V

Interval Complex Neutrosophic Graph of Type 1

Said Broumi[1] ▪ Assia Bakali[2] ▪ Mohamed Talea[3] ▪ Florentin Smarandache[4] ▪ V. Venkateswara Rao[5]

[1,3] Laboratory of Information Processing, Faculty of Science Ben M'Sik, University Hassan II, B.P 7955, Sidi Othman, Casablanca, Morocco. E-mail:broumisaid78@gmail.com, taleamohamed@yahoo.fr
[2]Ecole Royale Navale, Casablanca, Morocco. E-mail: assiabakali@yahoo.fr
[4]Department of Mathematics, University of New Mexico,705 Gurley Avenue, Gallup, NM 87301, USA .E-mail: fsmarandache@gmail.com
[5] Department of S&H, VFSTR University, Vadlamudi - 522 207, Guntur, India
E-mail: vunnamvenky@gmail.com

Abstract

The neutrosophic set theory, proposed by smarandache, can be used as a general mathematical tool for dealing with indeterminate and inconsistent information. By applying the concept of neutrosophic sets on graph theory, several studies of neutrosophic models have been presented in the literature. In this paper, the concept of complex neutrosophic graph of type 1 is extended to interval complex neutrosophic graph of type 1(ICNG1). We have proposed a representation of ICNG1 by adjacency matrix and studied some properties related to this new structure. The concept of ICNG1 generalized the concept of generalized fuzzy graphs of type 1 (GFG1), generalized single valued neutrosophic graphs of type 1 (GSVNG1) generalized interval valued neutrosophic graphs of type 1 (GIVNG1) and complex neutrosophic graph type 1(CNG1).

Keywords

Neutrosophic set; complex neutrosophic set; interval complex neutrosophic set; interval complex neutrosophic graph of type 1; adjacency matrix.

1 Introduction

Crisp set, fuzzy sets [14] and intuitionisitic fuzzy sets [13] already acts as a mathematical tool. But Smarandache [5, 6] gave a momentum by introducing the concept of neutrosophic sets (NSs in short). Neutrosophic sets came as a glitter in this field as their vast potential to intimate imprecise, incomplete, uncertainty and inconsistent information of the world. Neutrosophic sets associates a degree of membership (T) , indeterminacy(I) and non- membership (F) for an element each of which belongs to the non-standard unit interval]−0, 1+[. Due to this characteristics, the practical implement of NSs becomes difficult. So, for this reason, Smarandache [5, 6] and Wang et al. [10] introduced the concept of a single valued Neutrosophic sets (SVNS), which is an instance of a NS and can be used in real scientific and engineering applications. Wang et al. [12] defined the concept of interval valued neutrosophic sets as generalization of SVNS. In [11], the readers can found a rich literature on single valued neutrosophic sets and their applications in divers fields.

Graph representations are widely used for dealing with structural information, in different domains such as networks, image interpretation, pattern recognition operations research. In a crisp graphs two vertices are either related or not related to each other, mathematically, the degree of relationship is either 0 or 1. While in fuzzy graphs, the degree of relationship takes values from [0, 1].In [1] Atanassov defined the concept of intuitionistic fuzzy graphs (IFGs) with vertex sets and edge sets as IFS. The concept of fuzzy graphs and their extensions have a common property that each edge must have a membership value less than or equal to the minimum membership of the nodes it connects.

Fuzzy graphs and their extensions such as hesitant fuzzy graph, intuitionistic fuzzy graphs ..etc , deal with the kinds of real life problems having some uncertainty measure. All these graphs cannot handle the indeterminate relationship between object. So, for this reason, Smaranadache [3,9]defined a new form of graph theory called neutrosophic graphs based on literal indeterminacy (I) to deal with such situations. The same author[4]initiated a new graphical structure of neutrosphic graphs based on (T, I, F) components and proposed three structures of neutrosophic graphs such as neutrosophic edge graphs, neutrosophic vertex graphs and neutrosophic vertex-edge graphs. In [8] Smarandache defined a new classes of neutrosophic graphs including neutrosophic offgraph, neutrosophic bipolar/tripola/ multipolar graph. Single valued neutrosphic graphs with vertex sets and edge sets as SVN were first introduced by Broumi [33] and defined some of its properties. Also, Broumi et al.[34] defined certain degrees of SVNG and established some of their properties. The same author proved a necessary and sufficient condition for a single valued neutrosophic graph to be an isolated-SVNG [35]. In addition, Broumi et al. [47] defined the concept of the interval valued neutrosophic graph as a generalization of SVNG and analyzed some properties of it. Recently, Several extension of single valued neutrosophic graphs, interval valued neutrosophic graphs and their application have been studied deeply [17-19, 21-22, 36-45, 48-49,54-56].

In [7] Smarandache initiated the idea of removal of the edge degree restriction of fuzzy graphs, intuitionistic fuzzy graphs and single valued neutrosophic graphs. Samanta et al [53] discussed the concept of generalized fuzzy graphs (GFG) and studied some properties of it . The authors claim that fuzzy graphs and their extension defined by many researches are limited to represented for some systems such as social network. Employing the idea initiated by smarandache [7], Broumi et al. [46, 50,51]proposed a new structures of neutrosophic graphs such as generalized single valued neutrosophic graph of type1(GSVNG1), generalized interval valued neutrosophic graph of type1(GIVNG1), generalized bipolar neutrosophic graph of type 1, all these types of graphs are a generalization of generalized fuzzy graph of type1[53]. In [2], Ramot defined the concept of complex fuzzy sets as an extension of the fuzzy set in which the range of the membership function is extended from the subset of the real number to the unit disc. Later on, some extensions of complex fuzzy set have been studied well in the litteratur e[20,23,26,28,29,58-68].In [15],Ali and Smarandache proposed the concept of complex neutrosophic set in short CNS. The concept of complex neutrosophic set is an extension of complex intuitionistic fuzzy sets by adding by adding complex-valued indeterminate membership grade to the definition of complex intuitionistic fuzzy set. The complex-valued truth membership function, complex-valued indeterminacy membership function, and complex-valued falsity membership function are totally independent. The complex fuzzy set has only one extra phase term, complex intuitionistic fuzzy set has two additional phase terms while complex neutrosophic set has three phase terms. The complex neutrosophic sets (CNS) are used to handle the information of uncertainty and periodicity simultaneously. When the values of the membership function indeterminacy-membership function and the falsity-membership function in a CNS are difficult to be expressed as exact single value in many real-world problems, interval complex neutrosophic sets can be used to characterize the uncertain information more sufficiently and accurately. So for this purpose, Ali et al [16] defined the concept of interval complex neutrosophic sets (ICNs) and examined its characteristics. Recently, Broumi et al.[52]defined the concept of complex neutrosophic graphs of type 1 with vertex sets and edge sets as complex neutrosophic sets.

In this paper, an extended version of complex neutrosophic graph of type 1(ICNG1) is introduced. To the best of our knowledge, there is no research on interval complex neutrosophic graph of type 1 in literature at present.

The remainder of this paper is organized as follows. In Section 2, some fundamental and basic concepts regarding neutrosophic sets, single valued neutrosophic sets, complex neutrosophic set, interval complex neutrosophic set and complex neutrosophic graphs of type 1 are presented. In Section 3, ICNG1 is proposed and provided by a numerical example. In section 4 a representation matrix of ICNG1 is introduced and finally we draw conclusions in section 5.

