix} \begin{pmatrix} W_\mu^3 \\ B_\mu \end{pmatrix} , \tag{9}$$

where the weak mixing angle θ_W is defined by

$$\begin{aligned} \cos\theta_W &= \frac{g_W}{g_Z} \\ &= \frac{m_W}{m_Z} \end{aligned} \tag{10}$$

with $g_Z = \sqrt{g_W^2 + g_B^2}$, and the second line in (10) follows from (13) and (14). In terms of the physical fields, (5) takes the form

$$D_\mu = \partial_\mu - i\frac{g_W}{\sqrt{2}} \left(T_+ W_\mu^+ + T_- W_\mu^- \right) - i g_Z \left(T_3 - Q \sin^2\theta_W \right) Z_\mu - ieQA_\mu ,$$

where $T_\pm = T_1 \pm iT_2$ raise (+) or lower (-) the current in interactions between left-handed fermions and $W^\pm$, $Q = T_3 + \frac{Y}{2}$ is the Gell-Mann-Nishijima relation for the charge operator, and the electrical charge satisfies

$$e \equiv g_W \sin\theta_W = g_B \cos\theta_W \, . \tag{11}$$

Omitting higher-order non-mass terms, (4) becomes

$$\mathcal{L}_\mathrm{H}\left(m_W^2,\, m_Z^2,\, m_H^2\right) \simeq \frac{1}{2}\partial_\mu h \, \partial^\mu h + m_W^2 W_\mu^- W^{+\mu} + \frac{1}{2} \begin{pmatrix} Z_\mu & A_\mu \end{pmatrix} \begin{pmatrix} m_Z^2 & 0 \\ 0 & m_\gamma^2 \end{pmatrix} \begin{pmatrix} Z^\mu \\ A^\mu \end{pmatrix} - \frac{1}{2} m_H^2 h^2 , \tag{12}$$

where vector boson masses

$$m_W = \frac{1}{2} g_W v \text{ , and} \tag{13}$$

$$m_Z = \frac{1}{2} g_Z \, v , \tag{14}$$

generated via the Higgs mechanism [10, 11, 14], come from the kinetic part of (4), and the photon remains massless: $m_\gamma = 0$. The scalar boson mass (Higgs), resulting from an expansion of $V(\Phi)$ in (6) about v, is

$$m_H^2 = 2v^2 \lambda_\Phi , \tag{15}$$

where λ_Φ may be determined using the identity

$$v^2 = \frac{m_W^2 \sin^2\theta_W}{\sqrt{\pi\alpha}} \tag{16}$$

and experimental values [15] for m_W, $\sin^2\theta_W$, and m_H.

Suppressing the color attribute for quarks, the fermion part of the Lagrangian is given by

$$\mathcal{L}_F = \sum_j \overline{\psi}_L^j i\gamma^\mu D_\mu \psi_L^j + \sum_{j\sigma} \overline{\psi}_R^{j\sigma} i\gamma^\mu D_\mu \psi_R^{j\sigma} + \mathcal{L}_\mathrm{F}^\mathrm{Yukawa} \tag{17}$$

for each lepton or quark family (j), where γ^μ are Dirac matrices,

$$\psi_L^j = \begin{pmatrix} \psi_L^{j+} \\ \psi_L^{j-} \end{pmatrix}$$

is a left-handed fermion doublet with component index $\sigma = \pm$, and $\psi_R^{j\sigma}$ is a right-handed singlet for a fermion f indexed by $j\sigma$.

$$\mathcal{L}_{\mathrm{F}}^{\mathrm{Yukawa}} = -\sum_{j\sigma} g_{j\sigma} \left[\left(\overline{\psi}_L^j \Phi \right) \psi_R^{j\sigma} + \overline{\psi}_R^{j\sigma} \left(\Phi^\dagger \psi_L^j \right) \right] \tag{18}$$

is the Yukawa interaction term, and $g_{j\sigma}$ are fermion coupling constants. The complete set of fermions includes leptons:

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L, \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L, \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L, e_R,\ \mu_R,\ \tau_R$$

and quarks:

$$\begin{pmatrix} u \\ d \end{pmatrix}_L, \begin{pmatrix} c \\ s \end{pmatrix}_L, \begin{pmatrix} t \\ b \end{pmatrix}_L, u_R,\ d_R,\ c_R,\ s_R,\ t_R,\ b_R$$

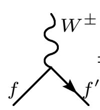
which carry color and belong to the fundamental representation of the color group $G = SU(N)$ of degree $N = 3$. Employing (7)

$$\mathcal{L}_{\mathrm{F}}^{\mathrm{Yukawa}} (m_{j\sigma}) = -m_{j\sigma} \left[\overline{\psi}_L^{j-} \psi_R^{j\sigma} + \overline{\psi}_R^{j\sigma} \psi_L^{j-} \right], \tag{19}$$

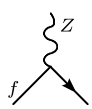
where the masses generated from the interaction between the fermion and Higgs fields are

$$m_{j\sigma} = \frac{1}{\sqrt{2}} g_{j\sigma} (v + h) \Big|_{h=0} . \tag{20}$$

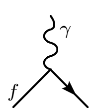
We will also need vertex factors and propagators below for later reference; these, along with propagators for the Higgs, ghost fields, and vertex factors for $SU(N)$ theories may be found in the literature and [13]. For fermions coupling to the W, Z, and γ, vertex factors are



$$= i \frac{e}{\sqrt{2}\, s_w} \gamma^\mu \frac{1}{2} (1 - \gamma_5) , \tag{21}$$



$$= ie\gamma^\mu (v_f - a_f \gamma_5) , \tag{22}$$



$$= ieQ_f \gamma^\mu , \tag{23}$$

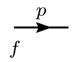
where $(f = j\sigma\,,\ \sigma = \pm\,,\ f' = j\sigma'\,,\ \sigma' = \mp)$, and the vector and axial vector coefficients

$$v_f = \frac{T_3^f - 2s_w^2 Q_f}{2 s_w c_w} , \tag{24}$$

$$a_f = \frac{T_3^f}{2 s_w c_w}$$

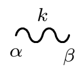
are neutral current (NC) coupling constants with $\{s_w \equiv \sin\theta_W \ , \ c_w \equiv \cos\theta_W\}$.

The fermion propagator [16] is



$$= S_F\left(p, m_f\right) = \frac{i}{\not{p} - m_f + i\varepsilon} \ , \tag{25}$$

where $\not{p} = \gamma^\mu p_\mu$, and anti-fermions are denoted by $\bar{f}$. The vector boson propagator is

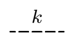


$$= D_F^{\alpha\beta}(k) = \frac{-ig^{\alpha\beta}}{k^2 - m_b^2 + i\varepsilon} \tag{26}$$

in the Feynman-'t Hooft gauge [17], where the metric tensor $g_{\alpha\beta} = g^{\alpha\beta}$ has non-zero components

$$g_{00} = -g_{11} = -g_{22} = -g_{33} = 1 \ ,$$

and $b \in \{W,\ Z,\ \gamma\}$. For the Higgs, we have



$$= \frac{i}{k^2 - m_H^2 + i\varepsilon} \ . \tag{27}$$

Finally, unphysical particles including gauge fixing Higgs $\{\phi^\pm, \chi\}$ and unitarity preserving Faddeev-Popov ghosts $\left\{u^\pm, u^Z, u^\gamma\right\}$ occur in loop corrections discussed in Sec. VII B.

B. Yang-Mills Theory

The SU(3) Yang-Mills theory involves $n_f = 6$ quarks interacting with $n_g = 8$ massless gluons in the adjoint representation $r = G$. Again omitting gauge fixing and Faddeev-Popov ghost terms, the QCD Lagrangian is

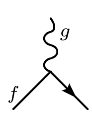
$$\mathcal{L}_{QCD} = \sum_{f=1}^{n_f} \bar{\psi}_f^j \left(i\gamma_\mu D_{jk}^\mu - m_f \delta_{jk}\right) \psi_f^k + \mathcal{L}_{YM} \ , \tag{28}$$

$$D_{jk}^\mu = \delta_{jk}\partial^\mu - ig_s\left(\overrightarrow{t} \cdot \overrightarrow{A}^\mu\right)_{jk} \ ,$$

$$\mathcal{L}_{YM} = -\frac{1}{4} F_{\mu\nu}^a F_a^{\mu\nu} \ , \tag{29}$$

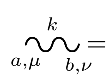
$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g_s f^{\,abc} A_\mu^b A_\nu^c \ ,$$

where ψ_f^k is a Dirac spinor for the quark field with flavor f and color state $k \in \{R,\ G,\ B\}$, g_s is the color charge, $t^{\,a} = \lambda^a/2\,,\ a = 1, ..., n_g$ are generators represented by 3×3 Gell-Mann matrices λ^a, A_μ^a are color-charged gluon fields, and $f^{\,abc}$ are structure constants of G. The t-matrices, which occur in a quark/gluon vertex



$$= ig_s\gamma^\mu t^a \ , \tag{30}$$

and gluon propagator



$$= \frac{-ig^{\mu\nu} t^a t^b}{k^2 + i\varepsilon} \tag{31}$$

rotate the quark in color space and generate the Lie algebra for G:

$$\left[t^{\,a},\ t^{\,b}\right] = if^{\,abc} t^{\,c} \ .$$

The structure constants occur in three- and four-gauge-boson vertices and satisfy

$$f^{\,acd} f^{\,bcd} = C_2\left(G\right) \delta^{ab} \ ,$$

where $C_2\left(G\right) = N$ is an eigenvalue of the quadratic Casimir operator.

III. FORMULATION

A. Physical Model for QED

Regarding an electron as a point particle [18], the classical electrostatic self-energy $e^2/2a \equiv \alpha\Lambda_\circ$ diverges linearly as the shell radius $a \to 0$, or energy cutoff $\Lambda_\circ \to \infty$, where $-e$ is the charge and $\alpha = e^2/4\pi$ is the fine-structure constant. However, Weisskopf [19, 20] showed using Dirac's theory [21] that the charge is effectively dispersed over a region the size of the Compton wavelength due to pair creation in the vacuum near an electron, and the self-energy only diverges logarithmically. Feynman's calculation [22] in covariant QED yields an electromagnetic mass-energy

$$m_{em} = \frac{3\alpha m}{2\pi}\left(\ln\frac{\Lambda_\circ}{m} + \frac{1}{4}\right) , \tag{32}$$

where m is the electron mass. In the absence of a compensating negative energy, (32) signals an energetically unstable electron. This is the fermion self-energy (SE) problem, whose general resolution will suggest a solution for boson SE processes as well resulting in finite amplitudes for all radiative corrections. In this section we derive an electron stability condition and apply it to develop a corresponding correction to the scattering amplitude.

To ensure that the total electron mass is its observed value, renormalization theory posits that a negatively infinite "bare" mass must exist to counterbalance m_{em}. For lack of physical evidence, negative matter is naturally met with some skepticism (see Dirac's discussion [23] of the classical problem, for example). Nevertheless, energies that hold an electron together are expected to be negative, and we can understand their origin by first considering the source for the electrical energy required to assemble a classical charge in the rest frame. Recall that the work done in assembling a charge from infinitesimal parts is equal to the electromagnetic field energy. Since the agents that do the work must draw an equivalent amount of energy from an external energy source (well), the well's energy is depleted and the total energy

$$\mathcal{E} = m + \mathcal{E}_{em}^{+} + \mathcal{E}_{w} \tag{33}$$

of the system including matter, electromagnetic field $\mathcal{E}_{em}^{+}$, and energy well $\mathcal{E}_{w}$ is constant. For an elementary particle, could the depleted energy well be the surrounding vacuum?

From another point of view, consider an electron and its neighboring vacuum treated as two distinct systems that can act on one another. Suppose the electron acts on the vacuum to polarize it creating a potential well, then there must be an opposing reaction of vacuum back on the electron. The resulting vacuum potential Φ_{vac} confines the physical (core) charge akin to a spherical capacitor depicted in Fig. 1, and the interaction energy

$$\mathcal{E}_{w} \to \mathcal{E}_{em}^{-} \equiv -e\Phi_{vac} \tag{34}$$

is assumed to just balance $\mathcal{E}_{em}^{+}$ resulting in a stability condition

$$m_{em}^{+} + m_{em}^{-} = 0 , \tag{35}$$

where the mass-energy equivalence $\mathcal{E}_{em}^{\pm} = m_{em}^{\pm}$ utilizing natural units has been applied. Therefore, the net mass-energy of a free electron is attributed entirely to the observed core mechanical mass m which is generated via the Higgs field interaction (20) in electroweak theory. In contrast to Poincaré's theory [24] wherein internal non-electromagnetic stresses hold an electron together, external vacuum electrical forces are assumed to provide charge stabilization and energy balance via a steady state polarization field surrounding the electron. Corresponding to a divergent self-action process, we require a mechanism whereby the core charge interacts locally with the polarized vacuum according to (34).

Apart from $\mathcal{E}_{em}^{+}$, the energy of the core charge in the potential well of Fig. 1 is shifted

$$\mathcal{E}_{core}^{-} = m + \mathcal{E}_{em}^{-} \equiv m_{bare} , \tag{36}$$

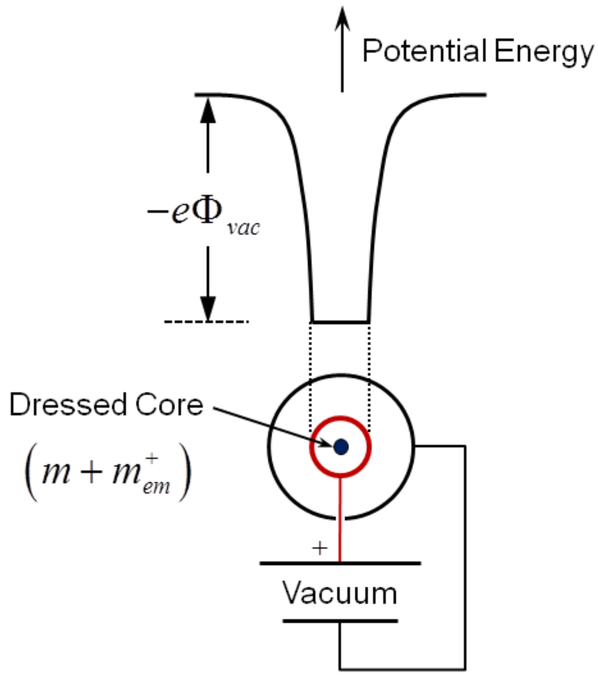


FIG. 1: Effective vacuum potential confines core electron charge similarly to spherical capacitor. Since the stability principle requires $\mathcal{E}^{+}_{em} - e\Phi_{vac} = 0$, the total energy of the electron in the well and dressed in its electromagnetic field is just its observed mass-energy.

where m_{bare} may be identified with the bare mass, and

$$m_{bare} + m^{+}_{em} = m \tag{37}$$

captures the mass renormalization condition which is equivalent to (33) with (34) and (35). However, notice that the bare mass corresponds to a core electron dressed in negative electromagnetic energy; hence, its characterization as a "mechanical mass" is a misnomer (see [25] for example). Only the core mass is observable, and only it is expected to appear in the Lagrangian if one takes (35) seriously. In renormalization theory, however, one starts with a bare electron, self-interaction dresses it with positive electromagnetic energy, and (37) is subsequently applied to redefine the mass. On the other hand, suppose we start with the observed electron charge; then taking into account (33), (34), and (35), m^{+}_{em} and m^{-}_{em} are always present, and the total mass reduces to the observed core mechanical mass. Starting with this premise, we can formulate a finite theory of radiative corrections that accounts for all possible electromagnetically dressed intermediate states, and no asymmetry necessitating a redefinition of mass and charge is introduced.

Equations (33) and (35) suggest that a stable electron consists of three rest mass components: a core mass m and two electromagnetic masses $m^{\pm}_{em}$ that are assumed large in magnitude but finite until the final step of the development. We can think of $m^{\pm}_{em}$ as components of an electromagnetic vacuum (zero net energy) which are tightly bound to the core mass and inseparable from the core and each other, at least for finite field actions. Considering all non-vanishing masses constructed from the set $\{m,\, m^{+}_{em},\, m^{-}_{em}\}$, we are led to define a complete set of mass levels $m + \lambda M$, where $\lambda = \{0, \pm 1\}$ and $M \equiv |m^{\pm}_{em}|$. In the following, an electromagnetically dressed core mass (DCM) refers to a composite particle with mass-energy levels $m \pm M$. Associated four-momenta are $p_{dcm} = p \pm P_M$, where $\{p,\, P_M\}$ correspond to $\{m,\, M\}$, respectively. DCM rules for fermion self-energy processes are defined by

$$m \to m + \lambda M \text{ , and} \tag{38}$$

$$p \to p + \lambda P_M \text{ .} \tag{39}$$

To transition this charge stability model into quantum theory, first consider a free particle state $|p,\, m\rangle$ satisfying $p^2 \equiv p_\mu p^\mu = m^2$, where $p^\mu = (p^\circ,\, \vec{p})$ and $p_\mu = g_{\mu\nu} p^\nu$ are contravariant and covariant momentum four-vectors, respectively. Spin is omitted in $|p,\, m\rangle$ since it is inessential to the subsequent development, and the rest mass is included because it is the fundamental particle characteristic which becomes dressed in stability corrections to the S-matrix; see (61). We employ the relativistic normalization

$$\langle p',\, m\, |p,\, m\rangle = 2E\,(\vec{p},\, m)\,(2\pi)^3\,\delta\left(\vec{p}-\vec{p}\,'\right)\ ,$$

where $E\,(\vec{p},\, m) = \sqrt{\vec{p}^{\,2}+m^2}$. Now construct a superposition

$$|\chi\rangle = \frac{1}{\sqrt{2}}\sum_{\lambda=\pm 1}\left|\Upsilon_{\lambda}^{dcm}\,(p)\right\rangle \tag{40}$$

of DCM states

$$\left|\Upsilon_{\lambda}^{dcm}\,(p)\right\rangle = |p+\lambda P_M\,,\, m+\lambda M\rangle\ , \tag{41}$$

where the core four-momentum is dispersed per an uncertainty $\Delta p \equiv \lambda P_M$. DCM states are normalized according to

$$\begin{aligned}\left\langle \Upsilon_{\lambda'}^{dcm}\,(p')\right|\,\Upsilon_{\lambda}^{dcm}\,(p)\big\rangle &= 2E\left(\vec{p}+\lambda\vec{P}_M\,,\, m+\lambda M\right)(2\pi)^3\,\delta\left(\vec{p}-\vec{p}\,'+(\lambda-\lambda')\,\vec{P}_M\right)\\ &\simeq 2E\left(\vec{P}_M\,,\, M\right)(2\pi)^3\,\delta\left(\vec{p}-\vec{p}\,'\right)\delta_{\lambda\lambda'}\ ,\end{aligned}$$

where the latter form follows upon assuming $M \gg m$ and requiring the vector components satisfy

$$\left|P_M^i\right| \gg \left|p^i - p'^i\right|\ ,\ i=1,2,3$$

thereby excluding a zero in the delta function argument at infinity for $\lambda' \neq \lambda$. While $\left|p^i - p'^i\right|$ is arbitrarily large in an integral over p'^i in the delta function, it is assumed small compared to $\left|P_M^i\right|$. The expected momentum and mass are given by

$$\frac{\langle\chi\,|\{p_{op}, m_{op}\}|\,\chi\rangle}{\langle\chi\,|\chi\rangle} = \{p, m\}\ , \tag{42}$$

where $\{\, p_{op},\, m_{op}\}$ are corresponding operators. Therefore, the composite state (40) is energetically equivalent to the core mass state $|p,\, m\rangle$ as required by (33) and (35). A core electron dressed with positive or negative energy as in (41) is a transient state that is sharply localized within a spacial interaction region $r \simeq \hbar/Mc$ in accordance with Heisenberg's uncertainty principle [26] $\Delta p^\mu \Delta x^\mu \geq \hbar/2$ (no implied sum over μ). Scattering amplitudes for low-energy processes are assumed unaffected because the energies are insufficient to induce a separation of tightly bundled states (41) in (40). For infinite field actions, however, DCM states may become separated in intermediate states with infinitesimally small lifetimes; in this case, we shall need to account for both core and DCM scattering amplitudes. To account for all possible intermediate states in QED and satisfy (35), both mass levels $m \pm M$ are required for interaction between the physical charge and vacuum potential; this generalizes the classical model in Fig. 1.