2 Fundamental and Basic Concepts

In this section we give some definitions regarding neutrosophic sets, single valued neutrosophic sets, complex neutrosophic set, interval complex neutrosophic set and complex neutrosophic graphs of type 1

Definition 2.1 [5, 6]

Let ζ be a space of points and let x $\in \zeta$. A neutrosophic set A $\in \zeta$ is characterized by a truth membership function T, an indeterminacy membership function I, and a falsity membership function F. The values of T, I, F are real standard or nonstandard subsets of $]^-0,1^+[$, and T, I, F: $\zeta \rightarrow]^-0,1^+[$. A neutrosophic set can therefore be represented as

$$A=\{(x, T_A(x), I_A(x), F_A(x)): x \in \zeta\} \quad (1)$$

Since $T, I, F \in [0, 1]$, the only restriction on the sum of T, I, F is as given below:

$$^-0 \leq T_A(x)+ I_A(x)+F_A(x) \leq 3^+. \quad (2)$$

From philosophical point of view, the NS takes on value from real standard or non-standard subsets of $]^-0,1^+[$. However, to deal with real life applications such as engineering and scientific problems, it is necessary to take values from the interval [0, 1] instead of $]^-0,1^+[$.

Definition 2.2 [10]

Let ζ be a space of points (objects) with generic elements in ζ denoted by x. A single valued neutrosophic set A (SVNS A) is characterized by truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$, and a falsity-membership function $F_A(x)$. For each point x in ζ, $T_A(x)$, $I_A(x)$, $F_A(x) \in [0, 1]$. The SVNS A can therefore be written as

$$A=\{(x, T_A(x), I_A(x), F_A(x)): x \in \zeta\} \quad (3)$$

Definition 2.3 [15]

A complex neutrosophic set A defined on a universe of discourse X, which is characterized by a truth membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$, and a falsity-membership function $F_A(x)$ that assigns a complex-valued membership grade to $T_A(x), I_A(x), F_A(x)$ for any $x \in X$. The values of $T_A(x), I_A(x), F_A(x)$ and their sum may be any values within a unit circle in the complex plane and is therefore of the form $T_A(x) = p_A(x)e^{i\mu_A(x)}, I_A(x) = q_A(x)e^{i\nu_A(x)}$, and $F_A(x) = r_A(x)e^{i\omega_A(x)}$. All the amplitude and phase terms are real-valued and $p_A(x), q_A(x), r_A(x) \in [0, 1]$, whereas $\mu_A(x), \nu_A(x), \omega_A(x) \in (0, 2\pi]$,such that the condition.

$$0 \leq p_A(x) + q_A(x) + r_A(x) \leq 3 \tag{4}$$

is satisfied. A complex neutrosophic set A can thus be represented in set form as:

$$A = \{\langle x, T_A(x) = a_T, I_A(x) = a_I, F_A(x) = a_F \rangle : x \in X\}, \tag{5}$$

Where$T_A: X \to \{a_T: a_T \in C, |a_T| \leq 1\}, I_A: X \to \{a_I: a_I \in C, |a_I| \leq 1\}, F_A: X \to \{a_F: a_F \in C, |a_F| \leq 1\}$, and also

$$|T_A(x) + I_A(x) + F_A(x)| \leq 3. \tag{6}$$

Let A and B be two CNSs in X, which are as defined as follow $A = \{(x, T_A(x), I_A(x), F_A(x)) : x \in X\}$ and $B = \{(x, T_B(x), I_B(x), F_B(x)) : x \in X\}$.

Definition 2.4 [15]

Let A and B be two CNSs in X. The union, intersection and complement of two CNSs are defined as:

The union of A and B denoted as $A \cup_N B$,is defined as:

$$A \cup_N B = \{(x, T_{A\cup B}(x), I_{A\cup B}(x), F_{A\cup B}(x)) : x \in X\}, \tag{7}$$

Where,$T_{A\cup B}(x), I_{A\cup B}(x), F_{A\cup B}(x)$ are given by

$$T_{A\cup B}(x) = \max(p_A(x), p_B(x)) \,.\, e^{i\,(\mu_A(x)\cup\mu_B(x))}$$

$$I_{A\cup B}(x) = \min(q_A(x), q_B(x)) \,.\, e^{i\,(\nu_A(x)\cup\nu_B(x))},$$

$$F_{A\cup B}(x) = \min(r_A(x), r_B(x)) \,.\, e^{i\,(\omega_A(x)\cup\omega_B(x))}.$$

The intersection of A and B denoted as $A \cap_N B$, is defined as:

$$A \cap_N B = \{(x, T_{A\cap B}(x), I_{A\cap B}(x), F_{A\cap B}(x)) : x \in X\}, \tag{8}$$

Where$T_{A\cap B}(x), I_{A\cap B}(x), F_{A\cap B}(x)$ are given by

$$T_{A\cup B}(x) = \min(p_A(x), p_B(x)) \,.\, e^{i\,(\mu_A(x)\cap\mu_B(x))} \tag{9}$$

$$I_{A\cup B}(x) = \max(q_A(x), q_B(x)) \,.\, e^{i\,(\nu_A(x)\cap\nu_B(x))}, \tag{10}$$

$$F_{A\cup B}(x) = \max(r_A(x), r_B(x)) \,.\, e^{i\,(\omega_A(x)\cap\omega_B(x))}. \tag{11}$$

The union and the intersection of the phase terms of the complex truth, falsity and indeterminacy membership functions can be calculated using any one of the following operations:

Sum:

$$\mu_{A\cup B}(x) = \mu_A(x) + \mu_B(x), \tag{12}$$

$$\nu_{A\cup B}(x) = \nu_A(x) + \nu_B(x), \tag{13}$$

$$\omega_{A\cup B}(x) = \omega_A(x) + \omega_B(x). \tag{14}$$

Max:

$$\mu_{A\cup B}(x) = \max(\mu_A(x), \mu_B(x)), \tag{15}$$

$$\nu_{A\cup B}(x) = \max(\nu_A(x), \nu_B(x)), \tag{16}$$

$$\omega_{A\cup B}(x) = \max(\omega_A(x), \omega_B(x)). \tag{17}$$

Min:

$$\mu_{A\cup B}(x) = \min(\mu_A(x), \mu_B(x)), \tag{18}$$

$$\nu_{A\cup B}(x) = \min(\nu_A(x), \nu_B(x)), \tag{19}$$

$$\omega_{A\cup B}(x) = \min(\omega_A(x), \omega_B(x)). \tag{20}$$

"The game of winner, neutral, and loser":

$$\mu_{A\cup B}(x) = \begin{cases} \mu_A(x) & if \quad p_A > p_B \\ \mu_B(x) & if \quad p_B > p_A \end{cases}, \tag{21}$$

$$\nu_{A\cup B}(x) = \begin{cases} \nu_A(x) & if \quad q_A < q_B \\ \nu_B(x) & if \quad q_B < q_A \end{cases}, \tag{22}$$

$$\omega_{A\cup B}(x) = \begin{cases} \omega_A(x) & if \quad r_A < r_B \\ \omega_B(x) & if \quad r_B < r_A \end{cases}. \tag{23}$$

Definition 2.5 [16]

An interval complex neutrosophic set A defined on a universe of discourse ζ, which is characterized by an interval truth membership function $\tilde{T}_A(x) = [T_A^L(x), T_A^U(x)]$, an interval indeterminacy-membership function $\tilde{I}_A(x)$, and an interval falsity-membership function$\tilde{F}_A(x)$that assigns a complex-valued membership grade to $\tilde{T}_A(x), \tilde{I}_A(x), \tilde{F}_A(x)$ for any $x \in \zeta$. The values of $\tilde{T}_A(x), \tilde{I}_A(x), \tilde{F}_A(x)$ and their sum may be any values within a unit circle in the complex plane and is therefore of the form

$$\tilde{T}_A(x) = [p_A^L(x), p_A^U(x)].e^{i[\mu_A^L(x),\ \mu_A^U(x)]}, \tag{24}$$

$$\tilde{I}_A(x) = [q_A^L(x), q_A^U(x)].e^{i[\nu_A^L(x),\ \nu_A^U(x)]} \tag{25}$$

and $\tilde{F}_A(x) = [r_A^L(x), r_A^U(x)].e^{i[\omega_A^L(x),\ \omega_A^U(x)]}$ (26)

All the amplitude and phase terms are real-valued and $p_A^L(x), p_A^U(x), q_A^L(x), q_A^U(x), r_A^L(x) and\ r_A^U(x) \in [0,1]$, whereas $\mu_A(x), \nu_A(x), \omega_A(x) \in (0, 2\pi]$,such that the condition

$$0 \le p_A^U(x) + q_A^U(x) + r_A^U(x) \le 3 \tag{27}$$

is satisfied. An interval complex neutrosophic set $\tilde{A}$ can thus be represented in set form as:

$$\tilde{A} = \{\langle x, T_A(x) = a_T, I_A(x) = a_I, F_A(x) = a_F\rangle : x \in \zeta\}, \tag{28}$$

Where$T_A: \zeta. \rightarrow \{a_T: a_T \in C, |a_T| \leq 1\}, I_A: \zeta. \rightarrow \{a_I: a_I \in C, |a_I| \leq 1\}, F_A: \zeta. \rightarrow \{a_F: a_F \in C, |a_F| \leq 1\}$, and also $\left|\mathrm{T}_{\mathrm{A}}^{\mathrm{U}}(x) + \mathrm{I}_{\mathrm{A}}^{\mathrm{U}}(x) + \mathrm{F}_{\mathrm{A}}^{\mathrm{U}}(x)\right| \leq 3.$ (29)

Definition 2.6 [16]

Let A and B be two ICNSs in ζ. The union, intersection and complement of two ICNSs are defined as:

The union of$A$ and B denoted as$A \cup_N B$,is defined as:

$$A \cup_N B = \left\{\left(x, \tilde{T}_{A\cup B}(x), \tilde{I}_{A\cup B}(x), \tilde{F}_{A\cup B}(x)\right) : x \in X\right\}, \quad (30)$$

Where,$\tilde{T}_{A\cup B}(x), \tilde{I}_{A\cup B}(x), \tilde{F}_{A\cup B}(x)$ are given by

$$\mathrm{T}_{\mathrm{A\cup B}}^{\mathrm{L}}(\mathrm{x})=\left[(\mathrm{p}_{\mathrm{A}}^{\mathrm{L}}(\mathrm{x}) \vee \mathrm{p}_{\mathrm{B}}^{\mathrm{L}}(\mathrm{x}))\right].\mathrm{e}^{\mathrm{j}.\mu_{\mathrm{T_{A\cup B}}}^{L}(\mathrm{x})},$$

$$\mathrm{T}_{\mathrm{A\cup B}}^{\mathrm{U}}(\mathrm{x})=\left[(\mathrm{p}_{\mathrm{A}}^{\mathrm{U}}(\mathrm{x}) \vee \mathrm{p}_{\mathrm{B}}^{\mathrm{U}}(\mathrm{x}))\right].\mathrm{e}^{\mathrm{j}.\mu_{\mathrm{T_{A\cup B}}}^{U}(\mathrm{x})} \quad (31)$$

$$\mathrm{I}_{\mathrm{A\cup B}}^{\mathrm{L}}(\mathrm{x})=\left[(\mathrm{q}_{\mathrm{A}}^{\mathrm{L}}(\mathrm{x}) \wedge \mathrm{q}_{\mathrm{B}}^{\mathrm{L}}(\mathrm{x}))\right].\mathrm{e}^{\mathrm{j}.\mu_{\mathrm{I_{A\cup B}}}^{L}(\mathrm{x})},$$

$$\mathrm{I}_{\mathrm{A\cup B}}^{\mathrm{U}}(\mathrm{x})=\left[(\mathrm{q}_{\mathrm{A}}^{\mathrm{U}}(\mathrm{x}) \wedge \mathrm{q}_{\mathrm{B}}^{\mathrm{U}}(\mathrm{x}))\right].\mathrm{e}^{\mathrm{j}.\mu_{\mathrm{I_{A\cup B}}}^{U}(\mathrm{x})}, \quad (32)$$

$$\mathrm{F}_{\mathrm{A\cup B}}^{\mathrm{L}}(\mathrm{x})=\left[(\mathrm{r}_{\mathrm{A}}^{\mathrm{L}}(\mathrm{x}) \wedge \mathrm{r}_{\mathrm{B}}^{\mathrm{L}}(\mathrm{x}))\right].\,\mathrm{e}^{\mathrm{j}.\mu_{\mathrm{F_{A\cup B}}}^{L}(\mathrm{x})},$$

$$\mathrm{F}_{\mathrm{A\cup B}}^{\mathrm{U}}(\mathrm{x})=\left[(\mathrm{r}_{\mathrm{A}}^{\mathrm{U}}(\mathrm{x}) \wedge \mathrm{r}_{\mathrm{B}}^{\mathrm{U}}(\mathrm{x}))\right].\,\mathrm{e}^{\mathrm{j}.\mu_{\mathrm{F_{A\cup B}}}^{U}(\mathrm{x})} \quad (33)$$

The intersection of$A$ and B denoted as $A \cap_N B$, is defined as:

$$A \cap_N B = \left\{\left(x, \tilde{T}_{A\cap B}(x), \tilde{I}_{A\cap B}(x), \tilde{F}_{A\cap B}(x)\right) : x \in X\right\}, \quad (34)$$

Where,$\tilde{T}_{A\cap B}(x), \tilde{I}_{A\cap B}(x), \tilde{F}_{A\cap B}(x)$ are given by

$$\mathrm{T}_{\mathrm{A\cap B}}^{\mathrm{L}}(\mathrm{x})=\left[(\mathrm{p}_{\mathrm{A}}^{\mathrm{L}}(\mathrm{x}) \wedge \mathrm{p}_{\mathrm{B}}^{\mathrm{L}}(\mathrm{x}))\right].\mathrm{e}^{\mathrm{j}.\mu_{\mathrm{T_{A\cup B}}}^{L}(\mathrm{x})},$$

$$\mathrm{T}_{\mathrm{A\cap B}}^{\mathrm{U}}(\mathrm{x})=\left[(\mathrm{p}_{\mathrm{A}}^{\mathrm{U}}(\mathrm{x}) \wedge \mathrm{p}_{\mathrm{B}}^{\mathrm{U}}(\mathrm{x}))\right].\mathrm{e}^{\mathrm{j}.\mu_{\mathrm{T_{A\cup B}}}^{U}(\mathrm{x})} \quad (35)$$

$$\mathrm{I}_{\mathrm{A\cap B}}^{\mathrm{L}}(\mathrm{x})=\left[(\mathrm{q}_{\mathrm{A}}^{\mathrm{L}}(\mathrm{x}) \vee \mathrm{q}_{\mathrm{B}}^{\mathrm{L}}(\mathrm{x}))\right].\mathrm{e}^{\mathrm{j}.\mu_{\mathrm{I_{A\cup B}}}^{L}(\mathrm{x})},$$

$$\mathrm{I}_{\mathrm{A\cap B}}^{\mathrm{U}}(\mathrm{x})=\left[(\mathrm{q}_{\mathrm{A}}^{\mathrm{U}}(\mathrm{x}) \vee \mathrm{q}_{\mathrm{B}}^{\mathrm{U}}(\mathrm{x}))\right].\mathrm{e}^{\mathrm{j}.\mu_{\mathrm{I_{A\cup B}}}^{U}(\mathrm{x})}, \quad (36)$$

$$\mathrm{F}_{\mathrm{A\cap B}}^{\mathrm{L}}(\mathrm{x})=\left[(\mathrm{r}_{\mathrm{A}}^{\mathrm{L}}(\mathrm{x}) \vee \mathrm{r}_{\mathrm{B}}^{\mathrm{L}}(\mathrm{x}))\right].\,\mathrm{e}^{\mathrm{j}.\mu_{\mathrm{F_{A\cup B}}}^{L}(\mathrm{x})},$$

$$\mathrm{F}_{\mathrm{A\cap B}}^{\mathrm{U}}(\mathrm{x})=\left[(\mathrm{r}_{\mathrm{A}}^{\mathrm{U}}(\mathrm{x}) \vee \mathrm{r}_{\mathrm{B}}^{\mathrm{U}}(\mathrm{x}))\right].\,\mathrm{e}^{\mathrm{j}.\mu_{\mathrm{F_{A\cup B}}}^{U}(\mathrm{x})} \quad (37)$$