Since the interaction region reduces to a point as $M \to \infty$ for DCM states, self-interaction effects vanish, and an electromagnetically dressed electron is assumed to interact only with the positive component of the polarized vacuum as indicated in Fig. 2. Suppose the dressed electron, located at space-time position x_1, has current density $j_\mu\,(x_1)$. The current at a neighboring point $x_2 \neq x_1$ within the interaction region is distinct from that of the dressed core and reversed in sign; that is,

$$\mathrm{sgn}\,[j_\mu\,(x_2)] = -\mathrm{sgn}\,[j_\mu\,(x_1)]\ , \tag{43}$$

where the core current is defined by the normal product [27, 28]

$$j_\mu\,(x_1) = -eN\left[\bar{\psi}\gamma_\mu\psi\right]_{x_1}\ .$$

Similarly to (43), the Hamiltonian density at nearby points must satisfy

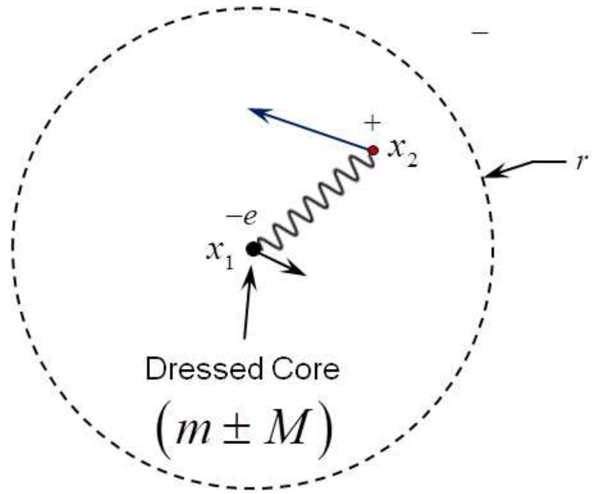


FIG. 2: Dressed electron interacts with opposing vacuum current in interaction region.

$$\text{sgn}\left[\mathcal{H}_{int}(x_2)\right] = -\text{sgn}\left[\mathcal{H}_{\text{int}}(x_1)\right] \ , \tag{44}$$

where $\mathcal{H}_{\text{int}}(x) = j_\mu(x) A^\mu(x)$ in the interaction representation [29], and $A^\mu(x)$ is the radiation field. From (44) we anticipate a sign reversal in the DCM scattering amplitude relative to that for the core mass since second-order S-matrix [30] corrections involve a product $\mathcal{H}_{int}(x_1)\mathcal{H}_{int}(x_2)$.

From (42), the averages of the dressed mass and momentum for charged fermions are just the core values. How do we apply this notion to calculation of radiative corrections in QFT? Consider a single fermion in QED whose Lagrangian is

$$\mathcal{L}^{qed}(m) = \mathcal{L}_F^{qed}(m) - \frac{1}{4}(F_{\mu\nu})^2 \ , \tag{45}$$

where

$$\mathcal{L}_F^{qed}(m) = \overline{\psi}\left(i\partial\!\!\!/ - eA\!\!\!/ - m\right)\psi \ ,$$
$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

is the electromagnetic field strength tensor, and $\partial\!\!\!/ = \gamma^\mu \partial_\mu$. Notice that the expectation

$$\frac{1}{2}\sum_{\lambda=\pm 1} \mathcal{L}_F^{qed}(m + \lambda M) = \mathcal{L}_F^{qed}(m) \tag{46}$$

is unchanged under (38) which suggests a general requirement: that the expectation of the Lagrangian for each field be invariant under DCM transformations. For radiative corrections containing primitive divergences in fermion self-energy or vertex diagrams, S-matrix charge stability corrections associated with DCM states are evaluated in the same manner – this entails a core mass replacement (38) in fermion lines internal to loops, that is, in each fermion propagator (25). Resulting loop-operator amplitudes are to be averaged over dressed mass levels; that is, $\lambda = \pm 1$. For an external line entering a loop as indicated in Fig. 3 (b,c), the momentum is similarly modified according to (39) since the propagator is required to have a pole at $m + \lambda M$. The same approach using (38) actually works for Fig. 3 (a), but is not generally valid for massive boson SE calculations in electroweak theory. Electroweak theory constrains DCM rules for boson self-energy processes.

B. Electroweak Application

The purpose of this section is to generalize and extend the DCM rules in Sec. III A to all particles of the electroweak Standard Model including interactions.

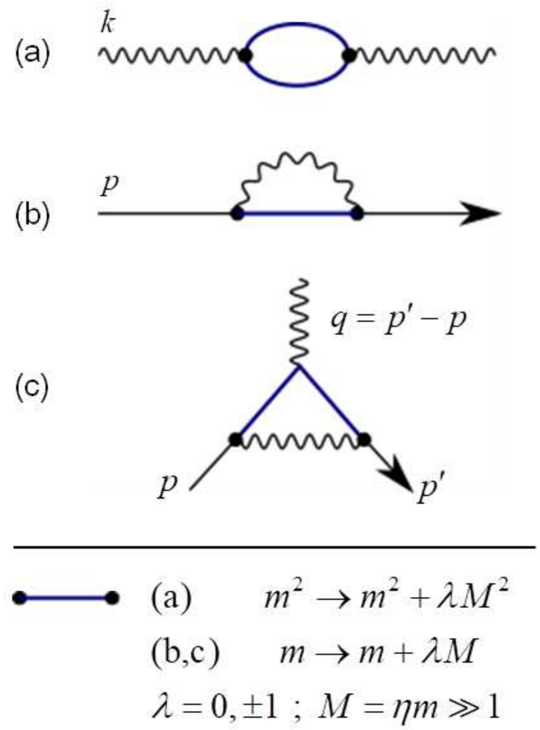


FIG. 3: Baseline radiative corrections in QED: (a) photon self-energy, (b) fermion self-energy, and (c) vertex involve the core mass only in internal fermion lines. Two additional diagrams, obtained by replacing the core mass with electromagnetically dressed mass levels, are required for each radiative process to account for interaction with an opposing vacuum current and ensure stability.

For each fermion mass $m_f \in \{m_{j\sigma}\}$ in $\mathcal{L}_{\mathrm{F}}^{\mathrm{Yukawa}}$, (19) is invariant under an expectation

$$\frac{1}{2}\sum_{\lambda=\pm1}\mathcal{L}_{\mathrm{F}}^{\mathrm{Yukawa}}\left(m_f+\lambda M_f\right)=\mathcal{L}_{\mathrm{F}}^{\mathrm{Yukawa}}\left(m_f\right) \tag{47}$$

similarly to (46). Assume identical scaling $M_f = \eta m_f$, then DCM levels are generated by

$$m_f \to m_f\left(1+\lambda\eta\right)\ . \tag{48}$$

Taking into account (20) with $h \neq 0$, we notice that DCM levels can be associated with a displacement of the vacuum from the ground state; that is,

$$\Delta v = h_\lambda\left(\eta\right) = \lambda\eta v\ . \tag{49}$$

For selected λ and η , the vacuum displacement (49) is the same for all fermions.

For bosons, $\mathcal{L}_{\mathrm{H}}$ in (12) is not invariant under a rule of the form (48). To determine the correct rule, let us follow the approach in Sec. III A and define [31]

$$M_b^2 \equiv \mathrm{Re}\left(\Sigma^b(m_b^2)\right)\ , \tag{50}$$

where Σ^b is a boson self-energy function (see Sec. VII B), and m_b is the core mass. Now assume that the vacuum response is a term Σ^b_{vac} satisfying a free particle stability condition

$$\Sigma^b + \Sigma^b_{vac} = 0 \tag{51}$$

analogous to (35). Noting that Σ^b_{vac} corresponds to a mass $M_{vac} = iM_b$, construct two squared DCM states

$$m_b^2 \to \left\{m_b^2 + M_b^2\ ,\ m_b^2 + \left(iM_b\right)^2\right\}\ , \tag{52}$$

wherein the second member of (52) corresponds to a complex mass state for an inherently unstable boson. Using our $\lambda-$index notation, the rule takes the form

$$m_b^2 \to m_b^2 + \lambda M_b^2 \, , \, b = W, \, Z \, , H \, . \tag{53}$$

Substituting (53) into (12), we find

$$\frac{1}{2} \sum_{\lambda=\pm 1} \mathcal{L}_{\mathrm{H}} \left(m_W^2 + \lambda M_W^2, \, m_Z^2 + \lambda M_Z^2, \, m_H^2 + \lambda M_H^2 \right) = \mathcal{L}_{\mathrm{H}} \left(m_W^2, \, m_Z^2, \, m_H^2 \right) \tag{54}$$

is invariant. Assume identical scaling

$$\{M_W, \, M_Z, \, M_H\} = \eta \, \{m_W, \, m_Z, \, m_H\} \ , \tag{55}$$

then each mass $m_b \in \{m_W, \, m_Z, \, m_H\}$ is dressed according to

$$m_b^2 \to m_b^2 \left(1 + \lambda \eta^2\right) \ . \tag{56}$$

Substituting (16) in (15) and applying (53) with $M_W^2 = \eta^2 m_W^2$, one obtains

$$m_H^2 \to m_H^2 \left(1 + \lambda \eta^2\right) \tag{57}$$

suggesting that DCM states of the W- or Z-bosons induce dressed states for the Higgs and vice versa. Substitution of (15) into (57) reveals that the boson vacuum is shifted $v^2 \to v^2 + \Delta v^2$ with

$$\Delta v^2 = \lambda \eta^2 v^2 \, ; \tag{58}$$

compare (58) with (49).

Vertex factors $\{(21), \, (22), \, (23)\}$, including the weak mixing angle (10), charge (11), and NC coupling constants (24) are all stationary under DCM transforms (56). On the other hand, propagators $\{(25), \, (26), \, (27)\}$ involving massive particles are not stationary under DCM transforms, and the resultant amplitudes constructed from them are either driven to zero or a stabilizing correction for finite tree or divergent loop processes, respectively. While the electrical charge (11) is an invariable according to (176), the couplings $\left\{g_W^2, \, g_Z^2\right\}$ and θ_W can vary due to finite on-shell mass shifts $\left\{\delta m_W^2, \, \delta m_Z^2\right\}$ derived from stabilized W- and Z-boson self-energy corrections discussed in Sec. VII B; see (201) and (202).

Construction of scattering amplitudes using fermion (25) and boson $\{(26), (27)\}$ propagators results in a mixing of fermion and boson masses. The regulation of infrared singularities for soft photon emissions provides a simple example: for fermion self-energy (FSE) and vertex processes, a small fictitious mass μ is introduced in the photon propagator [22]; that is, $m_b \to \mu$ in (26). Fermion and pseudo-boson masses m_f and μ mix in terms of form $\ln \frac{m_f}{\mu}$; for consistency, we define a DCM state

$$\mu \to \mu \left(1 + \lambda \eta\right) \ , \tag{59}$$

then $\ln \frac{m_f}{\mu}$ is invariant under (48) and (59). Generally, for massive bosons in FSE or vertex processes, we require $m_b \to m_b \left(1 + \lambda \eta\right)$. Similarly, when fermion and boson masses mix in boson self-energy (BSE) processes, we require $m_f^2 \to m_f^2 \left(1 + \lambda \eta^2\right)$.

Introduction of DCM states for massive bosons does not break gauge invariance of $\mathcal{L}_{EW}$ in (1) since it constitutes a displacement of the vacuum from the ground state and is therefore consistent with the Higgs mechanism; moreover, the Lagrangian is stationary under the expectations in (47) and (54).

C. Total Finite Scattering Amplitude

Rules for the total scattering amplitude take their simplest form if dimensional regularization is used to tame improper integrals; therefore, define the total loop-operator associated with a SE or vertex part by

$$\hat{\varOmega} = \varOmega_{core} \left(\mathbb{M}\right) + \varOmega_{dcm} \left(\mathbb{M}\right) \ , \tag{60}$$

where Ω_{core} accounts for self-interaction effects involving a core mass set

$$\mathbb{M} \subset \mathbb{S}_m = \{m_f,\, m_W,\, m_Z,\, m_H,\, \mu\} \text{ , and}$$

$$\Omega_{dcm}(\mathbb{M}) = -\frac{1}{2} \lim_{\eta\to\infty} \sum_{\lambda=\pm 1} \Omega_{core}(\mathbb{M}_{dcm}(\lambda,\eta)) \tag{61}$$

enforces stability via interaction of DCM states with an opposing vacuum current as required by (43). From (48), (56), and consistency conditions discussed near (59),

$$\mathbb{M}_{dcm}(\lambda,\eta) = \mathbb{M} \cdot \left[\eta_\lambda \equiv \begin{cases} 1+\lambda\eta & \text{FSE/vertex} \\ \sqrt{1+\lambda\eta^2} & \text{BSE} \end{cases}\right] \tag{62}$$

for FSE, vertex, and BSE diagrams in electroweak theory; for any $m \in \mathbb{M}$, the DCM state is

$$m_{dcm}(\eta_\lambda) = \eta_\lambda m \,. \tag{63}$$

In addition to mass m_b or m_f, Ω_{core} depends on external momenta $\{k,\, p\}$ for Feynman diagrams in Fig. 4 which may be on- or off-shell. Blobs in Fig. 4 involve one-particle irreducible (1PI) amputated correlation functions. For FSE/vertex processes, $p = m_f + \delta p_{os}$, where δp_{os} is an off-shell component; the corresponding DCM state is

$$p_{dcm} = m_f(1+\lambda\eta) + \delta p_{os} \;\; \text{FSE/vertex} \;\; . \tag{64}$$

For BSE processes, $k^2 = m_b^2 + \delta k_{os}^2$, and

$$k_{dcm}^2 = m_b^2\left(1+\lambda\eta^2\right) + \delta k_{os}^2 \;\; \text{BSE} \;; \tag{65}$$

of course, a massless photon ($b=\gamma$) is naked except for any off-shell term. For notational simplicity, any dependence on external momentum parameters has been suppressed during construction of Ω_{dcm} because $\{k,\, p,\, q\}$ are implicitly dependent on associated core masses.

If an energy cutoff $\Lambda_\circ$ is assumed in lieu of dimensional regularization, then we must include $\Lambda_\circ$ in the argument set of Ω_{core}. The cutoff scales in the same way as (63); that is,

$$\Lambda_{dcm} \equiv \eta_\lambda \Lambda_\circ \,. \tag{66}$$

Scaling rules for the cutoff are required for consistent definition of the integrals – they ensure that

$$\Lambda_{dcm} \gg m_{dcm}(\eta_\lambda)$$

for arbitrarily large η_λ, synchronize cutoff to $\Lambda_\circ$, and yield a well defined limit as $\eta \to \infty$ in (61). In the next section, we show that divergent integrals occurring in core and DCM terms are invariant under (63) and (66); as a result, the net amplitude is finite after cancellations.

Finally, we seek to apply the foregoing DCM rules to QCD. As with electroweak, all vertex factors are independent of mass and are therefore DCM invariant. Briefly, two modifications are required: First, the sign of the core and DCM amplitudes are reversed – this is because free quarks and gluons are confined and can not be experimentally isolated; therefore, apart from hadrons outside the region of confinement, the surrounding vacuum is effectively the primary observable. Relative to QED, this suggests an interchange of particles and vacuum and a sign reversal of the stabilized amplitude; thus, a factor

$$\lambda_c = -1 \tag{67}$$

is introduced for color confinement. Second, for diagrams involving massless gluons only in the pure gauge sector, we lack a mass reference – the solution is to introduce a small gluon mass μ_g via $k^2 \to k^2 - \mu_g^2$ in propagators (31) when constructing amplitudes, then (53) is used to define DCM states

$$m_{dcm}^2 = \mu_g^2 + \lambda M_g^2\big|_{\mu_g=0} \text{ with } M_g^2 \equiv \eta^2 \mu_\circ^2 \,, \tag{68}$$

where $\mu_\circ$ is a unit of mass measure. The rationale for (68) is discussed further in Sec. VII A.

In contrast to the regulator technique of Pauli and Villars [32], the above method employs physically meaningful dressed mass levels (albeit virtual only), and we assume that the same principle applies to all self-energy processes in QFT without introduction of auxiliary constraints.

IV. DIVERGENT INTEGRALS

Here we develop integration formulae required for evaluation of stability corrections using cutoff and dimensional regularization. In the p-representation, loop diagrams involve four-dimensional integrals over momentum space, and the real parts of scattering amplitudes contain integrals of the form [33]

$$D(\Delta) = \frac{1}{i\pi^2}\int \frac{d^4p}{(p^2-\Delta)^n} = \frac{(-1)^n}{\pi^2}\int \frac{d^4p_\varepsilon}{(p_\varepsilon^2+\Delta)^n} \,, \tag{69}$$

where Δ depends on the core mass m, momentum parameters external to the loop, and integration variables. On the right side of (69), a Wick rotation has been performed via a change of variables $p = (ip_\varepsilon^\circ, \vec{p}_\varepsilon)$, so that the integration can be performed in Euclidean space where $p_\varepsilon^2 = p_\varepsilon^\circ p_\varepsilon^\circ + \vec{p}_\varepsilon \cdot \vec{p}_\varepsilon$. Integrals for the divergent case ($n = 2$) must be regulated such that they are consistently defined for core and dressed core masses. For the core mass, D is regularized using a cutoff $\Lambda_\circ$ on $s = |p_\varepsilon|$. In four-dimensional polar coordinates, we have

$$D(\Delta, \Lambda_\circ) = \frac{1}{\pi^2}\int d\Omega \int_0^{\Lambda_\circ} ds\, \frac{s^3}{[s^2+\Delta]^2} \,. \tag{70}$$

For DCM states, Δ depends on m_{dcm}, and the domain of integration in (70) must be scaled according to (66); consequently, we need to evaluate

$$D_{dcm} = D\left[\Delta(m_{dcm}), \Lambda_{dcm}\right] \,.$$

With a change of variables $s = \eta_\lambda t$ and taking the limit $\eta \to \infty$, we obtain

$$D_{dcm} = D(\Delta_\circ, \Lambda_\circ) \,, \tag{71}$$

where

$$\Delta_\circ = \lim_{\eta\to\infty} \eta_\lambda^{-2}\Delta(\eta_\lambda m) \,. \tag{72}$$

For example, the standard divergent integral [33]

$$D_\circ \equiv D\left(\Delta = m^2, \Lambda_\circ\right) = \ln\frac{\Lambda_\circ^2}{m^2} - 1 + O\left(\frac{m^2}{\Lambda_\circ^2}\right) \tag{73}$$

is manifestly invariant under scaling rules (63) and (66); that is,

$$D_\circ = D\left(m_{dcm}^2, \Lambda_{dcm}\right) \,. \tag{74}$$

Note that the average of (32) over DCM states is stationary due to (74); this ensures that the FSE in QED is finite as shown in detail in Sec. V B.