The union and the intersection of the phase terms of the complex truth, falsity and indeterminacy membership functions can be calculated using any one of the following operations:

Sum:

$$\mu^{L}_{A\cup B}(x) = \mu^{L}_{A}(x) + \mu^{L}_{B}(x),$$
$$\mu^{U}_{A\cup B}(x) = \mu^{U}_{A}(x) + \mu^{U}_{B}(x), \quad (38)$$
$$\nu^{L}_{A\cup B}(x) = \nu^{L}_{A}(x) + \nu^{L}_{B}(x),$$
$$\nu^{U}_{A\cup B}(x) = \nu^{U}_{A}(x) + \nu^{U}_{B}(x), \quad (39)$$
$$\omega^{L}_{A\cup B}(x) = \omega^{L}_{A}(x) + \omega^{L}_{B}(x),$$
$$\omega^{U}_{A\cup B}(x) = \omega^{U}_{A}(x) + \omega^{U}_{B}(x), \quad (40)$$

Max:

$$\mu^{L}_{A\cup B}(x) = \max(\mu^{L}_{A}(x), \mu^{L}_{B}(x)),$$
$$\mu^{U}_{A\cup B}(x) = \max(\mu^{U}_{A}(x), \mu^{U}_{B}(x)), \quad (41)$$
$$\nu^{L}_{A\cup B}(x) = \max(\nu^{L}_{A}(x), \nu^{L}_{B}(x)),$$
$$\nu^{U}_{A\cup B}(x) = \max(\nu^{U}_{A}(x), \nu^{U}_{B}(x)), \quad (42)$$
$$\omega^{L}_{A\cup B}(x) = \max(\omega^{L}_{A}(x), \omega^{L}_{B}(x)),$$
$$\omega^{U}_{A\cup B}(x) = \max(\omega^{U}_{A}(x), \omega^{U}_{B}(x)), \quad (43)$$

Min:

$$\mu^{L}_{A\cup B}(x) = \min(\mu^{L}_{A}(x), \mu^{L}_{B}(x)),$$
$$\mu^{U}_{A\cup B}(x) = \min(\mu^{U}_{A}(x), \mu^{U}_{B}(x)), \quad (44)$$
$$\nu^{L}_{A\cup B}(x) = \min(\nu^{L}_{A}(x), \nu^{L}_{B}(x)),$$
$$\nu^{U}_{A\cup B}(x) = \min (\nu^{U}_{A}(x), \nu^{U}_{B}(x)), \quad (45)$$
$$\omega^{L}_{A\cup B}(x) = \min(\omega^{L}_{A}(x), \omega^{L}_{B}(x)),$$
$$\omega^{U}_{A\cup B}(x) = \min(\omega^{U}_{A}(x), \omega^{U}_{B}(x)), \quad (46)$$

"The game of winner, neutral, and loser":

$$\mu_{A\cup B}(x) = \begin{cases} \mu_A(x) & if \quad p_A > p_B \\ \mu_B(x) & if \quad p_B > p_A \end{cases}, \quad (47)$$

$$\nu_{A\cup B}(x) = \begin{cases} \nu_A(x) & if \quad q_A < q_B \\ \nu_B(x) & if \quad q_B < q_A \end{cases}, \quad (48)$$

$$\omega_{A\cup B}(x) = \begin{cases} \omega_A(x) & if \quad r_A < r_B \\ \omega_B(x) & if \quad r_B < r_A \end{cases}. \quad (49)$$

Definition 2.7 [52]

Consider V be a non-void set. Two function are considered as follows:

ρ=(ρ_T, ρ_I, ρ_F):V → $[0,1]^3$and

ω= (ω_T, ω_I, ω_F):VxV → $[0,1]^3$. We suppose

A= {($\rho_T(x)$,$\rho_T(y)$) | ω_T(x, y) ≥ 0}, (50)

B= {($\rho_I(x)$,$\rho_I(y)$) |ω_I(x, y) ≥ 0}, (51)

C= {($\rho_F(x)$,$\rho_F(y)$) |ω_F(x, y) ≥ 0}, (52)

considered ω_T, ω_I and $\omega_F \geq 0$ for all set A,B, C since its is possible to have edge degree = 0 (for T, or I, or F).

The triad (V, ρ, ω) is defined to be complex neutrosophic graph of type 1 (CNG1) if there are functions

α:A→ $[0,1]$, β:B→ $[0,1]$ and δ:C→ $[0,1]$ such that

$\omega_T(x,y) = \alpha((\rho_T(x),\rho_T(y)))$ (53)

$\omega_I(x,y) = \beta((\rho_I(x),\rho_I(y)))$ (54)

$\omega_F(x,y) = \delta((\rho_F(x),\rho_F(y)))$ where x, y∈ V. (55)

For each $\rho(x)$= ($\rho_T(x)$, $\rho_I(x)$, $\rho_F(x)$),,x∈ V are called the complex truth, complex indeterminacy and complex falsity-membership values, respectively, of the vertex x. likewise for each edge (x, y) : ω(x, y)=(ω_T(x, y), ω_I(x, y), ω_F(x, y)) are called the complex membership, complex indeterminacy membership and complex falsity values of the edge.

3 Interval Complex Neutrosophic Graph of Type 1

In this section, based on the concept of complex neutrosophic graph of type 1 [52], we define the concept of interval complex neutrosophic graph of type 1 as follows:

Definition 3.1.

Consider V be a non-void set. Two function are considered as follows:
ρ=($[\rho_T^L,\rho_T^U]$,$[\rho_I^L,\rho_I^U]$, $[\rho_F^L,\rho_F^U]$):V→ $[0,1]^3$and
ω=($[\omega_T^L,\omega_T^U]$, $[\omega_I^L,\omega_I^U]$, $[\omega_F^L,\omega_F^U]$):VxV → $[0,1]^3$. We suppose
A= {($[\rho_T^L(x),\rho_T^U(x)]$, $[\rho_T^L(y),\rho_T^U(y)]$) |$\omega_T^L$(x, y) ≥ 0
and ω_T^U(x, y) ≥0 }, (56)
B= {($[\rho_I^L(x),\rho_I^U(x)]$, $[\rho_I^L(y),\rho_I^U(y)]$) |$\omega_I^L$(x, y) ≥ 0
and ω_I^U(x, y) ≥0}, (57)
C= {($[\rho_F^L(x),\rho_F^U(x)]$, $[\rho_F^L(y),\rho_F^U(y)]$) |$\omega_F^L$(x, y) ≥ 0
and ω_F^U(x, y) ≥0}, (58)

We have considered ω_T, ω_I and $\omega_F \geq 0$ for all set A,B, C , since its is possible to have edge degree = 0 (for T, or I, or F).

The triad (V, ρ, ω) is defined to be an interval complex neutrosophic graph of type 1 (ICNG1) if there are functions

α:A→ [0, 1] , β:B→ [0, 1] and δ:C→ [0, 1] such that

$\omega_T(x,y)$=[$\omega_T^L(x,y)$,
$\omega_T^U(x,y)$]= α([$\rho_T^L(x)$,$\rho_T^U(x)$],[$\rho_T^L(y)$,$\rho_T^U(y)$]) (59)

$\omega_I(x,y)$=[$\omega_I^L(x,y)$,
$\omega_I^U(x,y)$]= β([$\rho_I^L(x)$,$\rho_I^U(x)$],[$\rho_I^L(y)$,$\rho_I^U(y)$]) (60)

$\omega_F(x,y)$=[$\omega_F^L(x,y)$,
$\omega_F^U(x,y)$]= δ([$\rho_F^L(x)$,$\rho_F^U(x)$],[$\rho_F^L(y)$,$\rho_F^U(y)$]) where x, y∈ V. (61)

For each $\rho(x)$=([$\rho_T^L(x)$,$\rho_T^U(x)$], [$\rho_I^L(x)$,$\rho_I^U(x)$],[$\rho_F^L(x)$,$\rho_F^U(x)$]),x∈ V are called the interval complex truth, interval complex indeterminacy and interval complex falsity-membership values, respectively, of the vertex x. likewise for each edge(x, y) :$\omega(x,y)$=($\omega_T(x,y)$, $\omega_I(x,y)$, $\omega_F(x,y)$) are called the interval complex membership, interval complex indeterminacy membership and interval complex falsity values of the edge.

Example 3.2

Consider the vertex set be V={x, y, z, t} and edge set be E= {(x, y),(x, z),(x, t),(y, t)}

	x	y	z	t
[ρ_T^L,ρ_T^U]	[0.5, 0.6]$e^{j.\pi[0.8,0.9]}$	[0.9 , 1]$e^{j.\pi[0.7,0.8]}$	[0.3, 0.4]$e^{j.\pi[0.2,0.5]}$	[0.8, 0.9]$e^{j.\pi[0.1,0.3]}$
[ρ_I^L,ρ_I^U]	[0.3, 0.4]$e^{j.\pi[0.1,0.2]}$	[0.2, 0.3]$e^{j.\pi[0.5,0.6]}$	[0.1, 0.2]$e^{j.\pi[0.3,0.6]}$	[0.5, 0.6]$e^{j.\pi[0.2,0.8]}$
[ρ_F^L,ρ_F^U]	[0.1, 0.2]$e^{j.\pi[0.5,0.7]}$	[0.6, 0.7]$e^{j.\pi[0.2,0.3]}$	[0.8, 0.9]$e^{j.\pi[0.2,0.4]}$	[0.4, 0.5]$e^{j.\pi[0.3,0.7]}$

Table 1. Interval Complex truth-membership, indeterminacy-membership and falsity-membership of the vertex set.

Given the following functions

$\alpha(m,n)$=[$m_T^L(u) \vee n_T^L(u), m_T^U(u) \vee n_T^U(u)$] . $e^{j.\pi\mu_{A\cup B}(u)}$ (62)

$\beta(m,n)$=[$m_I^L(u) \wedge n_I^L(u), m_I^U(u) \wedge n_I^U(u)$] . $e^{j.\pi\nu_{A\cup B}(u)}$ (63)

$\delta(m,n)$= [$m_F^L(u) \wedge n_F^L(u), m_F^U(u) \wedge n_F^U(u)$] . $e^{j.\pi\omega_{A\cup B}(u)}$ (64)

Here,

A={([0.5, 0.6]$e^{j.\pi[0.8,0.9]}$, [0.9, 1]$e^{j.\pi[0.7,0.8]}$), ([0.5, 0.6]$e^{j.\pi[0.8,0.9]}$, [0.3, 0.4]$e^{j.\pi[0.2,0.5]}$), ([0.5, 0.6]$e^{j.\pi[0.8,0.9]}$, [0.8, 0.9]$e^{j.\pi[0.1,0.3]}$),([0.9, 1.0]$e^{j.\pi[0.7,0.8]}$, [0.8, 0.9]$e^{j.\pi[0.1,0.3]}$)}

B={([0.3, 0.4]$e^{j.\pi[0.1,0.2]}$, [0.2, 0.3]$e^{j.\pi[0.5,0.6]}$), ([0.3, 0.4]$e^{j.\pi[0.1,0.2]}$, [0.1, 0.2]$e^{j.\pi[0.3,0.6]}$), ([0.3, 0.4]$e^{j.\pi[0.1,0.2]}$, [0.5, 0.6]$e^{j.\pi[0.2,0.8]}$), ([0.2, 0.3]$e^{j.\pi[0.5,0.6]}$, [0.5, 0.6]$e^{j.\pi[0.2,0.8]}$)}

C={([0.1, 0.2]$e^{j.\pi[0.5,0.7]}$, [0.6, 0.7]$e^{j.\pi[0.2,0.3]}$), ([0.1, 0.2]$e^{j.\pi[0.5,0.7]}$, [0.8, 0.9]$e^{j.\pi[0.2,0.4]}$), ([0.1, 0.2]$e^{j.\pi[0.5,0.7]}$, [0.4, 0.5]$e^{j.\pi[0.3,0.7]}$), ([0.6, 0.7]$e^{j.\pi[0.2,0.3]}$, [0.4, 0.5]$e^{j.\pi[0.3,0.7]}$)}.

Then

ω	(x,y)	(x,z)	(x,t)	(y,t)
$[\omega_T^L,\omega_T^U]$	[0.9, 1]$e^{j.\pi[0.8,0.9]}$	[0.5, 0.6]$e^{j.\pi[0.8,0.9]}$	[0.8,0.9]$e^{j.\pi[0.8,0.9]}$	[0.9,1]$e^{j.\pi[0.8,0.9]}$
$[\omega_I^L,\omega_I^U]$	[0.2,0.3]$e^{j.\pi[0.5,0.6]}$	[0.1,0.2]$e^{j.\pi[0.3,0.6]}$	[0.3,0.4]$e^{j.\pi[0.2,0.8]}$	[0.2, 0.3]$e^{j.\pi[0.5,0.8]}$
$[\omega_F^L,\omega_F^U]$	[0.1, 0.2]$e^{j.\pi[0.5,0.7]}$	[0.1,0.2]$e^{j.\pi[0.5,0.7]}$	[0.1,0.2]$e^{j.\pi[0.5,0.7]}$	[0.4,0.5]$e^{j.\pi[0.5,0.7]}$

Table 2. Interval Complex truth-membership, indeterminacy-membership and falsity-membership of the edge set.

The figure 2 show the interval complex neutrosophic graph of type 1

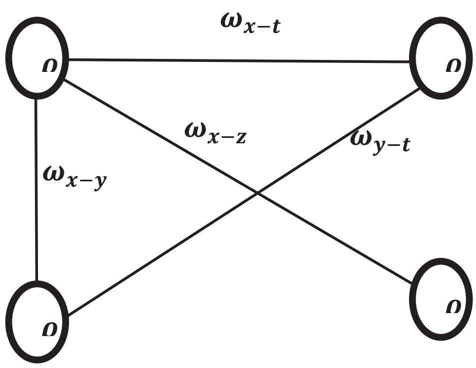

Fig 2. Interval complex neutrosophicgraph of type 1.

In classical graph theory, any graph can be represented by adjacency matrices, and incident matrices. In the following section ICNG1 is represented by adjacency matrix.