In contrast to the cutoff method, dimensional regularization evaluates a Feynman diagram as an analytic function of space-time dimension d. For $n = 2$ and $d^4p \to d^dp$ in (69), D may be evaluated using [34, 35]

$$\begin{aligned} D(\Delta, \sigma) &= \pi^{-\sigma}\Gamma(\sigma)\Delta^{-\sigma} \\ &= \frac{1}{\sigma} - \ln\Delta - \gamma + O(\sigma) \,, \end{aligned} \tag{75}$$

where $\sigma = 2 - d/2$ and $\gamma = 0.577...$ is Euler's constant. For $\sigma \neq 0$, the limit $\Lambda_\circ \to \infty$ may be taken since σ regulates the integral. For DCM states, D_{dcm} must yield consistent results for both cutoff and dimensional regularization methods. Considering the requirements used to derive (71) and employing appendix formulae in [34], we conclude

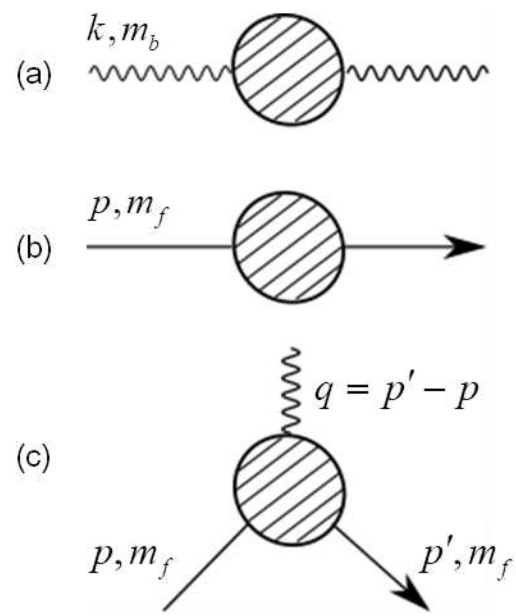


FIG. 4: Generic self-energy and vertex diagrams: (a) BSE, (b) FSE, and (c) vertex.

$$D_{dcm} = D\left(\Delta_\circ, \sigma\right) \ . \tag{76}$$

For the processes in Fig. 4, the argument Δ in (75) has the form

$$\Delta\left(m, \mu\right) = am^2 + b\ell^2 + c\mu^2 \ , \tag{77}$$

where $m = m_b | m_f$, $\ell^2 = k^2 \,|\, p^2 |\, q^2$, $\{a,\, b,\, c\}$ depend on Feynman parameters, and $c = 0$ for BSE processes. Applying (72) to (77) taking into account (64) and (65), the momenta go on-shell upon computing $\lim\limits_{\eta\to\infty} \eta_\lambda^{-2} \ell_{dcm}^2$; that is,

$$\begin{array}{cc} k^2 \to m_b^2 & \text{BSE} \\ p^2 \to m_f^2 & \text{FSE} \\ q^2 \to 0 & \text{Vertex} \end{array} \ , \tag{78}$$

which we recognize as on-shell renormalization conditions. For the vertex, the dressed momentum transfer $q_{dcm} = q + \lambda\left(P_M' - P_M\right)$ is assumed bounded, so $\lim\limits_{\eta\to\infty} \eta_\lambda^{-2} q_{dcm}^2 = 0$. As mentioned in Sec. III C, the case where particle masses internal and external to the blob in Fig. 4 (a) are both zero occurs for BSE processes in the pure-gauge sector of QCD. For this case, where $\Delta = bk^2$, choose $a = 1$ and replace $m^2 \to m_{dcm}^2$ in (77) using (68), then evaluate

$$\Delta_\circ = \lim_{\eta\to\infty} \eta_\lambda^{-2} \Delta\left(m = \eta_\lambda \mu_\circ\right) = \mu_\circ^2 \tag{79}$$

with $\eta_\lambda = \sqrt{\lambda}\,\eta$. Thus for all $m \geq 0$, the net S-matrix amplitude computed from (60) is well defined since it involves a factor

$$\frac{\Gamma\left(\sigma\right)}{\Delta^\sigma} - \frac{\Gamma\left(\sigma\right)}{\Delta_\circ^\sigma} = -\ln\left|\frac{\Delta}{\Delta_\circ}\right| \ . \tag{80}$$

The second term on the left side of (80) is associated with an opposing vacuum energy required for overall energy conservation and system stability. In addition to a divergent part, Ω_{dcm} in (61) may include a finite part, a constant, that cancels a like term in Ω_{core}.

V. QED APPLICATIONS

Let us apply the foregoing theory with integration formulae given above to verify that net amplitudes for second order radiative corrections in QED are convergent and agree with results obtained via renormalization theory. Cutoff and dimensional regularization approaches are used to illustrate the method. Since the complete amplitude $\hat{\Omega}$ is distinct from Ω_{core}, we drop the "core" subscript for unrenormalized (core) amplitudes to be consistent with notation in the literature.

A. Vacuum polarization

The photon self-energy associated with Fig. 3 (a) results in a propagator modification [30]

$$D_F^{\prime\alpha\beta} = D_F^{\alpha\beta} + D_F^{\alpha\mu} \left(i\hat{\Pi}_{\mu\nu} \right) D_F^{\nu\beta} ,$$

where

$$\hat{\Pi}_{\mu\nu} \equiv \Pi_{\mu\nu} + \Pi_{\mu\nu}^{dcm}$$

is a polarization tensor generalized to include the stability (aka DCM) correction, and

$$\Pi_{\mu\nu}\left(k,\, m\right) = -\frac{ie^2}{\left(2\pi\right)^4} \int d^4p \,\mathrm{tr}\left[\gamma_\mu S_F(p,m)\gamma_\nu S_F(p-k,m)\right]$$

follows from the Feynman-Dyson rules [5, 22]. In consequence of Lorentz and gauge invariance [7] or by direct calculation, it factors into

$$\Pi_{\mu\nu}\left(k,\, m\right) = \Pi\left(k^2,\, m^2\right)\left(g_{\mu\nu}k^2 - k_\mu k_\nu\right) .$$

As is well known, the contribution from terms $k_\mu k_\nu$ vanishes due to current conservation upon connection to an external fermion line. For a massless photon, k^2 is invariant under a DCM transform, and we need only focus on the scalar function $\Pi\left(k^2, m^2\right)$.

Since the scattering amplitude is in general a complex analytic function, it follows from Cauchy's formula that the real and imaginary parts are related by a dispersion relation [36]. The imaginary part is divergence free and may be obtained by replacing Feynman propagators with cut propagators on the mass shell according to Cutkosky's cutting rule [37] or, alternatively, via calculation in the Heisenberg representation as shown in Källén [38]. In particular for vacuum polarization, the real part for the core mass is given by

$$\Pi\left(k^2,\, m^2\right) = \frac{1}{\pi}\int_{4m^2}^{4\Lambda_\circ^2} ds\, \frac{g\left(\frac{4m^2}{s}\right)}{s-k^2} \tag{81}$$

with imaginary part

$$g\left(w\right) = -\frac{\alpha}{3}\sqrt{1-w}\left(1+w/2\right).$$

Applying (61) using (63) and (66) and performing a change of variables $s = \left(1+\lambda\eta^2\right)t$ in (81), we have

$$\begin{aligned}\Pi_{dcm} &= -\frac{1}{2}\lim_{\eta\to\infty}\sum_{\lambda=\pm1}\Pi\left(k^2,\, m^2+\lambda\eta^2 m^2\right) \qquad (82)\\ &= -\frac{1}{2\pi}\lim_{\eta\to\infty}\sum_{\lambda=\pm1}\int_{4m^2}^{4\Lambda_\circ^2} dt\, \frac{g\left(\frac{4m^2}{t}\right)}{t-\left(1+\lambda\eta^2\right)^{-1}k^2} .\end{aligned}$$

Letting $\eta \to \infty$, we see that (82) is equivalent to a subtracted core amplitude evaluated on the light cone

$$\Pi_{dcm} = -\Pi(k^2 = 0,\, m^2)\,.$$

Combining (81) and (82), we obtain a once-subtracted dispersion relation

$$\begin{aligned}\hat{\Pi}\left(k^2\right) &= \Pi\left(k^2,\, m^2\right) - \Pi\left(0,\, m^2\right) \\ &= \frac{k^2}{\pi}\int_{4m^2}^{\infty} ds\, \frac{g\left(\frac{4m^2}{s}\right)}{s\left(s-k^2\right)}\end{aligned} \tag{83}$$

in agreement with renormalized QED. From (83), $\hat{\Pi}\left(k^2 = 0\right) = 0$ epitomizes the free boson stability condition (51) for a photon. For an infinite sum of 1PI insertions, the generalized photon propagator is

$$= -\frac{ig_{\mu\nu}}{k^2}\hat{Z}_3\left(k^2\right)\ , \text{ where} \tag{84}$$

$$\hat{Z}_3\left(k^2\right) = \frac{1}{1-\hat{\Pi}\left(k^2\right)} \tag{85}$$

modifies the free photon propagator. Alternatively, one can define a running coupling constant

$$\alpha\left(k^2\right) = \hat{Z}_3\left(k^2\right)\alpha_\circ\,; \tag{86}$$

in this interpretation, the bare$\left(\alpha_\circ = \frac{e^2}{4\pi}\right)$ and effective couplings are equivalent on the light cone

$$\hat{Z}_3\left(0\right) = 1\,. \tag{87}$$

In terms of an external current $j_\mu^{ext}\left(x\right)$, the observable current is given by

$$j_\mu^{obs}\left(x\right) = j_\mu^{ext}\left(x\right) + \delta j_\mu\left(x\right)\ ,$$

where

$$\delta j_\mu\left(x\right) = \frac{1}{\left(2\pi\right)^4}\int d^4k\, e^{ikx} j_\mu^{ext}\left(k\right)\left[\Pi(k^2,\, m^2) - \Pi(0,\, m^2)\right]$$

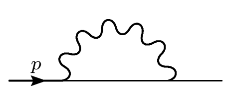
is the induced current. In standard renormalization theory (SRT), the last term in brackets is associated with a correction to a divergent bare charge ($e_\circ$), but here we suggest that the correction is a stability requirement associated with a vacuum reaction current. The physical and bare charges in SRT are related by

$$e^2 = \left(Z_3 = \frac{1}{1-\Pi\left(0,\, m^2\right)}\right)e_\circ^2\,,$$

where $\sqrt{Z_3}$ is the charge renormalization constant.

B. Fermion self-energy

The fermion self-energy operator for a core mass corresponding to the Feynman diagram in Fig. 3 (b) is



$$= -i\Sigma\left(p, m\right)\ , \text{ where}$$

$$\Sigma(p,m) = -\frac{e^2}{(2\pi)^4}\int d^4k\ \gamma_\mu\, S_F(p-k,m)\ \gamma^\mu \frac{1}{k^2-\mu^2}\ . \tag{88}$$

After standard reduction and dimensional regularization, Σ simplifies to

$$\Sigma(p,m) = \frac{\alpha}{2\pi}\left\{S_1 + \int_0^1 dx\ \left[2m - \not{p}x + \sigma\left(\not{p}x - m\right)\right]\ D(\Delta,\sigma)\right\}\ , \tag{89}$$

where $D(\Delta,\sigma)$ is given by (75) with

$$\Delta = (1-x)\left(m^2 - xp^2\right) + x\mu^2\ .$$

The integral expression in (89) is equivalent to a form given in [13], while the term

$$S_1 = -\frac{1-\sigma}{4}\not{p}$$

follows from appendix formulae in [33] and represents a surface contribution arising from a term linear in k during reduction of (88).

Evaluation of Σ_{dcm} using (61) reduces to negating (89) and replacing $\Delta \rightarrow \Delta_\circ$ according to (76); we obtain

$$\Sigma_{dcm}(p,m) = -\frac{\alpha}{2\pi}\left\{S_1 + \int_0^1 dx\ \left[2m - \not{p}x + \sigma\left(\not{p}x - m\right)\right]\ D(\Delta_\circ,\sigma)\right\}\ , \tag{90}$$

where

$$\Delta_\circ = m^2(1-x)^2 + x\mu^2$$

follows from (78). Terms involving $[(\lambda P_M, \lambda M)\ ;\ M = \eta m]$ have canceled in the average over DCM levels yielding a function of the observable mass and momentum only. The net correction, including all three mass levels in Fig. 3 (b), is given by (cf. [22])

$$\begin{aligned}\hat{\Sigma}(p) &= \Sigma + \Sigma_{dcm} \\ &= \frac{\alpha}{2\pi}\int_0^1 dx\ \left(2m - \not{p}x\right)\ln\left[\frac{m^2(1-x)^2 + x\mu^2}{\left(m^2 - xp^2\right)(1-x) + x\mu^2}\right]\ ,\end{aligned} \tag{91}$$

where the limit $\sigma \rightarrow 0$ has been taken to recover four-dimensional space-time. With a change of variables $x = 1 - z$, (91) is seen to be identical to the renormalized result given in Bjorken & Drell [39].

The processes in Fig. 3 (b), including iterations in the series

(92)

yields a modified propagator [5, 30]

$$\begin{aligned}S'_F &= S_F + S_F\left(-i\hat{\Sigma}(p)\right)\ S'_F \\ &= \frac{i}{\not{p} - m - \hat{\Sigma}(p) + i\varepsilon}\ ,\end{aligned} \tag{93}$$

which has the desired pole at $\not{p} = m$ since (91) vanishes on the mass shell

$$\hat{\Sigma}(p)\Big|_{p^2=m^2} = 0\ . \tag{94}$$

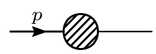
Using the general expression for the stabilized fermion self-energy (190)

$$\hat{\Sigma}(p) = \Sigma(p) - \Sigma(m) - \left.\frac{\partial \Sigma}{\partial \not{p}}\right|_{\not{p}=m} (\not{p} - m) \tag{95}$$

derived in Sec. VII B 2, we see that

$$\left.\frac{d\hat{\Sigma}(p)}{d\not{p}}\right|_{\not{p}=m} = 0\,,$$

and the residue of the pole is i. Note that one can write (93) in the form



$$= \frac{i}{\not{p} - m + i\varepsilon} \hat{Z}_2(\not{p})\,, \tag{96}$$

where

$$\hat{Z}_2 \equiv \left(1 - \frac{\hat{\Sigma}(\not{p})}{\not{p} - m + i\varepsilon}\right)^{-1} \tag{97}$$

is a finite stabilization parameter modifying the free field fermion propagator, and is analogous to the renormalization constant Z_2 in SRT relating the bare and renormalized fields via $\psi_\circ = \sqrt{Z_2}\psi$.

Upon identifying

$$m_{em}^{+} = \Sigma(\not{p} = m,\, \mu = 0)\,, \text{ and} \tag{98}$$

$$m_{em}^{-} = \Sigma_{dcm}(\not{p} = m,\, \mu = 0)\,, \tag{99}$$

we see that (94) is equivalent to the stability principle (35). Reverting to cutoff $\Lambda_\circ$ using (69), it follows that (98) reduces to Feynman's result (32); for derivation, see [33].

In the language of renormalization theory, the bare mass in the propagator [35]

$$S_F' = \frac{i}{\not{p} - m_{bare} - \Sigma + i\varepsilon}$$

must be renormalized using (37) with (98).

C. Vertex

A second-order correction to a corner (23) involves a replacement $ie\gamma^\mu \to ie\Gamma^\mu$, where

$$\begin{aligned}\Gamma^\mu &= \gamma^\mu + \Lambda^\mu \\ &= \gamma^\mu F_1\left(q^2\right) + \frac{i\sigma^{\mu\nu}q_\nu}{2m} F_2\left(q^2\right)\,,\end{aligned} \tag{100}$$

and $\sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu, \gamma^\nu]$ are spin matrices. Complete expressions for the form factors F_1 and F_2 can be found in [13]. For small q^2, the vertex function Λ^μ for the core mass corresponding to Fig. 3 (c) is given by the approximation [22]

$$\Lambda^\mu(q, m) = \gamma^\mu L + a^{(2)}\frac{i\sigma^{\mu\nu}q_\nu}{2m} + O\left(\frac{q^2}{m^2}\right)\,, \text{ where} \tag{101}$$

$$L = \frac{\alpha}{4\pi}\left(D_\circ + \frac{11}{2} - 4\ln\frac{m}{\mu}\right) \tag{102}$$

is a divergent constant. Note that $L = \frac{\alpha}{2\pi} r$, where r is given by Eq. (23) in [22]. The coefficient $a^{(2)} = \frac{\alpha}{2\pi}$ is the second-order contribution to the anomalous magnetic moment first derived by Schwinger [40] and verified experimentally by Foley & Kusch [41].

Inserting (101) into (61), using (59), and accounting for the invariance of $D_\circ$ (74) under scaling rules (63) and (66), the stability correction is $\Lambda^\mu_{dcm} = -\gamma^\mu L$, where finite terms in (101) of order $O\left(\frac{q}{m}\right)$ involving replacements $m \to m\left(1 + \lambda\eta\right)$ and $q \to q_{dcm}$ vanish in the limit $\eta \to \infty$ since q_{dcm} is bounded as we argued in Sec. IV. Therefore, the total vertex function

$$\hat{\Lambda}^\mu\left(q\right) = \Lambda^\mu + \Lambda^\mu_{dcm} \tag{103}$$

is convergent, and Λ^μ satisfies the usual renormalization condition for a vertex

$$\hat{\Lambda}^\mu \Big|_{q^2=0, \not{p}=\not{p}'=m} = 0 \, . \tag{104}$$

This completes verification that lowest-order S-matrix corrections are finite without renormalization.

VI. GENERALIZATION TO HIGHER ORDERS

Our next task is to show that stabilized higher-order radiative corrections in QED are finite and agree with renormalization. The proof closely follows renormalization arguments in the original references and [33]; therefore, we keep our remarks brief highlighting required modifications and differences of interpretation.

Irreducible (skeleton) diagrams include second-order self-energy (SE) and vertex (V) parts discussed in Sec. V plus infinitely many higher-order primitively divergent V-parts. Using Dyson's expansion method [30], second-order SE- and V-part operators for the core mass are

$$\Sigma = mA - \left(\not{p} - m\right) B + \hat{\Sigma} \, , \tag{105}$$

$$\Pi = C + \hat{\Pi} \, , \tag{106}$$

$$\Lambda^\mu = \gamma^\mu L + \hat{\Lambda}^\mu \, , \tag{107}$$

where $\{A, \, B \, , C \, , \, L\}$ are logarithmically divergent coefficients depending on $D_\circ$ – specifically, $A = \frac{3\alpha}{4\pi}\left(D_\circ + \frac{3}{2}\right)$ using (32), and Ward's identity [42] gives $B = L$ from (102). Innocuous finite terms can depend on the regularization method used; for example, compare $C = -\Pi_{dcm} = \frac{\alpha}{3\pi}\left(D_\circ + \ln 4 - \frac{2}{3}\right)$ from (82) with expressions in [33, 35]. Higher-order primitively divergent V-parts are also of the form (107) since the degree of divergence [30, 43]

$$K = 4 - \frac{3}{2} f_e - b_e$$

is zero (logarithmic), where $f_e\,(b_e)$ are the number of external fermion (boson) lines; in this case, $L\left(D_\circ\right)$ is a power series in α.