4 Representation of interval complex neutrosophic graph of Type 1 by adjacency matrix

In this section, interval truth-membership, interval indeterminate-membership and interval false-membership are considered independents. Based on the representation of complex neutrosophic graph of type 1 by adjacency matrix [52], we propose a matrix representation of interval complex neutrosophic graph of type 1 as follow:

The interval complex neutrosophic graph (ICNG1) has one property that edge membership values (T, I, F) depends on the membership values (T, I, F) of adjacent vertices. Suppose ξ=(V, ρ, ω) is a ICNG1 where vertex set $V=\{v_1,v_2,\ldots,v_n\}$. The functions

α :A→ (0, 1] is taken such that

$\omega_T^L(x,y) = \alpha((\rho_T^L(x),\rho_T^L(y)))$, $\omega_T^U(x,y) = \alpha((\rho_T^U(x),\rho_T^U(y)))$, where x, y∈ V and

A= $\{([\rho_T^L(x),\rho_T^U(x)], [\rho_T^L(y),\rho_T^U(y)]) \mid \omega_T^L(x, y) \geq 0$ and $\omega_T^U(x, y) \geq 0 \}$,

β :B→ (0, 1] is taken such that

$\omega_I^L(x,y) = \beta((\rho_I^L(x),\rho_I^L(y)))$, $\omega_I^U(x,y) = \beta((\rho_I^U(x),\rho_I^U(y)))$, where x, y∈ V and

B= $\{([\rho_I^L(x),\rho_I^U(x)], [\rho_I^L(y),\rho_I^U(y)]) \mid \omega_I^L(x, y) \geq 0$ and $\omega_I^U(x, y) \geq 0 \}$

and

δ :C→ (0, 1] is taken such that

$\omega_F^L(x,y) = \delta((\rho_F^L(x),\rho_F^L(y)))$, $\omega_F^U(x,y) = \delta((\rho_F^U(x),\rho_F^U(y)))$, where x, y∈ V and

C= $\{([\rho_F^L(x),\rho_F^U(x)], [\rho_F^L(y),\rho_F^U(y)]) \mid \omega_F^L(x, y) \geq 0$ and $\omega_F^U(x, y) \geq 0 \}$.

The ICNG1 can be represented by (n+1) x (n+1) matrix $M_{G_1}^{T,I,F}=[a^{T,I,F}(i, j)]$ as follows:

The interval complex truth membership (T), interval complex indeterminacy-membership (I) and the interval complex falsity-membership (F) values of the vertices are provided in the first row and first column. The (i+1, j+1)- th-entry are the interval complex truth membership (T), interval complex indeterminacy-membership (I) and the interval complex falsity-membership (F) values of the edge (x_i,x_j), i, j=1,…,n if i≠j.

The (i, i)-th entry is $\rho(x_i)=(\rho_T(x_i),\ \rho_I(x_i),\ \rho_F(x_i))$, where i=1,2,…,n. the interval complex truth membership (T), interval complex indeterminacy-membership (I) and the interval complex falsity-membership (F) values of the edge can be computed easily using the functions α, β and δ which are in (1,1)-position of the matrix. The matrix representation of ICNG1, denoted by $M_{G_1}^{T,I,F}$, can be written as three matrix representation $M_{G_1}^{T}$, $M_{G_1}^{I}$ and $M_{G_1}^{F}$. For convenience representation $v_i(\rho_T(v_i))$ =$[\rho_T^L(v_i),\ \rho_T^U(v_i)]$, $v_i(\rho_I(v_i))$ =$[\rho_I^L(v_i),\ \rho_I^U(v_i)]$ and $v_i(\rho_F(v_i))$ =$[\rho_F^L(v_i),\ \rho_F^U(v_i)]$, for i=1, …., n

The $M_{G_1}^T$ can be therefore represented as follows

α	$v_1(\rho_T(v_1))$	$v_2(\rho_T(v_2))$	$v_n(\rho_T(v_n))$
$v_1(\rho_T(v_1))$	$[\rho_T^L(v_1), \rho_T^U(v_1)]$	$\alpha(\rho_T(v_1),\rho_T(v_2))$	$\alpha(\rho_T(v_1),\rho_T(v_n))$
$v_2(\rho_T(v_2))$	$\alpha(\rho_T(v_2),\rho_T(v_1))$	$[\rho_T^L(v_2), \rho_T^U(v_2)]$	$\alpha(\rho_T(v_2),\rho_T(v_2))$
…	….	…	…
$v_n(\rho_T(v_n))$	$\alpha(\rho_T(v_n),\rho_T(v_1))$	$\alpha(\rho_T(v_n),\rho_T(v_2))$	$[\rho_T^L(v_n), \rho_T^U(v_n)]$

Table 3. Matrix representation of T-ICNG1

The $M_{G_1}^I$ can be therefore represented as follows

β	$v_1(\rho_I(v_1))$	$v_2(\rho_I(v_2))$	$v_n(\rho_I(v_n))$
$v_1(\rho_I(v_1))$	$[\rho_I^L(v_1), \rho_I^U(v_1)]$	$\beta(\rho_I(v_1),\rho_I(v_2))$	$\beta(\rho_I(v_1),\rho_I(v_n))$
$v_2(\rho_I(v_2))$	$\beta(\rho_I(v_2),\rho_I(v_1))$	$[\rho_I^L(v_2), \rho_I^U(v_2)]$	$\beta(\rho_I(v_2),\rho_I(v_2))$
…	….	…	…
$v_n(\rho_I(v_n))$	$\beta(\rho_I(v_n),\rho_I(v_1))$	$\beta(\rho_T(v_n),\rho_I(v_2))$	$[\rho_I^L(v_n), \rho_I^U(v_n)]$

Table 4. Matrix representation of I-ICNG1

The $M_{G_1}^I$ can be therefore represented as follows

δ	$v_1(\rho_F(v_1))$	$v_2(\rho_F(v_2))$	$v_n(\rho_F(v_n))$
$v_1(\rho_F(v_1))$	$[\rho_F^L(v_1), \rho_F^U(v_1)]$	$\delta(\rho_F(v_1),\rho_F(v_2))$	$\delta(\rho_F(v_1),\rho_F(v_n))$
$v_2(\rho_F(v_2))$	$\delta(\rho_F(v_2),\rho_F(v_1))$	$[\rho_F^L(v_2), \rho_F^U(v_2)]$	$\delta(\rho_F(v_2),\rho_F(v_2))$
…	….	…	…
$v_n(\rho_F(v_n))$	$\delta(\rho_F(v_n),\rho_F(v_1))$	$\delta(\rho_F(v_n),\rho_F(v_2))$	$[\rho_F^L(v_n), \rho_F^U(v_n)]$

Table 5. Matrix representation of F-ICNG1

Here the Interval complex neutrosophic graph of first type (ICNG1) can be represented by the matrix representation depicted in table 9. The matrix representation can be written as three interval complex matrices one containing the entries as T, I, F (see table 6, 7 and 8).