To determine the interaction of an electromagnetically dressed core with the polarized vacuum, we apply (61) with (74) to obtain

$$\Sigma_{dcm} = -\left[mA - \left(\not{p} - m\right) B\right] \, , \tag{108}$$

$$\Pi_{dcm} = -C \, , \tag{109}$$

$$\Lambda^\mu_{dcm} = -\gamma^\mu L \, , \tag{110}$$

where the stabilized second-order amplitudes (83), (91), and (103)

$$\hat{\Sigma}\left(p^2 = m^2\right) = 0 \tag{111}$$

$$\hat{\Pi}\left(k^2 = 0\right) = 0 \tag{112}$$

$$\hat{\Lambda}^\mu\left(q^2 = 0\right) = 0 \tag{113}$$

vanish on the mass shell. Higher-order primitively divergent V-parts also satisfy (113) since dressed stabilized amplitudes vanish and yield on-shell conditions. In this way, (60) yields unique finite results

$$\hat{\Sigma} = \Sigma + \Sigma_{dcm} \,, \tag{114}$$

$$\hat{\Pi} = \Pi + \Pi_{dcm} \,, \text{ and} \tag{115}$$

$$\hat{\Lambda}^{\mu} = \Lambda^{\mu} + \Lambda^{\mu}_{dcm} \tag{116}$$

for all irreducible diagrams; therefore, SE-part insertions

$$S_F \to S_F + S_F \left(-i\hat{\Sigma}\right) S_F \,, \text{ and} \tag{117}$$

$$D_F^{\alpha\beta} \to D_F^{\alpha\beta} + D_F^{\alpha\mu}\left(ig_{\mu\nu}k^2\hat{\Pi}\right) D_F^{\nu\beta} \tag{118}$$

into lines, and V-part insertions

$$\gamma^{\mu} \to \gamma^{\mu} + \hat{\Lambda}^{\mu} \tag{119}$$

into corners of a skeleton diagram yield no additional divergences.

For reducible vertex diagrams, the V-part resolves into a skeleton along with stabilized SE- and V-part insertions. With replacements (117), (118), and (119) in the skeleton, the vertex operator again reduces to the form (107), where $L \to L_s$ is the skeleton divergence. In general, L_s depends on multiple functions $D_\circ$ corresponding to all possible charged fermion masses arising from photon self-energy insertions which may in turn contain SE- and V-parts. Since each $D_\circ$ is invariant under (74), (113) holds, and (61) yields $\Lambda^{\mu}_{s,dcm} = -\gamma^{\mu} L_s$ similarly to (110); therefore, the complete reducible V-part given by (116) is convergent.

For reducible self-energy diagrams, a skeleton with SE insertions is handled in the same way as reducible vertex diagrams. However, vertex insertions into fermion and photon SE skeletons involve overlapping divergences that require further analysis [44, 45]. Integration of Ward's identities yields expressions of the same form as (105) and (106); in this case, the coefficients $\{A,\, B\,, C\}$ are all power series in α depending on $D_\circ$, and vertex insertions in SE-parts are convergent upon including stability corrections (108) and (109). We conclude that infinite field actions excite dressed mass levels uniformly in all connected fermion lines internal to overlapping loops; for a specific example, apply (60) to calculate the real part of the fourth-order vacuum polarization kernel [46] using the dispersion method given in Sec. V A. Therefore, a diagram with overlapping divergences is not a special case for implementation of stability corrections.

The complete propagators, replacing fermion and photon lines in a skeleton diagram, follow from Eqs. (63) and (64) of Dyson [30]; one obtains

$$S'_F(p) = \frac{i}{\not{p} - m - \hat{\Sigma}^* + i\varepsilon} \,, \text{ and}$$

$$D'^{\alpha\beta}_F(k) = \frac{-ig^{\alpha\beta}}{k^2\left[1 - \hat{\Pi}^*\right] + i\varepsilon} \,,$$

where $\left\{\hat{\Sigma}^*,\, \hat{\Pi}^*\right\}$ are given by sums over all proper SE-parts. Similarly, the most general vertex replacing a corner in a skeleton diagram is given by a sum over all proper V-parts. Since both core and DCM contributions are included in each sub-diagram, the complete propagators and vertices are well defined (convergent).

VII. NON-ABELIAN APPLICATIONS

Stabilized radiative corrections are computed for diagrams in Yang-Mills and electroweak theories.

A. Yang-Mills Theory Corrections

In the examples below, we focus on a key subset of one-loop diagrams [47, 48] that occur in the $SU(3)$ Yang-Mills theory discussed in Sec. II B.

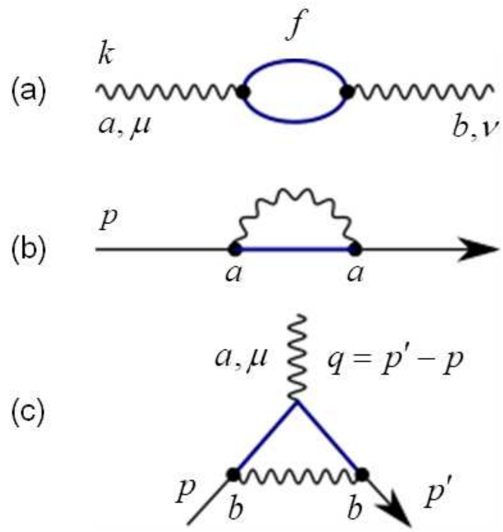


FIG. 5: Gluon/quark self-energies and vertex diagrams

For diagrams in Fig. 5, core amplitudes differ from QED only by group factors and the confinement factor λ_c from (67); therefore, finite S-matrix amplitudes (60), including stability corrections, are

$$\hat{\Pi}_1^{ab} = \lambda_c \mathrm{tr}\left(t^a t^b\right) \hat{\Pi}\left[QED\right] \ , \tag{120}$$

$$\hat{\Sigma}^{aa} = \lambda_c t^a t^a \hat{\Sigma}\left[QED\right] \ , \text{and} \tag{121}$$

$$\hat{\Lambda}_1^{a,\mu} = \lambda_c t^b t^a t^b \hat{\Lambda}^\mu \left[QED\right] \ . \tag{122}$$

Group factors are given by

$$\mathrm{tr}\left(t^a t^b\right) = C\left(r\right)\delta^{ab} \ ,$$
$$t^a t^a = C_2\left(r\right) \ ,$$
$$t^b t^a t^b = \left[C_2\left(r\right) - \frac{1}{2}C_2\left(G\right)\right] t^a \ ,$$

where $C(N) = \frac{1}{2}$, and $C_2\left(N\right) = \frac{N^2-1}{2N} = \frac{4}{3}$ are normalization and quark color charge factors, respectively.

In addition to the fermion (quark) loop diagram in Fig. 5 (a), gluon SE corrections in Fig. 6 yield [13]

$$\left[\text{Fig. } 6\right]_{core} = iT_{\mu\nu}\left(k^2\right)\delta^{ab}\Pi_2\left(k^2\right) \ , \tag{123}$$

$$T_{\mu\nu}\left(k^2\right) = g_{\mu\nu}k^2 - k_\mu k_\nu \ ,$$

$$\Pi_2\left(k^2\right) = \lambda_c \frac{\alpha_s C_2\left(G\right)}{4\pi}\int_0^1 dx \, \frac{\Gamma\left(\sigma\right)}{\Delta^\sigma}\left[\left(-1+\sigma\right)\left(1-2x\right)^2 + 2\right] \ , \tag{124}$$

where $\alpha_s = g_s^2/4\pi$ is the strong coupling constant, $\Delta = -k^2 x\left(1-x\right)$, and x is a Feynman parameter. While individual gluons are massless to ensure gauge invariance of $\mathcal{L}_{YM}$, systems of gluons depicted in Fig. 6 are expected to have a non-zero mass defined by (50) with self-energy function $\Sigma_2^g \sim k^2\Pi_2$. The generation of such systems must draw energy from the vacuum leaving it with a squared energy deficit $\Sigma_{vac} = -M_g^2$ such that (51) is satisfied. Consequently, we need to include a stability correction for an opposing vacuum response involving DCM states, and for this we need a mass term in Δ. If we appeal to massive Yang-Mills theories [49], we get unwanted particles and ghosts, and it might seem that we have an impasse. While gauge invariance demands that mass be acquired via a Higgs mechanism, introduction of DCM states in (29) yielding

$$\mathcal{L}'_{YM} \to \mathcal{L}_{YM} - \frac{1}{2}\sum_{\lambda=\pm 1}\left[\mu_g^2 + \lambda M_g^2\right]_{\mu_g=0}\left(A_\mu^a\right)^2$$

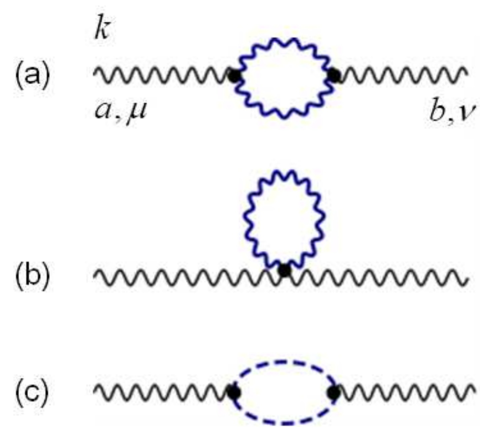


FIG. 6: Gluon SE corrections in pure-gauge sector: (a) gluon loop, (b) four-gluon vertex, and (c) ghost loop.

does not break gauge invariance of $\mathcal{L}_{YM}$ since the expectation over DCM states is zero. Therefore, let us temporarily assign a small mass μ_g to the gluon, then propagators in the loops are modified

$$\frac{1}{p^2-\mu_g^2}\frac{1}{(p+k)^2-\mu_g^2}=\int_0^1\frac{dx}{\left[P^2-\Delta\left(\mu_g\right)\right]^2},$$

where the usual change of variables $P=p+xk$ has been made for loop integration parameter p, and

$$\Delta\left(\mu_g\right)=\mu_g^2-k^2x\left(1-x\right).$$

To evaluate the stability contribution, make the replacement (68)

$$\mu_g^2\rightarrow\left[\mu_g^2+\lambda\eta^2\mu_\circ^2\right]_{\mu_g=0}$$

in $\Delta\left(\mu_g\right)$; then from (79), we have $\Delta_\circ=\mu_\circ^2$, and the stability correction is obtained simply by negating (124) and replacing

$$\frac{1}{\Delta^\sigma}\rightarrow\frac{1}{\Delta_\circ^\sigma}.$$

From (60), the net amplitude

$$\hat{\mathit{\Pi}}_2\left(k^2\right)=-\lambda_c\frac{\alpha_sC_2\left(G\right)}{4\pi}\int_0^1dx\ \ln\left[\frac{-k^2x\left(1-x\right)}{\mu_\circ^2}\right]\left[-\left(1-2x\right)^2+2\right]\tag{125}$$

is finite. If we define a reference mass M_s by

$$\frac{5}{3}\ln\left(\frac{\mu_\circ^2}{M_s^2}\right)\equiv\int_0^1dx\ \ln\left[x\left(1-x\right)\right]\left[-\left(1-2x\right)^2+2\right],\ \text{then}$$

$$\hat{\mathit{\Pi}}_2\left(\rho_s\equiv-\frac{k^2}{M_s^2}\right)=-\lambda_c\frac{\alpha_sC_2\left(G\right)}{4\pi}\frac{5}{3}\ln\rho_s\tag{126}$$

vanishes at spacelike $k^2=-M_s^2$.

In the stabilized theory, it is invalid to neglect quark masses m_f in the calculations since they are required for defining DCM corrections; in contrast, QCD calculations in the usual theory often omit m_f in processes where the momentum transfer q is presumed much larger than physical masses involved in the problem.

Therefore, following Peskin & Schroeder [13], but assuming $m \equiv m_f \neq 0$, the core amplitude for the quark/three-gluon vertex shown in Fig. 7 is

$$\Lambda_2^{a,\mu} = \lambda_c \frac{g_s^2 f^{abc} t^b t^c}{(2\pi)^4} \int d^4k \, \frac{\gamma_\nu \left(\not{k} + m\right) \gamma_\rho N^{\mu\nu\rho}\left(k, p', p\right)}{\left(k^2 - m^2 + i\varepsilon\right) \left[\left(p' - k\right)^2 - \mu^2 + i\varepsilon\right] \left[\left(p - k\right)^2 - \mu^2 + i\varepsilon\right]} \; , \tag{127}$$

where

$$N^{\mu\nu\rho}\left(k, p', p\right) = g^{\mu\nu}\left(q + p' - k\right)^\rho + g^{\nu\rho}\left(2k - p' - p\right)^\mu + g^{\rho\mu}\left(p - k - q\right)^\nu$$

is the tensor part of the 3-gluon vertex function. Introducing Feynman parameters (x, y, z) for the factors in the denominator, letting $\ell = k - px - p'y$, dropping terms linear in ℓ that vanish upon symmetrical integration, and using the identity $f^{abc} t^b t^c = \frac{i}{2} C_2(G)\, t^a$, (127) reduces to

$$\Lambda_2^{a,\mu} = i\lambda_c \frac{g_s^2 C_2(G)\, t^a}{(2\pi)^4} \int dx dy dz \, \delta\left(x + y + z - 1\right) I^\mu \; , \tag{128}$$

where

$$I^\mu = I_1^\mu + I_2^\mu \; , \tag{129}$$

$$\begin{aligned} I_1^\mu \; &= \int^{\Lambda_\circ} d^d\ell \, \frac{\gamma_\nu \not{\ell} \gamma_\rho \left(-g^{\mu\nu}\ell^\rho + 2g^{\nu\rho}\ell^\mu - g^{\rho\mu}\ell^\nu\right)}{\left(\ell^2 - \Delta\right)^3} \\ &= -3i\pi^2 \frac{\Gamma(\sigma)}{\Delta^\sigma} \gamma^\mu \; , \end{aligned}$$

$$I_2^\mu = \int^{\Lambda_\circ} d^4\ell \, \frac{\gamma_\nu \left(\not{p}x + \not{p}'y + m\right) \gamma_\rho N^{\mu\nu\rho}\left(px + p'y, \, p', \, p\right)}{\left(\ell^2 - \Delta\right)^3} \; , \text{ and}$$

$$\Delta = m^2 z + \left(px + p'y\right)^2 - p^2 x - p'^2 y + \mu^2 \left(1 - z\right) \; .$$

The cutoff is retained in the divergent and finite parts $\{I_1^\mu, I_2^\mu\}$ as a reminder that for computation of the stability correction, $\Lambda_\circ \to \eta_\lambda \Lambda_\circ$ followed by a change of variables $\ell \to \eta_\lambda \ell$. Now apply (61), (76), and integration formulae in [13] to obtain

$$\begin{aligned} I_{dcm}^\mu &= I_{1,dcm}^\mu + I_{2,dcm}^\mu \; , \\ I_{1,dcm}^\mu &= 3i\pi^2 \frac{\Gamma(\sigma)}{\Delta_\circ^\sigma} \gamma^\mu \; , \\ I_{2,dcm}^\mu &= i\pi^2 \frac{m^2}{\Delta_\circ} z\left(2 - z\right)\left(\gamma^\mu - 4\right) \; , \end{aligned} \tag{130}$$

where from (72)

$$\Delta_\circ = m^2 z^2 + \mu^2 \left(1 - z\right) \; ,$$

and we have used

$$\begin{aligned} p^\nu(m) &= m + \delta p_{os}^\nu \; , \text{ and} \\ \lim_{\eta \to \infty} \eta_\lambda^{-1} p^\nu\left(\eta_\lambda m\right) &= m \end{aligned}$$

for evaluation of $I_{2,dcm}^\mu$. Finally, the stabilized integral is given by

$$\begin{aligned} \hat{I}^\mu &= I^\mu + I_{dcm}^\mu \\ &= 3i\pi^2 \ln \frac{\Delta}{\Delta_\circ} + finite \; , \end{aligned} \tag{131}$$

and the complete amplitude

$$\hat{\Lambda}_2^{a,\mu} = i\lambda_c \frac{g_s^2 C_2(G)\, t^a}{(2\pi)^4} \int dxdydz\, \delta(x+y+z-1)\, \hat{I}^\mu \tag{132}$$

is finite without renormalization and only involves physical parameters.

From standard renormalization theory (SRT), the seven diagrams in Figs. 5, 6, and 7 yield a running coupling constant characterized by a Callan-Symanzik [50, 51] beta function [52, 53]

$$\beta_{QCD}(\alpha_s) = \frac{\partial \alpha_s}{\partial \ln M_s^2} = -\frac{\alpha_s^2}{4\pi}\left[\frac{11}{3}C_2(G) - \frac{4}{3}n_f C(r)\right] , \tag{133}$$

where α_s is the renormalized coupling. Compared to QED where $\beta_{QED}(\alpha) = \frac{\alpha^2}{3\pi}$ with charge screening, $\beta_{QCD}(\alpha_s) < 0$ leads to an anti-screening effect or asymptotic freedom resulting in a weaker coupling for high energies. The dependence of α_s on momentum transfer q is

$$\alpha_s\left(q^2\right) = \frac{\alpha_s\left(M_s^2\right)}{1 + \frac{\alpha_s\left(M_s^2\right)}{4\pi}\left(11 - \frac{2}{3}n_f\right)\ln\left|\frac{q^2}{M_s^2}\right|} , \tag{134}$$

where M_s is usually chosen to be the $Z-$boson mass. The running of the strong coupling constant in SRT models experimental data [54] well; therefore, our remaining task is to show that the stabilized theory yields an effective running of the coupling constant in agreement with SRT and experiment. Well known formulae from SRT are used with stabilized amplitude parameters.