α = max(x, y)	x([0.5, 0.6] $e^{j.\pi[0.8,0.9]}$)	y([0.9 , 1]. $e^{j.\pi[0.7,0.8]}$)	z([0.3, 0.4]. $e^{j.\pi[0.2,0.5]}$)	t([0.8, 0.9]. $e^{j.\pi[0.1,0.3]}$)
x([0.5, 0.6].$e^{j.\pi[0.8,0.9]}$)	[0.5, 0.6].$e^{j.\pi[0.8,0.9]}$	[0.9 , 1]. $e^{j.\pi[0.8,0.9]}$	[0.5, 0.6].$e^{j.\pi[0.8,0.9]}$	[0.8, 0.9]. $e^{j.\pi[0.8\ 0.9]}$
y([0.9 , 1]. $e^{j.\pi[0.7,0.8]}$)	[0.9 , 1]. $e^{j.\pi[0.8,0.9]}$	[0.9 , 1]. $e^{j.\pi[0.7,0.8]}$	**[0, 0]**	[0.9 , 1]. $e^{j.\pi[0.7,0.8]}$
z([0.3, 0.4]. $e^{j.\pi[0.2,0.5]}$)	[0.5, 0.6].$e^{j.\pi[0.8,0.9]}$	**[0, 0]**	[0.3, 0.4]. $e^{j.\pi[0.2,0.5]}$	**[0, 0]**
t([0.8, 0.9]. $e^{j.\pi[0.1,0.3]}$)	[0.8, 0.9]. $e^{j.\pi[0.8,0.9]}$	[0.9 , 1]. $e^{j.\pi[0.7,0.8]}$	**[0, 0]**	[0.8, 0.9]. $e^{j.\pi[0.1,0.3]}$

Table 6: Lower and upper Truth- matrix representationof ICNG1

β = min(x, y)	x([0.3, 0.4]. $e^{j.\pi[0.1,0.2]}$)	y([0.2, 0.3]. $e^{j.\pi[0.5,0.6]}$)	z([0.1, 0.2]. $e^{j.\pi[0.3,0.6]}$	t([0.5, 0.6]. $e^{j.\pi[0.2,0.8]}$)
x([0.3, 0.4]. $e^{j.\pi[0.1,0.2]}$)	[0.3, 0.4]. $e^{j.\pi[0.1,0.2]}$	[0.2, 0.3]. $e^{j.\pi[0.5,0.6]}$	[0.1, 0.2]. $e^{j.\pi[0.3,0.6]}$	[0.3, 0.4]. $e^{j.\pi[0.3,0.6]}$
y([0.2, 0.3]. $e^{j.\pi[0.5,0.6]}$)	[0.2, 0.3]. $e^{j.\pi[0.5,0.6]}$	[0.2, 0.3]. $e^{j.\pi[0.5,0.6]}$	**[0, 0]**	[0.2, 0.3]. $e^{j.\pi[0.5,0.8]}$
z([0.1, 0.2]. $e^{j.\pi[0.3,0.6]}$	[0.1, 0.2]. $e^{j.\pi[0.3,0.6]}$	**[0, 0]**	[0.1, 0.2]. $e^{j.\pi[0.3,0.6]}$	**[0, 0]**
t([0.5, 0.6]. $e^{j.\pi[0.2,0.8]}$)	[0.3, 0.4]. $e^{j.\pi[0.2,0.8]}$	[0.2, 0.3]. $e^{j.\pi[0.5,0.8]}$	**[0, 0]**	[0.5 0.6]. $e^{j.\pi[0.2,0.8]}$

Table 7: Lower and upper Indeterminacy- matrix representation of ICNG1

δ= min(x, y)	x([0.1, 0.2]. $e^{j.\pi[0.5,0.7]}$)	y([0.6, 0.7]. $e^{j.\pi[0.2,0.3]}$)	z([0.8, 0.9]. $e^{j.\pi[0.2,0.4]}$)	t([0.4, 0.5]. $e^{j.\pi[0.3,0.7]}$)
x([0.1, 0.2].$e^{j.\pi[0.5,0.7]}$)	[0.1, 0.2]. $e^{j.\pi[0.5,0.7]}$	[0.1, 0.2]. $e^{j.\pi[0.5,0.7]}$	[0.1, 0.2]. $e^{j.\pi[0.8,0.9]}$	[0.1, 0.2]. $e^{j.\pi[0.5,0.7]}$
y[0.6, 0.7].$e^{j.\pi[0.2,0.3]}$)	[0.1, 0.2]. $e^{j.\pi[0.5,0.7]}$	[0.6, 0.7]. $e^{j.\pi[0.2,0.3]}$	**[0, 0]**	[0.4, 0.5]. $e^{j.\pi[0.3,0.7]}$
z([0.8, 0.9].$e^{j.\pi[0.2,0.4]}$)	[0.1, 0.2]. $e^{j.\pi[0.8,0.9]}$	**[0, 0]**	[0.8, 0.9]. $e^{j.\pi[0.2,0.4]}$	**[0, 0]**
t[0.4, 0.5]. $e^{j.\pi[0.3,0.7]}$)	[0.1, 0.2]. $e^{j.\pi[0.5,0.7]}$	[0.4, 0.5]. $e^{j.\pi[0.3,0.7]}$	**[0, 0]**	[0.4, 0.5]. $e^{j.\pi[0.3,0.7]}$

Table 8: Lower and upper Falsity- matrix representation of ICNG1

The matrix representation of ICNG1 can be represented as follows:

(α, β, δ)	**X(**<[0.5, 0.6]. $e^{j.\pi[0.8,0.9]}$, [0.3, 0.4]. $e^{j.\pi[0.1,0.2]}$, [0.1, 0.2]. $e^{j.\pi[0.5,0.7]}$>)	y(<[0.5, 0.6]. $e^{j.\pi[0.8,0.9]}$, [0.3, 0.4]. $e^{j.\pi[0.1,0.2]}$, [0.1, 0.2]. $e^{j.\pi[0.5,0.7]}$>)	z(<[0.5, 0.6]. $e^{j.\pi[0.8,0.9]}$, [0.3, 0.4]. $e^{j.\pi[0.1,0.2]}$, [0.1, 0.2]. $e^{j.\pi[0.5,0.7]}$>)	t(<[0.5, 0.6]. $e^{j.\pi[0.8,0.9]}$, [0.3, 0.4]. $e^{j.\pi[0.1,0.2]}$, [0.1, 0.2]. $e^{j.\pi[0.5,0.7]}$>)
X(<[0.5, 0.6]. $e^{j.\pi[0.8,0.9]}$, [0.3, 0.4]. $e^{j.\pi[0.1,0.2]}$, [0.1, 0.2]. $e^{j.\pi[0.5,0.7]}$>)	<[0.5, 0.6]. $e^{j.\pi[0.8,0.9]}$, [0.3, 0.4]. $e^{j.\pi[0.1,0.2]}$, [0.1, 0.2]. $e^{j.\pi[0.5,0.7]}$>	<[0.9, 0.1]. $e^{j.\pi[0.8,0.9]}$, [0.2, 0.3]. $e^{j.\pi[0.5,0.6]}$, [0.1, 0.2]. $e^{j.\pi[0.5,0.7]}$>	<[0.5, 0.6]. $e^{j.\pi[0.8,0.9]}$, [0.1, 0.2]. $e^{j.\pi[0.3,0.6]}$, [0.1, 0.2]. $e^{j.\pi[0.8,0.9]}$>	<[0.8, 0.9]. $e^{j.\pi[0.8,0.9]}$, [0.3, 0.4]. $e^{j.\pi[0.2,0.8]}$, [0.1, 0.2]. $e^{j.\pi[0.5,0.7]}$>
y(<[0.5, 0.6]. $e^{j.\pi[0.8,0.9]}$, [0.3, 0.4]. $e^{j.\pi[0.1,0.2]}$, [0.1, 0.2]. $e^{j.\pi[0.5,0.7]}$>)	<[0.9, 0.1]. $e^{j.\pi[0.8,0.9]}$, [0.2, 0.3]. $e^{j.\pi[0.5,0.6]}$, [0.1, 0.2]. $e^{j.\pi[0.5,0.7]}$>	<[0.5, 0.6]. $e^{j.\pi[0.8,0.9]}$, [0.3, 0.4]. $e^{j.\pi[0.1,0.2]}$, [0.1, 0.2]. $e^{j.\pi[0.5,0.7]}$>	**<[0, 0],** **[0, 0],** **[0, 0]>**	<[0.9 1]. $e^{j.\pi[07,0.8]}$, [0.2, 0.3]. $e^{j.\pi[0.5,0.8]}$, [0.4, 0.5]. $e^{j.\pi[0.3,0.7]}$>
z(<[0.5, 0.6]. $e^{j.\pi[0.8,0.9]}$, [0.3, 0.4]. $e^{j.\pi[0.1,0.2]}$, [0.1, 0.2]. $e^{j.\pi[0.5,0.7]}$>)	<[0.5, 0.6]. $e^{j.\pi[0.8,0.9]}$, [0.1, 0.2]. $e^{j.\pi[0.3,0.6]}$, [0.1, 0.2]. $e^{j.\pi[0.8,0.9]}$>	**<[0, 0],** **[0, 0],** **[0, 0]>**	<[0.5, 0.6]. $e^{j.\pi[0.8,0.9]}$, [0.3, 0.4]. $e^{j.\pi[0.1,0.2]}$, [0.1, 0.2]. $e^{j.\pi[0.5,0.7]}$>	**<[0, 0],** **[0, 0],** **[0, 0]>**
t(<[0.5, 0.6]. $e^{j.\pi[0.8,0.9]}$, [0.3, 0.4]. $e^{j.\pi[0.1,0.2]}$, [0.1, 0.2]. $e^{j.\pi[0.5,0.7]}$>)	<[0.8, 0.9]. $e^{j.\pi[0.8,0.9]}$, [0.3, 0.4]. $e^{j.\pi[0.2,0.8]}$, [0.1, 0.2]. $e^{j.\pi[0.5,0.7]}$>	<[0.9 1]. $e^{j.\pi[07,0.8]}$, [0.2, 0.3]. $e^{j.\pi[0.5,0.8]}$, [0.4, 0.5]. $e^{j.\pi[0.3,0.7]}$>	**<[0, 0],** **[0, 0],** **[0, 0]>**	<[0.5, 0.6]. $e^{j.\pi[0.8,0.9]}$, [0.3, 0.4]. $e^{j.\pi[0.1,0.2]}$, [0.1, 0.2]. $e^{j.\pi[0.5,0.7]}$>