Leading terms of stabilized amplitudes for the asymptotic case of high energy yield an effective color charge

$$\begin{aligned} \overline{g}_s(\rho_s) &= g_s \frac{\hat{Z}_1}{\hat{Z}_2\sqrt{\hat{Z}_3}} \\ &\simeq g_s\left[1 + \lambda_c \frac{\alpha_s}{8\pi}\left(11 - \frac{2}{3}n_f\right)\ln\rho_s\right] \text{, where} \end{aligned} \tag{135}$$

$$\hat{Z}_1^{-1} = 1 + \hat{\Lambda}_1(\rho_s) + \hat{\Lambda}_2(\rho_s) , \tag{136}$$

$$\hat{Z}_2^{-1} = 1 - \left.\frac{d\hat{\Sigma}}{d\not{p}}\right|_{\rho=\rho_s} \text{, and} \tag{137}$$

$$\hat{Z}_3^{-1} = 1 - \left[\hat{\Pi}_1(\rho_s) + \hat{\Pi}_2(\rho_s)\right] \tag{138}$$

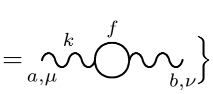
are finite running stabilization parameters that modify the vertex (30), fermion field propagator (25), and gluon field propagator (31), respectively. With $n_f = 6$ the confinement factor $\lambda_c = -1$ in (135) leads to asymptotic freedom; note that the need for λ_c arises from opposing signs of the first two terms of (75). For loops including quarks, asymptotic amplitudes involve spacelike momenta ℓ in ratios

$$\rho = -\frac{\ell^2}{m_f^2} \gg 1 \, ; \, \ell^2 \in \left\{k^2,\, p^2,\, q^2\right\} , \tag{139}$$

where we have reinstated $m = m_f$. Setting $p^2 = q^2 = k^2$, energy ratios ρ in (139) may be approximated by

$$\begin{aligned} \ln\rho &= \ln\rho_s + O(1) \\ &\simeq \ln\rho_s \, . \end{aligned} \tag{140}$$

With (140), the sum over fermions in Fig. 5(a) becomes trivial. Neglecting $O(1)$ terms, we have

$$\sum_{f=1}^{n_f} \left\{ [\text{Fig. 5(a)}] = \right.$$


$$\left. \right\} \simeq iT_{\mu\nu}\left(k^2\right)\delta^{ab}\left[\hat{\Pi}_1(\rho_s) \equiv \lambda_c \frac{\alpha_s}{3\pi} n_f\, C(r) \ln\rho_s\right] , \tag{141}$$

$$[\text{Fig. 5(b)}] \simeq -i\left[\hat{\Sigma}\left(\not{p},\rho_s\right) \equiv \lambda_c \frac{\alpha_s}{4\pi} C_2\left(r\right)\left(\not{p}-4m_f\right)\ln\rho_s\right] , \tag{142}$$

$$[\text{Fig. 5(c)}] \simeq i g_s t^a \gamma^\mu \left\{\hat{\Lambda}_1\left(\rho_s\right) \equiv -\lambda_c \frac{\alpha_s}{4\pi}\left[C_2\left(r\right)-\frac{1}{2}C_2\left(G\right)\right]\ln\rho_s\right\} , \tag{143}$$

$$[\text{Fig. 6}] = i T_{\mu\nu}\left(k^2\right)\delta^{ab}\left[\hat{\Pi}_2\left(\rho_s\right) \equiv -\lambda_c \frac{\alpha_s C_2\left(G\right)}{4\pi}\frac{5}{3}\ln\rho_s\right] \tag{144}$$

from (126), and

$$[\text{Fig. 7}] \simeq i g_s t^a \gamma^\mu \left[\hat{\Lambda}_2\left(\rho_s\right) \equiv -\lambda_c \frac{\alpha_s}{4\pi}\frac{3}{2}C_2\left(G\right)\ln\rho_s\right] . \tag{145}$$

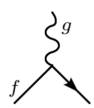
With the approximation $-k^2 \gg m_f^2$, $\hat{\Pi}_1$ in (141) follows from (120) using (83). Similarly, $\hat{\Sigma}$ in (142) is obtained from (121) using (91), and $\hat{\Lambda}_1$ in (143) is derived using (122) with (100) to obtain

$$\hat{\Lambda}^\mu\left[QED\right] = \hat{F}_1\left(q^2\right)\gamma^\mu + ... ,$$

where the stabilized form factor

$$\begin{aligned}\hat{F}_1\left(q^2\right) &= F_1\left(q^2\right) - F_1\left(0\right) \\ &= \frac{\alpha}{2\pi}\int_0^1 dxdydz\,\delta\left(x+y+z-1\right)\ln\left[\frac{m_f^2\left(1-z\right)^2}{m_f^2\left(1-z\right)^2 - q^2xy}\right] + ... \\ &\simeq -\frac{\alpha}{4\pi}\ln\left(\rho = -\frac{q^2}{m_f^2}\right) = -\frac{\alpha}{4\pi}\ln\rho_s + O\left(1\right)\end{aligned}$$

follows from (78) and results given in [13] assuming $-q^2 \gg m_f^2$. The stabilization parameters $\hat{Z}_1$, $\hat{Z}_2$, and $\hat{Z}_3$ are defined similarly to their SRT counterparts. $\hat{Z}_1^{-1}$ is the coefficient of $i g_s \gamma^\mu t^a$ for the sum of proper vertex diagrams in (30), Fig. 5(c), and Fig. 7



$$\cdot\left(1+\hat{\Lambda}_1+\hat{\Lambda}_2\right) \equiv i g_s \gamma^\mu t^a \hat{Z}_1^{-1} .$$

Using (137) or (97) and assuming $-p^2 \gg m_f^2$, we have

$$\hat{Z}_2^{-1} = 1 - \lambda_c \frac{\alpha_s}{4\pi} C_2\left(r\right)\left[\ln\rho = \ln\rho_s + O\left(1\right)\right] .$$

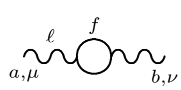
For $\hat{Z}_3$ we employ (85) with $\hat{\Pi} = \hat{\Pi}_1 + \hat{\Pi}_2$ using gluon SE contributions (141) and (144).

An estimate of M_s may be obtained by synchronizing the energy ratios for ρ in (139) across diagrams in Fig. 5 (a) with that for ρ_s in (126) for Fig. 6: let $k \to \ell$ in Fig. 5 (a), and require

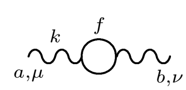
$$\frac{\ell^2}{m_f^2} = \rho_s \equiv \frac{k^2}{M_s^2} . \tag{146}$$

Noting that $\hat{\Pi}\left[QED\right]$ is a function of ρ only from (83) and using (146), the sum over fermions is given by

$$\sum_{f=1}^{n_f}$$



$$= \left[\left(\frac{1}{n_f}\sum_{f=1}^{n_f}\frac{m_f^2}{M_s^2}\right) \equiv 1\right] n_f \tag{147}$$



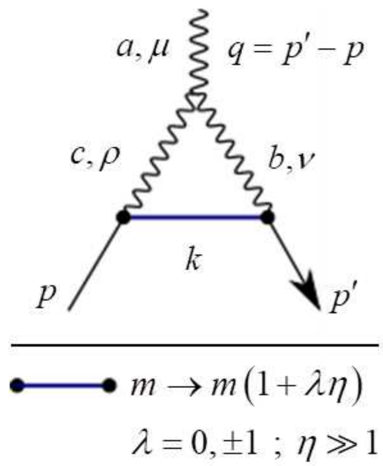


FIG. 7: Quark/three-gluon vertex.

The condition in brackets is obtained by factoring

$$T_{\mu\nu}\left(\ell^2\right) = \frac{m_f^2}{M_s^2} T_{\mu\nu}\left(k^2\right)$$

and comparing (147) with (141), then we have

$$M_s^2 = \frac{1}{n_f} \sum_{f=1}^{n_f} m_f^2 \tag{148}$$

for the reference mass. Evaluating (148) for quarks gives $M_s = 70.65\, GeV/c^2$; compare with Z-boson mass given in Appendix Table II.

Finally, the beta function is given by

$$\beta\left(\overline{g}\right) = 2 \frac{\partial \overline{g}}{\partial \ln \rho_s} = -\overline{g}^3 \left(11 - \frac{2}{3} n_f\right)$$

from which the running coupling constant (134) follows; see [55] for example.

B. Electroweak Corrections

We compute finite electroweak amplitudes using dimensionally regularized radiative corrections for unrenormalized (core) functions [12, 56, 57]. Core one-loop SE functions include $\Sigma^{ab}\,(ab = \gamma\gamma,\, \gamma Z,\, ZZ,\, WW)$ for bosons, Σ^f for fermions ($f = j\sigma$ for family j and doublet index $\sigma = \pm$), and vertex $\Lambda_\mu^{\gamma f}$. For repeated indices $a = b$, we abbreviate $\Sigma^b \equiv \Sigma^{bb}$, $b = \gamma,\, Z,\, W$; in general formulae applicable to $\gamma - Z$ mixing, we admit $b = \gamma Z$ as well for brevity. A subscript "sa" is appended to a stabilized amplitude $\hat{\Sigma}^b \equiv \hat{\Sigma}^b_{sa}$ when it is necessary to distinguish it from a corresponding renormalized amplitude $\hat{\Sigma}^b_{ra}$.

In Hollik's notation [12], the basic singular function

$$\Delta_\kappa = \frac{1}{\sigma} - \gamma - \ln \frac{m_\kappa^2}{\mu_\circ^2} + \ln 4\pi \tag{149}$$

differs from (75) by finite terms. For consistency, the input momentum to a loop is k with $s \equiv k^2$ for both bosons and fermions. Abbreviations for squared boson masses

$$z = m_Z^2, \quad w = m_W^2, \quad h = m_H^2$$

are used. In addition to (149), core amplitudes involve finite functions

$$\overline{B}_{\circ}\left(s,m_1,m_2\right)=-\int_0^1 dx\,\ln\left[\frac{x^2s-x\left(s+m_1^2-m_2^2\right)+m_1^2-i\epsilon}{m_1m_2}\right]\ , \tag{150}$$

$$F\left(s,m_1,m_2\right)=-1+\frac{m_1^2+m_2^2}{m_1^2-m_2^2}\ln\frac{m_1}{m_2}+\overline{B}_{\circ}\left(s,m_1,m_2\right)\ , \tag{151}$$

$$\overline{B}_1\left(s,m_1,m_2\right)=-\frac{1}{4}+\frac{m_1^2}{m_1^2-m_2^2}\ln\frac{m_1}{m_2}+\frac{m_2^2-m_1^2-s}{2s}F\left(s,m_1,m_2\right)\ , \tag{152}$$

and singular expressions

$$B_{\circ}\left(s,m_1,m_2\right)=\frac{1}{2}\left(\Delta_{m_1}+\Delta_{m_2}\right)+\overline{B}_{\circ}\left(s,m_1,m_2\right)\ \text{, and} \tag{153}$$

$$B_1\left(s,m_1,m_2\right)=-\frac{1}{2}\left(\Delta_{m_2}+\frac{1}{2}\right)+\overline{B}_1\left(s,m_1,m_2\right)\ . \tag{154}$$

Scalar one-loop integrals, including (150), are defined in [58].

1. *Boson SE corrections*

For these corrections, it is useful to expand the core boson SE

$$\Sigma^b\left(s\right)=\Sigma^b\left(m_b^2\right)+\sum_{n=1}^{\infty}\left.\frac{\partial^n\Sigma^b}{\partial s^n}\right|_{s=m_b^2}\left(s-m_b^2\right)^n\ . \tag{155}$$

From core amplitudes below, it can be seen by inspection and dimensional analysis that averages of $\Sigma^b\left(m_b^2\right)$ and $\left.\frac{\partial\Sigma^b}{\partial s}\right|_{s=m_b^2}$ over DCM states in (61) are invariant similarly to (54) – this may be shown in detail by applying (62) for the mass set $\{m_\kappa\}$ $\forall\kappa\epsilon I_m$, where

$$I_m\equiv\{f,\,l,\,W,\,Z\,,H,+,-\}$$

is a complete set of mass indices occurring in (169), (173), (178), and (180); the DCM transform is

$$\left\{m_\kappa^2\right\}\rightarrow\left\{m_\kappa^2\right\}\cdot\left(1+\lambda\eta^2\right)\ . \tag{156}$$

On the mass shell, the self-energy function has the general form

$$\Sigma^b\left(m_b^2\right)=\sum_\kappa\alpha_\kappa^b m_\kappa^2\,,$$

where α_κ^b are dimensionless coefficients which may depend on invariant mass ratios. Therefore, under (156)

$$\Sigma^b\left(m_b^2\right)\rightarrow\left(1+\lambda\eta^2\right)\Sigma^b\left(m_b^2\right)\ ,$$

and the average over DCM states

$$\begin{aligned}\left\langle\Sigma^b\left(m_b^2\right)\right\rangle_{dcm}&=\frac{1}{2}\sum_{\lambda=\pm1}\left(1+\lambda\eta^2\right)\Sigma^b\left(m_b^2\right)\\&=\Sigma^b\left(m_b^2\right)\end{aligned} \tag{157}$$

is stationary. Since the derivative $\left.\frac{\partial \Sigma^b}{\partial s}\right|_{s=m_b^2}$ is dimensionless, it is invariant under (156); in particular, partials

$$F'(s, m_1, m_2) = \frac{\partial F(s, m_1, m_2)}{\partial s} \tag{158}$$

occurring in $\left.\frac{\partial \Sigma^b}{\partial s}\right|_{s=m_b^2}$ transform as

$$F'\left(m_b^2, m_1, m_2\right) \to \left(1 + \lambda\eta^2\right)^{-1} F'\left(m_b^2, m_1, m_2\right) ,$$

and terms of form $g = m^2 F'\left(m_b^2, m_1, m_2\right)$ are again invariant under (156). Finally, higher order derivatives are either zero outright, or

$$\left.\frac{\partial^n \Sigma^b}{\partial s^n}\right|_{s=m_b^2} \sim \left(m_b^2\right)^{1-n} \to O\left(\eta^{2(1-n)}\right) \tag{159}$$

vanishes under (156) as $\eta \to \infty$ for $n \geq 2$. Therefore, (61) yields

$$\Sigma_{dcm}^b(s) = -\Sigma^b\left(m_b^2\right) - \left.\frac{\partial \Sigma^b}{\partial s}\right|_{s=m_b^2} \left(s - m_b^2\right) . \tag{160}$$

Since the off-shell factor $\left(s - m_b^2 = \delta k_{os}^2\right)$ is invariant under (156), the entire expression (160) is stationary under an average over DCM states similarly to (157). The net stabilized amplitude

$$\hat{\Sigma}^b(s) = \Sigma^b(s) + \Sigma_{dcm}^b(s) \tag{161}$$

from (60) satisfies

$$\hat{\Sigma}^b\left(m_b^2\right) = 0 \text{ , and} \tag{162}$$

$$\left.\frac{\partial \hat{\Sigma}^b(s)}{\partial s}\right|_{s=m_b^2} = 0 \, . \tag{163}$$

Taking the real part of (162) and (163) yields Denner's alternative renormalization conditions [59] to those given in [12]. For stabilized amplitude (161), (162) and (163) yield a propagator residue of unity so there is no need for external wave function corrections as in the on-shell renormalization scheme proposed by Ross and Taylor [60]; however, inclusion of Δr corrections discussed in Sec. VII B 4 leads to finite wave field corrections. Splitting off singular terms (149), the core boson SE can be expressed in the form

$$\Sigma^b(s) = \sum_\kappa \left[\alpha_\kappa^b s\Delta_\kappa + \beta_\kappa^b m_\kappa^2 \Delta_\kappa\right] + \Sigma_{finite}^b(s) \ , \tag{164}$$

where the sum over κ is $\forall \kappa \epsilon I_m$, and $\left\{\alpha_\kappa^b, \beta_\kappa^b\right\}$ are constant coefficients. Singular terms involving $\left\{s\Delta_\kappa,\, m_\kappa^2 \Delta_\kappa\right\}$ in Σ_{dcm}^b cancel those in Σ^b, and (161) reduces to

$$\hat{\Sigma}^b(s) = \Sigma_{finite}^b(s) - \Sigma_{finite}^b\left(m_b^2\right) - \left.\frac{\partial \Sigma_{finite}^b}{\partial s}\right|_{s=m_b^2} \left(s - m_b^2\right) . \tag{165}$$

For a free boson, the squared mass shift

$$\delta m_b^2 \equiv \mathrm{Re}\left[\Sigma_{finite}^b\left(m_b^2\right)\right] \tag{166}$$

represents the residual boson self-energy of the core after divergent parts of the vacuum response in $\Sigma_{dcm}^b\left(m_b^2\right)$ have canceled those in (164). For later reference, the polarization function is

$$\hat{\Pi}^b(s) = \frac{\hat{\Sigma}^b(s)}{s - m_b^2} = \frac{\Sigma^b(s) - \Sigma^b\left(m_b^2\right)}{s - m_b^2} - \left.\frac{\partial \Sigma^b}{\partial s}\right|_{s=m_b^2} . \tag{167}$$

Neglecting Δr corrections [31], mixing angle functions $c_w = \cos\theta_W$ (10) and $s_w = \sin\theta_W$, and neutral current constants (24) are invariant under (56). See (181) and (182) for inclusion of Δr.

Application of (161) to photon SE corrections shown in Fig. 8 yields

$$\begin{aligned}\hat{\Sigma}^{\gamma}(s) &= \Sigma^{\gamma}(s) + \Sigma^{\gamma}_{dcm}(s) \\ &= \frac{\alpha}{4\pi}\left\{\frac{4}{3}\sum_f Q_f^2\left[\left(s+2m_f^2\right)F(s,m_f,m_f) - \frac{s}{3}\right]\right. \\ &\quad \left. - (3s+4w)F(s,m_W,m_W) + \frac{2}{3}s\right\},\end{aligned} \tag{168}$$

$$\begin{aligned}\Sigma^{\gamma}(s) = \frac{\alpha}{4\pi}&\left\{\frac{4}{3}\sum_f Q_f^2\left[s\Delta_f + \left(s+2m_f^2\right)F(s,m_f,m_f) - \frac{s}{3}\right]\right. \\ &\left. - 3s\Delta_W - (3s+4w)F(s,m_W,m_W)\right\},\end{aligned} \tag{169}$$

$$\Sigma^{\gamma}_{dcm}(s) = -\Sigma^{\gamma}(0) - \left.\frac{\partial\Sigma^{\gamma}}{\partial s}\right|_{s=0} s\,, \tag{170}$$

where

$$\begin{aligned}\Sigma^{\gamma}(0) &= 0\,, \\ \left.\frac{\partial\Sigma^{\gamma}}{\partial s}\right|_{s=0} &= \frac{\alpha}{4\pi}\left\{\frac{4}{3}\sum_f Q_f^2\Delta_f - 3\Delta_W - \frac{2}{3}\right\},\end{aligned}$$

and the sum over fermions includes color for the case of quarks. Both $\hat{\Sigma}^{\gamma}(s)$ and $\hat{\Pi}^{\gamma}(s)$ vanish in the Thomson limit $s \to 0$, and physically meaningful corrections in (168) are due to incomplete cancellation for $s = k^2 \neq 0$. Singular terms in Σ^{γ}_{dcm} exactly cancel those in Σ^{γ} for all s, and there remains a term

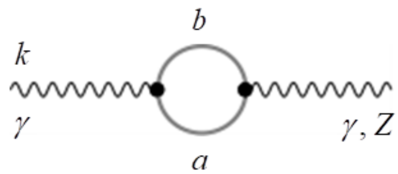
$$\left[\Sigma^{\gamma}_{dcm}\right]_{finite} = \left(\delta\alpha_{finite} \equiv \frac{\alpha}{6\pi}\right)s \tag{171}$$

in the vacuum response, where $\delta\alpha_{finite}$ is the finite part of renormalization constant δZ_2^{γ} in the usual theory.

For $\gamma - Z$ mixing corrections also represented in Fig. 8, we have

$$\begin{aligned}\hat{\Sigma}^{\gamma Z}(s) &= \Sigma^{\gamma Z} + \Sigma^{\gamma Z}_{dcm} \\ &= \frac{\alpha}{4\pi}\left\{-\frac{4}{3}\sum_f Q_f v_f\left[\left(s+2m_f^2\right)F(s,m_f,m_f) - \frac{s}{3}\right]\right. \\ &\quad \left. + \frac{1}{c_w s_w}\left[\left(3c_w^2+\frac{1}{6}\right)s + \left(4c_w^2+\frac{4}{3}\right)w\right]F(s,m_W,m_W) - \frac{s}{6c_w s_w}\left(4c_w^2+\frac{4}{3}\right)\right\},\end{aligned} \tag{172}$$

$$\begin{aligned}\Sigma^{\gamma Z}(s) = \frac{\alpha}{4\pi}&\left\{-\frac{4}{3}\sum_f Q_f v_f\left[s\Delta_f + \left(s+2m_f^2\right)F(s,m_f,m_f) - \frac{s}{3}\right]\right. \\ &+ \frac{1}{c_w s_w}\left[\left(3c_w^2+\frac{1}{6}\right)s + 2w\right]\Delta_W \\ &\left. + \frac{1}{c_w s_w}\left[\left(3c_w^2+\frac{1}{6}\right)s + \left(4c_w^2+\frac{4}{3}\right)w\right]F(s,m_W,m_W) + \frac{s}{9c_w s_w}\right\},\end{aligned} \tag{173}$$



a	b
$\bar{f}$	f
W	W
ϕ	W
ϕ	ϕ
$u^{\pm}$	$u^{\mp}$
†	W,ϕ

† **Four gauge boson vertex with internal W,ϕ**

FIG. 8: Photon self-energy and photon–Z mixing diagrams.

$$\Sigma^{\gamma Z}_{dcm} = -\Sigma^{\gamma Z}(0) - \left.\frac{\partial \Sigma^{\gamma Z}}{\partial s}\right|_{s=0} s\,, \tag{174}$$

where

$$\Sigma^{\gamma Z}(0) = \frac{\alpha}{4\pi}\left\{\frac{2w}{c_w s_w}\Delta_W\right\}\,,$$

$$\left.\frac{\partial \Sigma^{\gamma Z}}{\partial s}\right|_{s=0} = \frac{\alpha}{4\pi}\left\{-\frac{4}{3}\sum_f Q_f v_f \Delta_f + \frac{1}{c_w s_w}\left[\left(3c_w^2+\frac{1}{6}\right)\Delta_W + \frac{1}{6}\left(4c_w^2+\frac{4}{3}\right)+\frac{1}{9}\right]\right\}\,,$$

and $\Sigma^{\gamma Z}(0) \neq 0$ is due to non-Abelian boson loops in Fig. 8. For both photon SE and $\gamma - Z$ mixing corrections, we used the approximation [12]

$$F(s,m,m) \simeq \frac{s}{6m^2}$$

for small $s \ll m^2$, then the partials in (160) yield terms of the form $m^2 F'(s,m,m) = \frac{1}{6}$. Reverting to (155), the expansion of $F(s,m,m)$ about $s=0$ yields higher-order terms $m^2 O\left(\left[\frac{s}{m^2}\right]^2\right)$ in Σ^b for $b=\gamma,\gamma Z$ which vanish when dressed according to (156) in agreement with (159).

Renormalization starts with a bare charge $e_\circ$, and the correction [56]

$$\begin{aligned}\delta e\left(\Pi^\gamma, \Sigma^{\gamma Z}\right) &= e_\circ\left[\delta Z_1^\gamma - \frac{3}{2}\delta Z_2^\gamma\right] \\ &= e_\circ\left[\frac{1}{2}\Pi^\gamma(0) - \frac{s_w}{c_w}\frac{\Sigma^{\gamma Z}(0)}{m_Z^2}\right]\end{aligned} \tag{175}$$

renormalizes the charge $e = e_\circ + \delta e$, where

$$\begin{aligned}\delta Z_1^\gamma &= -\Pi^\gamma(0) - \frac{s_w}{c_w}\frac{\Sigma^{\gamma Z}(0)}{m_Z^2}\,, \text{ and} \\ \delta Z_2^\gamma &= -\Pi^\gamma(0)\end{aligned}$$

are the charge and photon field renormalization constants, respectively. In the usual theory, arguments on the left in (175) are core functions $\{\Pi^{\gamma}, \varSigma^{\gamma Z}\}$; however, in the stabilized theory, we utilize the complete amplitudes (168) and (172) to obtain

$$\delta e\left(\hat{\Pi}^{\gamma}, \hat{\Sigma}^{\gamma Z}\right) = 0 \,. \tag{176}$$

Therefore, $e = e_{\circ}$, and there is no charge renormalization. Any redistribution of vacuum charge which shifts the energy is just a natural consequence of the stability requirement: Charge e acts on the vacuum $|0\rangle \rightarrow |0_p\rangle$, and the polarized vacuum $|0_p\rangle$ acts back on e shifting its energy as illustrated in Fig. 1.

For the remaining amplitudes, we list only the net and core since DCM expressions, although lengthy, are easily evaluated using analytic expressions for partials (158) derived from (150), (151), and integral tables. For Z-boson self-energy corrections shown in Fig. 9, we have

$$\hat{\Sigma}^{Z}(s) = \Sigma^{Z}(s) - \Sigma^{Z}(z) - \left.\frac{\partial \varSigma^{Z}}{\partial s}\right|_{s=z}(s-z) \,, \tag{177}$$

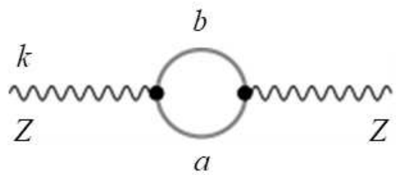
where

$$\begin{aligned}
\Sigma^{Z}(s) = \frac{\alpha}{4\pi}\Bigg\{ & \frac{4}{3}\sum_{l=e,\mu,\tau} 2a_l^2 s\left[\varDelta_l + \frac{5}{3} - \ln\left(-\frac{s}{m_l^2} - i\epsilon\right)\right] \\
& + \frac{4}{3}\sum_{f\neq\nu}\left[\left(v_f^2 + a_f^2\right)\left(s\varDelta_f + \left(s + 2m_f^2\right)F(s, m_f, m_f) - \frac{s}{3}\right)\right. \\
& \left. - \frac{3}{8c_w^2 s_w^2} m_f^2\left(\varDelta_f + F(s, m_f, m_f)\right)\right] \\
& + \left[\left(3 - \frac{19}{6s_w^2} + \frac{1}{6c_w^2}\right)s + \left(4 + \frac{1}{c_w^2} - \frac{1}{s_w^2}\right)m_Z^2\right]\varDelta_W \\
& + \left[\left(-c_w^4\left(40s + 80w\right) + \left(c_w^2 - s_w^2\right)^2\left(8w + s\right) + 12w\right)F(s, m_W, m_W)\right. \\
& + \left(10z - 2h + s + \frac{(h-z)^2}{s}\right)F(s, m_H, m_Z) - 2h\ln\frac{h}{w} - 2z\ln\frac{z}{w} \\
& + (10z - 2h + s)\left(1 - \frac{h+z}{h-z}\ln\frac{m_H}{m_Z} - \ln\frac{m_H m_Z}{w}\right) \\
& \left. + \frac{2}{3}s\left(1 + \left(c_w^2 - s_w^2\right)^2 - 4c_w^2\right)\right]\frac{1}{12c_w^2 s_w^2}\Bigg\} \,.
\end{aligned} \tag{178}$$

The first sum in (178) includes leptons (l) only, and the second excludes neutrinos (ν).

For W-boson self-energy corrections shown in Fig. 10,

$$\hat{\Sigma}^{W}(s) = \Sigma^{W}(s) - \Sigma^{W}(w) - \left.\frac{\partial \varSigma^{W}}{\partial s}\right|_{s=w}(s-w) \,, \tag{179}$$



a	b
$\bar{f}$	f
W	W
ϕ	W
H	Z
χ	H
ϕ	ϕ
$u^{\pm}$	$u^{\mp}$
†	W, H, χ, ϕ

FIG. 9: Z-boson self-energy.

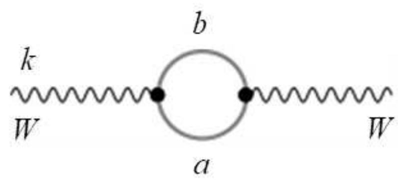
where

$$\begin{aligned}\Sigma^{W}(s) = \frac{\alpha}{4\pi}\frac{1}{s_w^2}&\left\{\frac{1}{3}\sum_{l=e,\mu,\tau}\left[\left(s-\frac{3}{2}m_l^2\right)\Delta_l+\left(s-\frac{m_l^2}{2}-\frac{m_l^4}{2s}\right)F(s,0,m_l)+\frac{2}{3}s-\frac{m_l^2}{2}\right]\right.\\
&+\sum_{q-doublets}\frac{1}{3}\left[\frac{\Delta_+}{2}\left(s-\frac{5}{2}m_+^2+\frac{m_-^2}{2}\right)+\frac{\Delta_-}{2}\left(s-\frac{5}{2}m_-^2-\frac{m_+^2}{2}\right)\right.\\
&+\left(s-\frac{m_+^2+m_-^2}{2}-\frac{\left(m_+^2-m_-^2\right)^2}{2s}\right)F(s,m_+,m_-)\\
&-\left.\left(s-\frac{m_+^2+m_-^2}{2}\right)\left(1-\frac{m_+^2+m_-^2}{m_+^2-m_-^2}\ln\frac{m_+}{m_-}\right)-\frac{s}{3}\right]-\left[\frac{19}{2}s+3w\left(1-\frac{s_w^2}{c_w^2}\right)\right]\frac{\Delta_W}{3}\\
&+\left[s_w^4 z-\frac{c_w^2}{3}\left(7z+7w+10s-2\frac{(z-w)^2}{s}\right)-\frac{1}{6}\left(w+z-\frac{s}{2}-\frac{(z-w)^2}{2s}\right)\right]F(s,m_Z,m_W)\\
&+\frac{s_w^2}{3}\left(-4w-10s+\frac{2w^2}{s}\right)F(s,0,m_W)+\frac{1}{6}\left(5w-h+\frac{s}{2}+\frac{(h-w)^2}{2s}\right)F(s,m_H,m_W)\\
&+\left[\frac{c_w^2}{3}\left(7z+7w+10s-4(z-w)\right)-s_w^4 z+\frac{1}{6}\left(2w-\frac{s}{2}\right)\right]\frac{3z}{z-w}\ln\frac{z}{w}\\
&-\left(\frac{2}{3}w+\frac{s}{12}\right)\frac{h}{h-w}\ln\frac{h}{w}-\frac{c_w^2}{3}\left(7z+7w+\frac{32}{3}s\right)+s_w^4 z\\
&\left.+\frac{1}{6}\left(\frac{5}{3}s+4w-z-h\right)-\frac{s_w^2}{3}\left(4w+\frac{32}{3}s\right)\right\},\end{aligned} \tag{180}$$

and m_+ and m_- are masses for upper and lower components of a quark doublet, respectively.

Self-energies for diagrams with $b=\{\gamma Z,\, Z,\, W\}$ require adjustments

$$\left\{\hat{\Sigma}^b(s)\to\hat{\Sigma}^b(s)+\left(s-m_b^2\right)\Delta r^b\,,\; b=W,Z\right\}\,,\ \text{and} \tag{181}$$



a	b
f^-	f^+
W	Z
ϕ	Z
W	γ
ϕ	γ
H	W
ϕ	H
ϕ	χ
u^Z	$u^\pm$
u^γ	$u^\pm$
$\dagger$	Z, W, H, χ, ϕ

FIG. 10: W-boson self-energy.

$$\hat{\Sigma}^{\gamma Z}(s) \to \hat{\Sigma}^{\gamma Z}(s) + s\Delta r^{\gamma Z} \tag{182}$$

for Δr corrections [31] which account for variations of $\{g_W, g_Z\}$ with respect to m_W and m_Z; we have

$$\left\{\Delta r^{\gamma Z},\, \Delta r^Z,\, \Delta r^W\right\} = \left\{-\frac{c_w}{s_w},\, \frac{c_w^2 - s_w^2}{s_w^2},\, \frac{c_w^2}{s_w^2}\right\}\left(\frac{\delta m_Z^2}{m_Z^2} - \frac{\delta m_W^2}{m_W^2}\right), \tag{183}$$

wherein finite-on-shell-mass shifts from (166) are

$$\delta m_Z^2 = \mathrm{Re}\left[\Sigma_{finite}^Z\left(m_Z^2\right)\right], \text{ and} \tag{184}$$
$$\delta m_W^2 = \mathrm{Re}\left[\Sigma_{finite}^W\left(m_W^2\right)\right]. \tag{185}$$

In Sec. VII B 4, we derive Δr^b using stability arguments. Values for squared mass ratios $\left\{\frac{\delta m_Z^2}{m_Z^2},\, \frac{\delta m_W^2}{m_W^2}\right\}$ and Δr are given in Appendix Table II.

Net amplitudes in (168), (172), (177), and (179) for boson self-energies are finite and satisfy required mass shell conditions (162) and (163) for $b = \{\gamma,\, \gamma Z,\, Z,\, W\}$. Amplitude $\hat{\Sigma}^\gamma$ agrees with the result given in Hollik [12]; however, $\left\{\hat{\Sigma}^{\gamma Z},\, \hat{\Sigma}^Z,\, \hat{\Sigma}^W\right\}$ including Δr corrections differ from Hollik's results in two respects:

a) a small finite charge renormalization $\frac{\alpha}{6\pi} = 3.87 \times 10^{-4}$ from (171) is absent in $\left\{\hat{\Sigma}^Z,\, \hat{\Sigma}^W\right\}$, and
b) they include polarization derivative shifts in (167) – finite parts are given in Appendix Table II.

As regards item a), inclusion of any charge renormalization would be inconsistent with the stability approach and result (176) in particular. For item b), finite parts differ depending on the renormalization scheme, and $\left\{\hat{\Sigma}^{\gamma Z},\, \hat{\Sigma}^Z,\, \hat{\Sigma}^W\right\}$ are consistent with the scheme given in [59]; moreover, all four boson self-energies are unified under the same formula (161). Numerical results for boson polarization functions are given in the Appendix.

For fermion self-energy corrections, we again expand the core amplitude

$$\Sigma^f(k) = \Sigma^f(m_f) + \left.\frac{\partial \Sigma^f}{\partial \not{k}}\right|_{\not{k}=m_f} (\not{k} - m_f) + H.O.T.\,. \tag{186}$$

From (62), the DCM transform is

$$\{m_\kappa\} \to \{m_\kappa\} \cdot (1 + \lambda\eta) \ , \tag{187}$$

where the mass set $\{m_\kappa\} \subset \{m_f,\, m_W,\, m_Z,\, \mu\}$ corresponds to terms in (192). Upon applying (61) to (186) and noting that $\left\{B_i\left(m_f^2, m_1, m_2\right);\ i = 0, 1\right\}$ occurring in (192) are invariant under (187) applied to all mass arguments, we obtain

$$\left.\Sigma^f_{dcm}(k)\right|_{\not{k}=m_f} = -\Sigma^f(m_f) \ . \tag{188}$$

From arguments similar to those for boson self-energies above, $\not{k} - m_f$ and its dimensionless coefficient (first partial) in (186) are also invariant under (187). The first partial involves derivatives of $B_\circ$ (153) and B_1 (154). Finally, higher-order terms in (186) vanish under (187); therefore, the DCM amplitude is

$$\Sigma^f_{dcm}(k) = -\Sigma^f(m_f) - \left.\frac{\partial \Sigma^f}{\partial \not{k}}\right|_{\not{k}=m_f} (\not{k} - m_f) \ ; \tag{189}$$

compare with (160). The net amplitude

$$\hat{\Sigma}^f(k) = \Sigma^f(k) + \Sigma^f_{dcm}(k) \tag{190}$$

satisfies the expected mass shell condition

$$\left.\hat{\Sigma}^f(k)\right|_{\not{k}=m_f} = 0\,. \tag{191}$$

For the corrections shown in Fig. 11, the core amplitude [56] is

$$\Sigma^f(k) = \not{k}\Sigma^f_V\left(k^2\right) + \not{k}\gamma_5\Sigma^f_A\left(k^2\right) + m_f\Sigma^f_S\left(k^2\right) \ , \tag{192}$$

where

$$\Sigma^f_V = -\frac{\alpha}{4\pi}\left\{Q_f^2\left[2B_1\left(k^2; m_f, \mu\right) + 1\right] + \left(v_f^2 + a_f^2\right)\left[2B_1\left(k^2; m_f, m_Z\right) + 1\right] + \frac{1}{4s_w^2}\left[2B_1\left(k^2; m_f, m_W\right) + 1\right]\right\} \ ,$$

$$\Sigma^f_A = -\frac{\alpha}{4\pi}\left\{2v_f a_f\left[2B_1\left(k^2; m_f, m_Z\right) + 1\right] - \frac{1}{4s_w^2}\left[2B_1\left(k^2; m_{f'}, m_W\right) + 1\right]\right\} \ , \text{ and}$$

$$\Sigma^f_S = -\frac{\alpha}{4\pi}\left\{Q_f^2\left[4B_\circ\left(k^2; m_f, \mu\right) - 2\right] + \left(v_{i\sigma}^2 - a_{i\sigma}^2\right)\left[4B_\circ\left(k^2; m_f, m_Z\right) - 2\right]\right\} \ .$$

Substituting the vector $\left(V_{core} = \not{k}\Sigma^f_V\right)$, axial $\left(A_{core} = \not{k}\gamma_5\Sigma^f_A\right)$, and scalar $\left(S_{core} = m_f\Sigma^f_S\right)$ parts of (192) into (189) , we obtain

$$\Sigma^f_{dcm}(k) = V_{dcm}(k) + A_{dcm}(k) + S_{dcm}(k) \ , \tag{193}$$

where

$$
\begin{aligned}
V_{dcm} &= -\not{k}\Sigma_V^f\left(m^2\right) - 2m_f^2 \left.\frac{\partial \Sigma_V^f}{\partial k^2}\right|_{k^2=m_f^2}\left(\not{k}-m_f\right) , \\
A_{dcm} &= \gamma_5 \not{k} \Sigma_A^f\left(m_f^2\right) \text{ , and} \\
S_{dcm} &= -m_f \Sigma_S^f\left(m^2\right) - 2m_f^2 \left.\frac{\partial \Sigma_S^f}{\partial k^2}\right|_{k^2=m_f^2}\left(\not{k}-m_f\right) .
\end{aligned}
$$

The identity

$$
\frac{\partial \Sigma_J^f}{\partial \not{k}} = 2\not{k}\frac{\partial \Sigma_J^f}{\partial k^2}
$$

has been used to evaluate derivatives for $J = \{V, A, S\}$. For the derivative of A_{core}, we have replaced $\not{k}\gamma_5 = -\gamma_5\not{k}$ so $\not{k}$ stands to the right as required by (189); one finds

$$
\frac{\partial}{\partial \not{k}}\left[-\gamma_5\not{k}\Sigma_A^f\right] = -\gamma_5\Sigma_A^f ,
$$

where the symmetrized expression for the derivative

$$
\begin{aligned}
\frac{\partial}{\partial \not{k}}\left[\gamma_5\Sigma_A^f\right] &= \frac{1}{2}\frac{\partial}{\partial \not{k}}\left[\gamma_5\Sigma_A^f + \Sigma_A^f\gamma_5\right] \\
&= \frac{\partial \Sigma_A^f}{\partial k^2}\left(\gamma_5\not{k} + \not{k}\gamma_5\right) = 0
\end{aligned}
$$

has also been employed. Collecting terms, the net amplitude (190) reduces to

$$
\hat{\Sigma}^f(k) = \not{k}\hat{\Sigma}_V\left(k^2\right) + \not{k}\gamma_5\hat{\Sigma}_A\left(k^2\right) + m_f\hat{\Sigma}_S\left(k^2\right) , \tag{194}
$$

where

$$
\begin{aligned}
\hat{\Sigma}_V\left(k^2\right) &= \Sigma_V^f\left(k^2\right) - \Sigma_V^f\left(m_f^2\right) - 2m_f^2 \left.\frac{\partial \Sigma_{VS}^f}{\partial k^2}\right|_{k^2=m_f^2} , \\
\hat{\Sigma}_A\left(k^2\right) &= \Sigma_A^f\left(k^2\right) - \Sigma_A^f\left(m_f^2\right) , \\
\hat{\Sigma}_S\left(k^2\right) &= \Sigma_S^f\left(k^2\right) - \Sigma_S^f\left(m_f^2\right) + 2m_f^2 \left.\frac{\partial \Sigma_{VS}^f}{\partial k^2}\right|_{k^2=m_f^2} ,
\end{aligned}
$$

and $\Sigma_{VS}^f = \Sigma_V^f + \Sigma_S^f$. Using formulae in [12, 56], the renormalization constants are

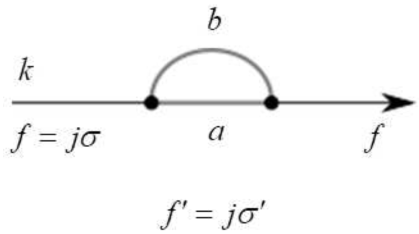
$$
\begin{aligned}
\delta Z_V &= -\Sigma_V^f\left(m_f^2\right) - 2m_f^2 \left.\frac{\partial \Sigma_{VS}^f}{\partial k^2}\right|_{k^2=m_f^2} , \\
\delta Z_A &= \Sigma_A^f\left(m_f^2\right) , \\
\delta m_f &= m_f\Sigma_S^f\left(m_f^2\right) ,
\end{aligned}
$$

and it can be seen that the result (194) agrees precisely with that obtained from renormalization. Numerical results for fermion self-energy functions for an electron are given in Appendix Fig. 17.

3. Vertex corrections

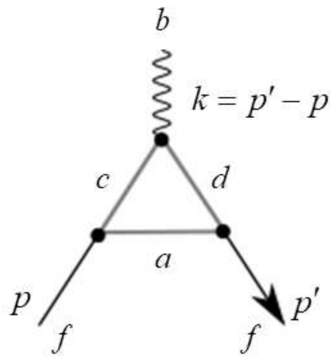
Consider the vertex corrections shown in Fig. 12; in the small fermion mass limit [57], only vector and axial vector terms contribute, and the core amplitude is

$$
\Lambda_\mu^{\gamma f}\left(k^2, m_f\right) = \gamma_\mu\Lambda_V^{\gamma f}\left(k^2, m_f\right) - \gamma_\mu\gamma_5\Lambda_A^{\gamma f}\left(k^2, m_f\right) , \tag{195}
$$



a	b
f	γ
f	Z
f′	W⁺
f′	φ⁺

FIG. 11: Fermion self-energy.



a	b	c	d
γ	γ	f	f
Z	γ	f	f
γ	Z	f	f
Z	Z	f	f
W⁺	γ	f′	f′
f′	Z	W⁻	W⁺
φ⁺	γ	f′	f′
f′	γ	φ⁻	φ⁺
f′	γ	φ⁻	W⁺
f′	γ	W⁻	φ⁺

FIG. 12: Vertex corrections.

where $k^2 = (p' - p)^2$. The functions

$$\Lambda_{V,A}^{\gamma f}\left(k^2, m_f\right) = \Lambda_{V,A}^{\gamma f}\left(0, m_f\right) + F_{V,A}^{\gamma f}\left(\frac{k^2}{m_f^2}\right) \tag{196}$$

involve singular parts at $k^2 = 0$ and finite form factors $F_{V,A}^{\gamma f}$ which vanish at $k^2 = 0$. Detailed expressions for the functions are given in [12]. Applying (61) and (62), the form factors $F_{V,A}^{\gamma f}\left(\frac{k^2}{m_f^2}\right)$ in (196) vanish as $\eta \to \infty$ in $m_f(\eta) = m_f(1 + \lambda\eta)$; therefore, the DCM vertex is

$$\left[\Lambda_\mu^{\gamma f}\right]_{dcm} = -\left[\gamma_\mu \Lambda_V^{\gamma f}\left(0, m_f\right) - \gamma_\mu\gamma_5 \Lambda_A^{\gamma f}\left(0, m_f\right)\right] , \tag{197}$$

and the net vertex amplitude (60) reduces to the expected result from renormalization

$$\hat{\Lambda}_\mu^{\gamma f} = \gamma_\mu F_V^{\gamma f}\left(\frac{k^2}{m_f^2}\right) - \gamma_\mu\gamma_5 F_A^{\gamma f}\left(\frac{k^2}{m_f^2}\right) . \tag{198}$$

4. *Wave field renormalization and Δr corrections*

In the stabilized theory, Δr factors for $\{W,\, Z\}$ follow easily from the constancy of the electrical charge; squaring (11), taking variations

$$\delta e^2 = \delta g_W^2 s_w^2 + g_W^2 \delta s_w^2 = 0 \tag{199}$$

$$= \delta g_B^2 c_w^2 + g_B^2 \delta c_w^2 = 0\,, \tag{200}$$

and using (10), the quadratic coupling deltas are

$$\delta g_W^2 = -g_W^2 \left[\Delta r^W \equiv \frac{c_w^2}{s_w^2} \left(\frac{\delta m_Z^2}{m_Z^2} - \frac{\delta m_W^2}{m_W^2} \right) \right]\,, \tag{201}$$

$$\begin{aligned} \delta g_Z^2 &= \delta g_W^2 + \delta g_B^2 \\ &= -g_Z^2 \left[\Delta r^Z \equiv \frac{c_w^2 - s_w^2}{s_w^2} \left(\frac{\delta m_Z^2}{m_Z^2} - \frac{\delta m_W^2}{m_W^2} \right) \right]\,, \end{aligned} \tag{202}$$

where

$$\delta g_B^2 = g_B^2 \left(\frac{\delta m_Z^2}{m_Z^2} - \frac{\delta m_W^2}{m_W^2} \right)\,.$$

A similar correction for $\gamma - Z$ mixing may be derived in the manner of [31] by varying the coupling constants in the Lagrangian term

$$\mathcal{L}_{\mathrm{Z}}^{\mathrm{m}}\left(g_W, g_B\right) = \frac{v^2}{2}\left(g_W W_\mu^3 + g_B B_\mu\right)^2$$

taking care to vary only the factors $\{g_W,\, g_B\}$ of the baseline fields $\left\{W_\mu^3,\, B_\mu\right\}$. Upon taking the variation

$$\delta\mathcal{L}_{\mathrm{Z}}^{\mathrm{m}}\left(g_W, g_B\right) = \frac{\partial \mathcal{L}_{\mathrm{Z}}^{\mathrm{m}}}{\partial g_W}\delta g_W + \frac{\partial \mathcal{L}_{\mathrm{Z}}^{\mathrm{m}}}{\partial g_B}\delta g_B$$

using (9) to re-express in terms of $\{Z_\mu,\, A_\mu\}$, and considering only terms involving $Z_\mu A^\mu$, one obtains

$$\delta\mathcal{L}_{\gamma \mathrm{Z}}^{\mathrm{m}} = \frac{1}{2}\delta m_{\gamma Z}^2 Z_\mu A^\mu\,,$$

where

$$\delta m_{\gamma Z}^2 = -m_Z^2 \left[\Delta r^{\gamma Z} \equiv -\frac{c_w}{s_w} \left(\frac{\delta m_Z^2}{m_Z^2} - \frac{\delta m_W^2}{m_W^2} \right) \right]\,.$$

Using (14) and defining $\delta m_{\gamma Z}^2 \equiv \frac{1}{4}\delta g_{\gamma Z}^2 v^2$, we have

$$\delta g_{\gamma Z}^2 = -g_Z^2 \Delta r^{\gamma Z}\,. \tag{203}$$

Therefore, we expect free field propagator modifications of the form

$$\frac{1}{k^2 - m_b^2} \rightarrow \frac{1 - \Delta r^b}{k^2 - m_b^2}\,,\ b = W,\, Z$$

resulting in small departures of the propagator residue from unity. To nail down the propagator modification for $\gamma - Z$ mixing, one tries an average

$$\frac{1}{2}\left(\frac{\delta Z_{Z\gamma}}{k^2} + \frac{\delta Z_{\gamma Z}}{k^2 - m_Z^2} \right)$$

and uses the renormalization method reviewed below to relate the coefficients $\{\delta Z_{Z\gamma},\, \delta Z_{\gamma Z}\}$ to $\Delta r^{\gamma Z}$.

Standard renormalization theory (SRT) introduces mass and wave field renormalization constants to construct finite S-matrix elements and Green's functions. Boson self-energy and $\gamma - Z$ mixing propagators are

$$D^b_{\mu\nu}(k) = -ig_{\mu\nu}\left\{\frac{1}{k^2-m_b^2} - \frac{1}{k^2-m_b^2}\left[\hat{\Sigma}^b_{ra}\left(k^2,\,\Sigma^b\right)\right]\frac{1}{k^2-m_b^2}+...\right\},\; b=\gamma,\,W,\,Z \tag{204}$$
$$= \frac{-ig_{\mu\nu}}{k^2-m_b^2+\hat{\Sigma}^b_{ra}\left(k^2,\,\Sigma^b\right)}\text{ , and}$$

$$D^{\gamma Z}_{\mu\nu}(k) = ig_{\mu\nu}\left\{\frac{1}{2}\left(\frac{\delta Z_{Z\gamma}}{k^2}+\frac{\delta Z_{\gamma Z}}{k^2-m_Z^2}\right)+\frac{1}{k^2}\Sigma^{\gamma Z}\left(k^2\right)\frac{1}{k^2-m_Z^2}\right\} \tag{205}$$
$$= ig_{\mu\nu}\frac{1}{k^2}\left[\hat{\Sigma}^{\gamma Z}_{ra}\left(k^2,\,\Sigma^{\gamma Z}\right)\right]\frac{1}{k^2-m_Z^2}\,,$$

where (204) includes iterations with $b \equiv bb$, and the renormalized amplitudes are given by [12, 61]

$$\hat{\Sigma}^{\gamma}_{ra}\left(k^2,\,\Pi^\gamma\right) = \Sigma^\gamma\left(k^2\right)+k^2\delta Z_\gamma \equiv k^2\left[\Pi^\gamma\left(k^2\right)+\delta Z_\gamma\right]\,, \tag{206}$$
$$\hat{\Sigma}^{\gamma Z}_{ra}\left(k^2,\,\Sigma^{\gamma Z}\right) = \Sigma^{\gamma Z}\left(k^2\right)+\frac{1}{2}\left[\delta Z_{\gamma Z}k^2+\delta Z_{Z\gamma}\left(k^2-m_Z^2\right)\right]\,, \tag{207}$$
$$\hat{\Sigma}^{Z}_{ra}\left(k^2,\,\Sigma^{Z}\right) = \Sigma^{Z}\left(k^2\right)-\delta M_Z^2+\delta Z_Z\left(k^2-m_Z^2\right)\text{ , and} \tag{208}$$
$$\hat{\Sigma}^{W}_{ra}\left(k^2,\,\Sigma^{W}\right) = \Sigma^{W}\left(k^2\right)-\delta M_W^2+\delta Z_W\left(k^2-m_W^2\right)\,, \tag{209}$$

where δZ_Z and δZ_W are displacements of the field renormalization constants

$$Z_b = 1+\delta Z_b\,,\; b = Z, W$$

from unity. From field renormalization relations

$$W_{\circ\mu} = \left[Z_W^{1/2} \simeq 1+\frac{1}{2}\delta Z_W\right]W_\mu\,,$$
$$B_{\circ\mu} = \left[Z_B^{1/2} \simeq 1+\frac{1}{2}\delta Z_B\right]B_\mu\,,$$

and (9), the physical fields satisfy

$$\begin{bmatrix} Z_{\circ\mu} \\ A_{\circ\mu}\end{bmatrix} = \begin{bmatrix} 1+\frac{1}{2}\delta Z_Z & \frac{1}{2}\delta Z_{Z\gamma} \\ \frac{1}{2}\delta Z_{\gamma Z} & 1+\frac{1}{2}\delta Z_\gamma\end{bmatrix}\begin{bmatrix} Z_\mu \\ A_\mu\end{bmatrix}\,,$$

where the subscript "$\circ$" denotes bare, as opposed to renormalized quantities, and the renormalization constants satisfy [60]

$$\begin{bmatrix} \delta Z_Z \\ \delta Z_\gamma\end{bmatrix} = \begin{bmatrix} c_w^2 & s_w^2 \\ s_w^2 & c_w^2\end{bmatrix}\begin{bmatrix} \delta Z_W \\ \delta Z_B\end{bmatrix}\,,$$
$$\delta Z_{Z\gamma} = -s_w c_w\left(\delta Z_W-\delta Z_B\right)-\Delta r^{\gamma Z}\text{ , and}$$
$$\delta Z_{\gamma Z} = -s_w c_w\left(\delta Z_W-\delta Z_B\right)+\Delta r^{\gamma Z}\,.$$

Ordinarily $\Sigma^b \equiv \Sigma^b_{core}$ in (206)-(209); however, with the stabilized amplitudes at our disposal, we are free to replace Σ^b with $\hat{\Sigma}^b_{sa} = \hat{\Sigma}^b$ from (165) to determine all renormalization constants. Applying mass shell renormalization (stability) conditions

$$\hat{\Pi}^\gamma_{ra}\left(0,\,\hat{\Pi}^\gamma_{sa}\right) = 0\,,$$
$$\hat{\Sigma}^{\gamma Z}_{ra}\left(0,\,\hat{\Sigma}^{\gamma Z}_{sa}\right) = 0\,,$$
$$\hat{\Sigma}^{Z}_{ra}\left(m_Z^2,\,\hat{\Sigma}^{Z}_{sa}\right) = 0\text{ , and}$$
$$\hat{\Sigma}^{W}_{ra}\left(m_W^2,\,\hat{\Sigma}^{W}_{sa}\right) = 0\,,$$

Table I: Renormalization constants

	Σ^b	δM_b^2 , $b = Z, W$	δZ_γ	$\delta Z_{Z\gamma}$	$\delta Z_{\gamma Z}$	δZ_W	δZ_Z
Stability	$\hat{\Sigma}^b_{sa}$	0	0	0	$2\Delta r^{\gamma Z}$	Δr^W	Δr^Z
SRT	Σ^b_{core}	$Re\left[\Sigma^b\left(m_b^2\right)\right]$	$-\Pi^\gamma\left(0\right)$	$\frac{2\Sigma^{\gamma Z}(0)}{m_Z^2}$	$\frac{2\Sigma^{\gamma Z}(0)}{m_Z^2} + 2\Delta r^{\gamma Z}$	$-\Pi^\gamma\left(0\right) + \Delta r^W + \frac{c_w}{s_w}\frac{2\Sigma^{\gamma Z}(0)}{m_Z^2}$	$-\Pi^\gamma\left(0\right) + \Delta r^Z + \frac{c_w^2 - s_w^2}{s_w c_w}\frac{2\Sigma^{\gamma Z}(0)}{m_Z^2}$

only finite wave field corrections for Δr shown in Table I are non-zero; SRT results are included for comparison. Referring to (205), the stability result $\delta Z_{Z\gamma} = 0$ means that the photon propagator has no Z-component

$$\frac{\delta Z_{Z\gamma}}{k^2} = 0\,;$$

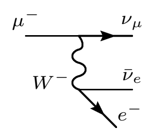
consequently, there is no direct coupling between the photon and a neutral current J_{NC} for $\gamma - Z$ mixing – not even an infinite one. On the other hand, an electromagnetic current couples to J_{NC} via the Z with amplitude

$$\frac{1}{2}\delta Z_{\gamma Z} = \Delta r^{\gamma Z}$$

as originally suggested by (203).

5. *Muon Decay and Δr corrections*

In the Born approximation, the muon decay amplitude corresponds to a Feynman diagram



in the Standard Model. The resulting decay rate [12]

$$\Gamma_\mu^\circ = \frac{\alpha^2}{384\pi}\frac{m_\mu^5}{s_w^4 m_W^4}\left(1 - \frac{8m_e^2}{m_\mu^2}\right)$$

when reconciled with the Fermi contact model prediction

$$\Gamma_\mu^F = \frac{G_F^2 m_\mu^5}{192\pi^3}\left(1 - \frac{8m_e^2}{m_\mu^2}\right)\,,$$

yields the Fermi constant in lowest order

$$G_F^\circ = \frac{\pi\alpha}{\sqrt{2}s_w^2 m_W^2}\,. \tag{210}$$

With higher-order QED corrections [62, 63],

$$\frac{1}{\tau_\mu} = \frac{G_F^2 m_\mu^5}{192\pi^3} f\left(\frac{m_e^2}{m_\mu^2}\right)\left(1 + \Delta_{QED}\right)$$

defines G_F in terms of the precisely measured muon lifetime τ_μ, where

$$f\left(x\right) = 1 - 8x - 12x^2\ln x + 8x^3 - x^4\,,\text{ and}$$
$$\Delta_{QED} = \frac{\alpha}{2\pi}\left(\frac{25}{4} - \pi^2\right) + O\left(\alpha^2\right)\,.$$

In addition to the one-loop correction shown in Δ_{QED}, $O\left(\alpha^2\right)$ corrections for two-loops are also known [64–66]. These QED corrections involve several renormalization schemes; however, the corresponding stabilized QED corrections are finite without renormalization as shown in Sections V-VI. Stability corrections for vacuum polarization involve a subtraction of the form (83) at $k^2 = 0$ and are therefore equivalent to the on-shell renormalization scheme. For other renormalization schemes; for example, the modified minimal subtraction $\overline{MS}$, Δ_{QED} involves a coupling constant renormalization. Ritbergen [64] gives a prescription

$$\alpha\left(m_\mu\right) = \frac{\alpha}{1 - \frac{\alpha}{3\pi}\ln\frac{m_\mu^2}{m_e^2}} + O\left(\alpha^3\right) \tag{211}$$

relating the $\overline{MS}$ coupling constant $\alpha\left(m_\mu\right)$ to the on-shell value $\alpha = 1/137.035999139(31)$ [15]. However, from foregoing results (87) and (176), the prescription (211) does not represent an intrinsic renormalization of electrical charge in the stabilized theory.

Electroweak corrections to the muon lifetime involve Δr corrections to the Fermi constant [12, 31, 67]

$$G_F = G_F^\circ\left[1 + \Delta r\right] , \tag{212}$$

where after renormalization

$$\Delta r = -\Delta r^W - \frac{\delta m_W^2}{m_W^2} + \frac{\hat{\Sigma}^w(0)}{m_W^2} + \Delta r^{[\text{vertex, box}]} \text{ with} \tag{213}$$

$$\Delta r^{[\text{vertex, box}]} = \frac{\alpha}{4\pi s_w^2}\left(6 + \frac{7 - 4s_w^2}{2s_w^2}\ln c_w^2\right) .$$

From a stability perspective, the first two terms of (213) are due to finite mass shifts (184) and (185); taking into account (176), variation of (210) yields

$$\delta G_F^\circ = -G_F^\circ\left[\Delta r^W + \frac{\delta m_W^2}{m_W^2}\right] . \tag{214}$$

In standard renormalization theory, bare parameters $\{\alpha_\circ,\, s_w^\circ,\, m_w^\circ\}$ replace those in (210), and the expression for Δr includes a charge renormalization term $\delta\alpha_\circ$ which is subsequently incorporated into a renormalized coupling.

VIII. CONCLUDING REMARKS

In this paper, we developed a model for a stable electrical charge wherein a hidden interaction between the electromagnetically dressed charge and an opposing vacuum current offsets the positive electromagnetic field energy. The model was generalized to apply to all Standard Model interactions by defining stability conditions for fermion and boson self-energy processes which result in intermediate dressed core mass states of infinitesimally short duration for radiative corrections. Concise rules for constructing S-matrix corrections for the dressed core were developed and applied to resolve divergence issues in Abelian QED and non-Abelian QCD and electroweak theories. The stabilized amplitudes, including core and dressed core contributions, are finite and agree with renormalized QFT for all cases considered.

Since mass and charge were maintained as observed fundamental constants throughout in both the Lagrangian and subsequent radiative corrections, there is no mass or charge renormalization in this approach. Fundamentally, the electromagnetic and strong coupling constants $\{\alpha, \alpha_s\}$ are independent of the energy scale, and QFT is scale-invariant. For a collection of diagrams, however, renormalization methods remain essential for deriving an effective running coupling constant [68, 69] with an energy scale signature consistent with QCD's prediction of asymptotic freedom [47, 48] and its excellent agreement with experimental results [54].

In conclusion, we remark that the stability approach is simpler compared to renormalization and offers several other advantages:

1. stabilized amplitudes are uniquely determined in contrast to multiple renormalization schemes,

2. no separation of left- and right-handed fermion fields in electroweak theory is required, and
3. only finite wave field corrections for Δr are non-zero for electroweak corrections $\left\{\hat{\Sigma}^{\gamma Z},\, \hat{\Sigma}^{Z},\, \hat{\Sigma}^{W}\right\}$.

Overall, our results suggest that elementary electromagnetic and color charges are rock-solid constants, and any energy dependence of predictions arising from radiative corrections is better attributed to modifications to the field propagation and vertex functions rather than fundamental physical constants. Finally, we believe that it more accurately characterizes the physics involved in radiative processes since it includes the vacuum reaction (61) that stabilizes the system.

ACKNOWLEDGMENTS

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APPENDIX: NUMERICAL RESULTS

Values for Δr are tabulated in Table II using $\sin^2(\theta_W) = 0.23122(4)$ and other physical constants [15].

Table II: Numerical results for Δr and derivative shifts.

$b =$	γ	γZ	Z	W
$m_b\,(GeV/c^2)$	0	$\{0,\, m_Z\}$	91.1876	80.379
$\frac{\delta m_b^2}{m_b^2}$	–	–	-0.1061	-0.0920
Δr^b	0	0.0258	-0.0329	-0.0470
$\frac{\partial \Sigma^b}{\partial s}\left(m_b^2\right)\Big\vert_{finite}$	$-\frac{\alpha}{6\pi}$	0.001165	-0.1142	-0.1252

Real parts of boson polarization functions (167) are plotted in Figs. 13-16. Stability profiles use amplitudes (161) or, equivalently, (165) exclusive of Δr. Results in Fig. 13 agree with those in Fig. 8 of [56] notwithstanding updated physical constants [15]; QED results are added for comparison using an analytic result for (83) given in [70]. For numerical evaluation of photon SE and $\gamma - Z$ mixing profiles shown in Figs. 13 and 14, the stability value at $s = 0$ is not represented; but analytically, $\hat{\Pi}^{\gamma}(0) = \hat{\Pi}^{\gamma Z}(0) = 0$ from (167). Differences between "$Stability + \Delta r$" profiles shown in Figs. 14-16 and Figs. 9-11 of [56] are due to

1. Δr impacts arising from updates to the core functions for $\left\{\varSigma^{Z},\, \varSigma^{W}\right\}$ in [12] relative to [56],
2. derivative shifts in Table II, and
3. updated physical constants including a Higgs mass measurement $125.18 \pm 0.16\, GeV/c^2$ [15].

Analytic expressions for $F(s, m_1, m_2)$ given in [56] and its partials (158) were verified against numerical integration results for all mass arguments m_1 and m_2 over the range $0 < \sqrt{|k^2|} < 200\, GeV$.

Electron self-energy function profiles $\left\{\hat{\Sigma}_V,\, \hat{\Sigma}_A,\, \hat{\Sigma}_S\right\}$ shown in Fig. 17 agree with those in Fig. 18a of [56].

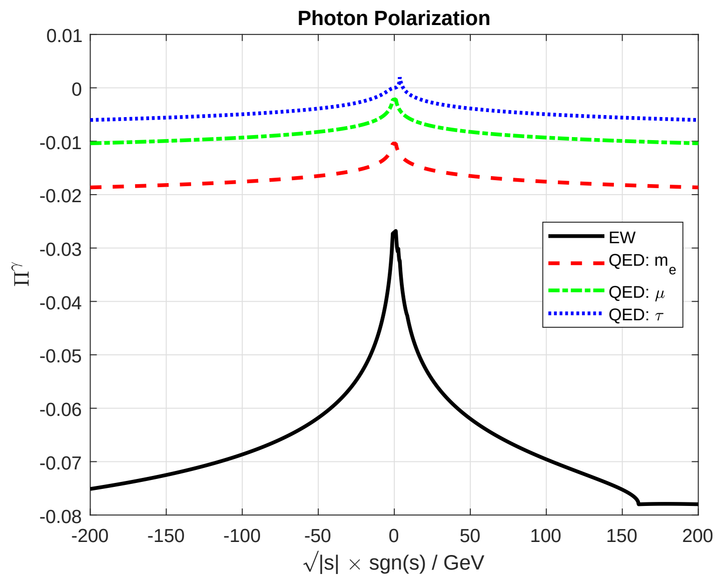

FIG. 13: Stabilized electroweak photon polarization is compared with QED for electron, muon, and tau.

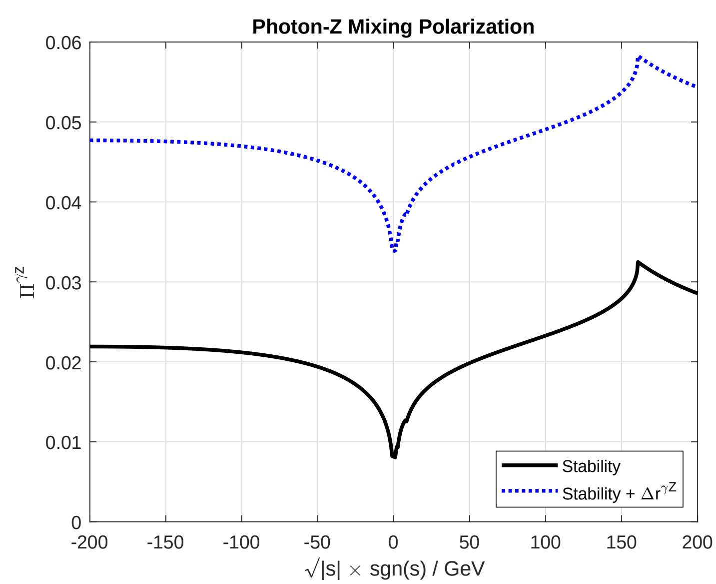

FIG. 14: Stabilized photon–Z mixing profiles with/without adjustments for Δr.

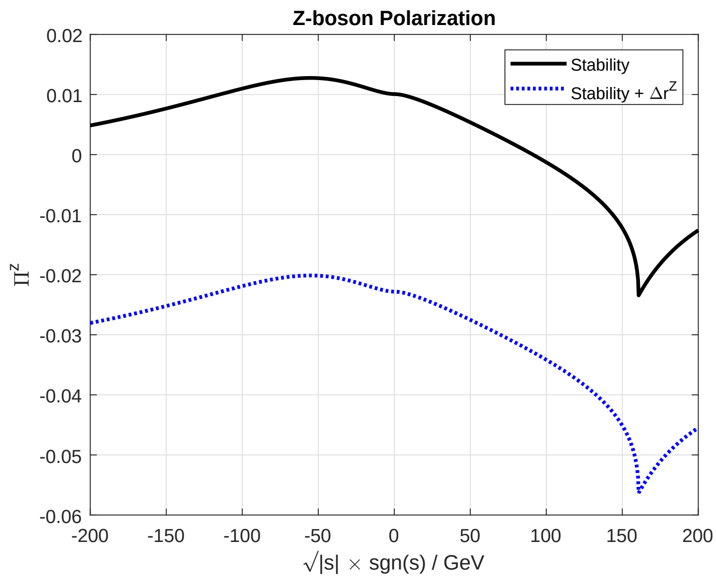

FIG. 15: Stabilized Z-boson polarization profiles with/without adjustments for Δr.

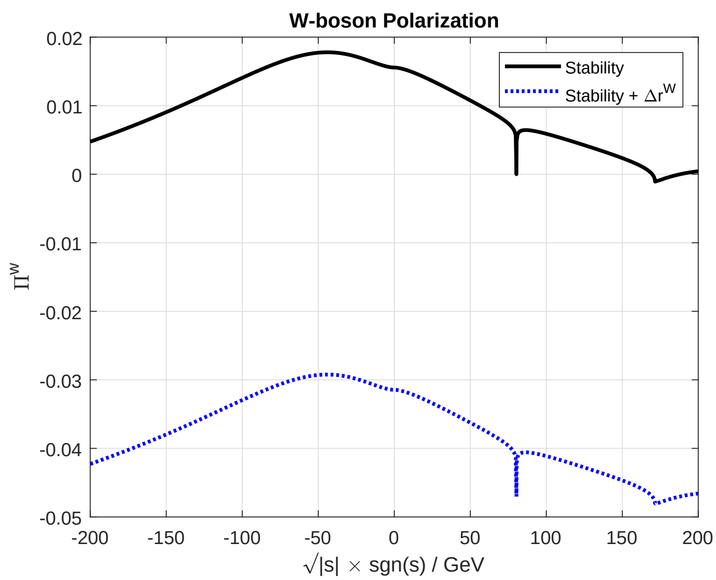

FIG. 16: Stabilized W-boson polarization profiles with/without adjustments for Δr.

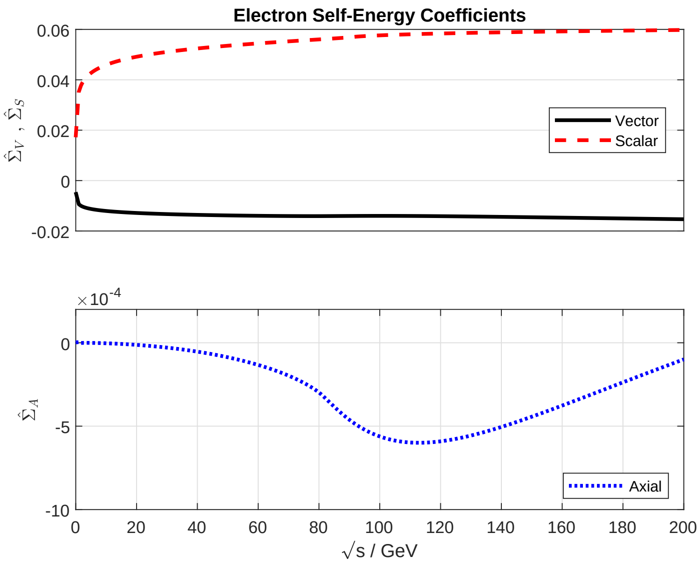

FIG. 17: Electron self-energy coefficients for vector, axial, and scalar contributions.

[1] F. Rohrlich, Am. J. Phys **28**, 639 (1960).
[2] M. Janssen and M. Mecklenburg, in *Interactions* (Springer, 2006) pp. 65–134.
[3] T. Aoyama, M. Hayakawa, T. Kinoshita, and M. Nio, Phys. Rev. D **85**, 093013 (2012).
[4] D. Hanneke, S. Fogwell Hoogerheide, and G. Gabrielse, Phys. Rev. A **83**, 052122 (2011).
[5] F. J. Dyson, Phys. Rev. **75**, 486 (1949).
[6] H. A. Bethe, Phys. Rev. **72**, 339 (1947).
[7] S. S. Schweber, H. A. Bethe, and F. de Hoffmann, *Mesons and Fields*, Vol. I (Row, Peterson and Co., 1955) Ch. 21a discusses H. A. Kramer's mass renormalization principle. See Ch. 21e for polarization tensor factorization.
[8] C. N. Yang and R. L. Mills, Phys. Rev. **96**, 191 (1954).
[9] S. Glashow, Nucl. Phys. B **22**, 579 (1961).
[10] S. Weinberg, Phys. Rev. Lett. **19**, 1264 (1967).
[11] A. Salam, in *Elementary Particle Theory*, edited by N. Svartholm (New York : Wiley – Stockholm : Almqvist and Wiksell, 1968).
[12] W. F. Hollik, Fortschr. Phys. **38**, 165 (1990).
[13] M. E. Peskin and D. V. Schroeder, *An Introduction to Quantum Field Theory* (Addison-Wesley, New York, 1995) p. 332ff, Sec. 16, A.4.
[14] P. W. Higgs, Phys. Rev. Lett. **13**, 508 (1964).
[15] M. Tanabashi *et al.*, Phys. Rev. D **98**, 030001 (2018).
[16] R. P. Feynman, Phys. Rev. **76**, 749 (1949).
[17] G. 't Hooft and M. Veltman, Nucl. Phys. B **50**, 318 (1972).
[18] L. D. Landau and E. M. Lifshitz, *The classical theory of fields* (Pergamon, Oxford, 1975) Sec. 15.
[19] V. F. Weisskopf, Z. Angew. Phys. **89**, 27 (1934), english translation given in A. I. Miller, *Early Quantum Electrodynamics: A Source Book* (Cambridge U. Press, New York, 1994).
[20] V. F. Weisskopf, Phys. Rev. **56**, 72 (1939).
[21] P. A. M. Dirac, Proc. R. Soc. London, Ser. A **117**, 610 (1928).
[22] R. P. Feynman, Phys. Rev. **76**, 769 (1949).
[23] P. A. M. Dirac, Proc. R. Soc. London, Ser. A **167**, 148 (1938).
[24] H. Poincaré, Rendiconti del Circolo Matematico di Palermo (1884-1940) **21**, 129 (1906).
[25] S. S. Schweber, Rev. Mod. Phys. **58**, 449 (1986).
[26] W. Heisenberg, Zeitschrift für Physik **43**, 172 (1927).
[27] W. Heisenberg, Zeitschrift für Physik **90**, 209 (1934).
[28] G. C. Wick, Phys. Rev. **80**, 268 (1950).
[29] J. Schwinger, Phys. Rev. **74**, 1439 (1948).
[30] F. J. Dyson, Phys. Rev. **75**, 1736 (1949).
[31] A. Sirlin, Phys. Rev. D **22**, 971 (1980).
[32] W. Pauli and F. Villars, Rev. Mod. Phys. **21**, 434 (1949).
[33] J. M. Jauch and F. Rohrlich, *The Theory of Photons and Electrons* (Springer-Verlag, New York, 1976) Ch. 9-10, App. A5-2, Supp. S2.
[34] G. 't Hooft and M. Veltman, Nucl. Phys. B **44**, 189 (1972).
[35] F. Mandl and G. Shaw, *Quantum Field Theory* (Wiley, New York, 1984) pp. 188, 227, 231.
[36] M. Gell-Mann, M. L. Goldberger, and W. Thirring, Phys. Rev. **95**, 1612 (1954).
[37] R. E. Cutkosky, J. Math. Phys. **1**, 429 (1960).
[38] G. Källén, Helv. Phys. Acta. **25**, 417 (1952).
[39] J. D. Bjorken and S. D. Drell, *Relativistic Quantum Mechanics* (McGraw-Hill, New York, 1964) p. 163.
[40] J. Schwinger, Phys. Rev. **73**, 416 (1948).
[41] H. M. Foley and P. Kusch, Phys. Rev. **73**, 412 (1948).
[42] J. C. Ward, Phys. Rev. **78**, 182 (1950).
[43] S. Weinberg, Phys. Rev. **118**, 838 (1960).
[44] A. Salam, Phys. Rev. **82**, 217 (1951).
[45] J. C. Ward, Proc. Phys. Society **A64**, 54 (1951).
[46] G. Källén and A. Sabry, Dan. Mat. Fys. Medd. **29**, 1 (1955).
[47] D. J. Gross and F. Wilczek, Phys. Rev. D **8**, 3633 (1973).
[48] H. D. Politzer, Physics Reports **14**, 129 (1974).
[49] G. 't Hooft, Nucl. Phys. B **35**, 167 (1971).
[50] C. G. Callan, Phys. Rev. D **2**, 1541 (1970).
[51] K. Symanzik, Commun. Math. Phys. **18**, 227 (1970).
[52] H. D. Politzer, Phys. Rev. Lett. **30**, 1346 (1973).
[53] D. J. Gross and F. Wilczek, Phys. Rev. Lett. **30**, 1343 (1973).
[54] G. Dissertori, "The determination of the strong coupling constant," (The Standard Theory of Particle Physics,

2016) Chap. 6, pp. 113–128.
[55] L. H. Ryder, *Quantum Field Theory* (Cambridge University Press, 1985) p. 371.
[56] M. Böhm, H. Spiesberger, and W. Hollik, Fortschritte Der Physik **34**, 687 (1986).
[57] G. Passarino and M. Veltman, Nuclear Physics B **160**, 151 (1979).
[58] G. 't Hooft and M. Veltman, Nuclear Physics B **153**, 365 (1979).
[59] A. Denner, Fortschr. Phys. **41**, 307 (1993).
[60] D. Ross and J. Taylor, Nuclear Physics B **51**, 125 (1973).
[61] Y. Nagashima, *Elementary Particle Physics: Foundations of the Standard Model*, Vol. 2 (Wiley, 2013) sec. 5.2.2.
[62] T. Kinoshita and A. Sirlin, Phys. Rev. **113**, 1652 (1959).
[63] M. Roos and A. Sirlin, Nucl. Phys. B **29**, 296 (1971).
[64] T. van Ritbergen and R. G. Stuart, Nucl. Phys. B **564**, 343 (2000).
[65] M. Steinhauser and T. Seidensticker, Phys. Lett. B **467**, 271 (1999).
[66] A. Pak and A. Czarnecki, Phys. Rev. Lett. **100**, 241807 (2008).
[67] W. Hollik, Journal of Physics: Conference Series **53**, 7 (2006).
[68] M. Gell-Mann and F. E. Low, Phys. Rev. **95**, 1300 (1954).
[69] K. Huang, International Journal of Modern Physics A **28**, 1330050 (2013).
[70] G. Källén, *Quantum Electrodynamics* (Springer-Verlag, New York, 1972) p. 149.