Table 9: Matrix representation of ICNG1.

Remark 1

If $\rho_T^L(x)$=$\rho_T^U(x)$,$\rho_I^L(x)$=$\rho_I^U(x)$=0 and $\rho_F^L(x)$=$\rho_F^U(x) = 0$ and the interval valued phase terms equals zero, the interval complex neutrosophic graphs type 1 is reduced to generalized fuzzy graphs type 1 (GFG1).

Remark 2

If $\rho_T^L(x)$=$\rho_T^U(x)$,$\rho_I^L(x)$=$\rho_I^U(x)$ and $\rho_F^L(x)$=$\rho_F^U(x)$and the interval valued phase terms equals zero, the interval complex neutrosophic graphs type 1 is reduced to generalized single valued graphs type 1 (GSVNG1).

Remark 3

If $\rho_T^L(x)$=$\rho_T^U(x)$,$\rho_I^L(x)$=$\rho_I^U(x)$ and $\rho_F^L(x)$=$\rho_F^U(x)$ the interval complex neutrosophic graphs type 1 is reduced to complex neutrosophic graphs type 1 (CNG1).

Remark 4

If $\rho_T^L(x) \neq \rho_T^U(x)$, $\rho_I^L(x) \neq \rho_I^U(x)$and $\rho_F^L(x) \neq \rho_F^U(x)$ *and* the interval valued phase terms equals zero, the interval complex neutrosophic graphs type 1 is reduced to generalized interval valued graphs type 1 (GIVNG1).

Theorem 1

Given the $M_{G_1}^T$ be matrix representation of T-ICNG1, then the degree of vertex $D_T(x_k)$ =[$\sum_{j=1,j\neq k}^{n} a_T^L(k+1,j+1)$,$\sum_{j=1,j\neq k}^{n} a_T^U(k+1,j+1)$],$x_k$ ∈ V or

$D_T(x_p)$ =[$\sum_{i=1,i\neq p}^{n} a_T^L(i+1,p+1)$,$\sum_{i=1,i\neq p}^{n} a_T^U(i+1,p+1)$ x_p ∈ V.

Proof

Similar to that of theorem 1 of [52].

Theorem 2

Given the $M_{G_1}^I$ be a matrix representation of I-ICNG1, then the degree of vertex $D_I(x_k)$ =[$\sum_{j=1,j\neq k}^{n} a_I^L(k+1,j+1)$,$\sum_{j=1,j\neq k}^{n} a_I^U(k+1,j+1)$],$x_k$ ∈ V

or $D_I(x_p)$ =[$\sum_{i=1,i\neq p}^{n} a_I^L(i+1,p+1)$,$\sum_{i=1,i\neq p}^{n} a_I^U(i+1,p+1)$], x_p ∈ V.

Proof

Similar to that of theorem 1 of [52].

Theorem 3

Given the $M_{G_1}^F$ be a matrix representation of ICNG1, then the degree of vertex

$D_F(x_k)$ =[$\sum_{j=1,j\neq k}^{n} a_F^L(k+1,j+1)$,$\sum_{j=1,j\neq k}^{n} a_F^U(k+1,j+1)$],$x_k$ ∈ V or

$D_F(x_p)$ =[$\sum_{i=1,i\neq p}^{n} a_F^L(i+1,p+1)$,$\sum_{i=1,i\neq p}^{n} a_F^U(i+1,p+1)$], x_p ∈ V.

Proof

Similar to that of theorem 1 of [52].

Theorem 4

Given the $M_{G_1}^{T,I,F}$ be a matrix representation of ICNG1, then the degree of vertex D(x_k) =($D_T(x_k)$,$D_I(x_k)$,$D_F(x_k)$) where

$D_T(x_k)$ =[$\sum_{j=1,j\neq k}^{n} a_T^L(k+1,j+1)$, $\sum_{j=1,j\neq k}^{n} a_T^U(k+1,j+1)$] , x_k ∈ V.

$D_I(x_k)$ ==[$\sum_{j=1,j\neq k}^{n} a_I^L(k+1,j+1), \sum_{j=1,j\neq k}^{n} a_I^U(k+1,j+1)], x_k \in$ V.

$D_F(x_k)$ ==[$\sum_{j=1,j\neq k}^{n} a_F^L(k+1,j+1),\ \sum_{j=1,j\neq k}^{n} a_F^U(k+1,j+1)], x_k \in$ V.

Proof

The proof is obvious.

5 Conclusion

In this article, we have introduced the concept of interval complex neutrosophic graph of type1 as generalization of the concept of single valued neutrosophic graph type 1 (GSVNG1), interval valued neutrosophic graph type 1 (GIVNG1) and complex neutrosophic graph of type1(CNG1). Next, we processed to presented a matrix representation of it. In the future works, we plan to study some more properties and applications of ICNG type 1 define the concept of interval complex neutrosophic graphs type 2.

Acknowledgements

The authors are very grateful to the chief editor and reviewers for their comments and suggestions, which is helpful in improving the chapter.

References

[1] Shannon A, Atanassov K, "A First Step to a Theory of the Intuitionistic Fuzzy Graphs." Proc. of the First Workshop on Fuzzy Based Expert Systems (D. akov, Ed.), Sofia, (1994):59-61.
[2] Ramot D, Menahem F, Gideon L, and Abraham K, "Complex Fuzzy Set." IEEE Transactions on Fuzzy Systems ,Vol 10, No 2, 2002.
[3] Smarandache F," Refined Literal Indeterminacy and the Multiplication Law of Sub-Indeterminacies." Neutrosophic Sets and Systems 9, (2015):58.63.
[4] Smarandache F, "Types of Neutrosophic Graphs and neutrosophic Algebraic Structures together with their Applications in Technology." seminar, UniversitateaTransilvania din Brasov, Facultatea de Design de ProdussiMediu, Brasov, Romania 06 June 2015.
[5] Smarandache F, "Neutrosophic set - a generalization of the intuitionistic fuzzy set." Granular Computing, 2006 IEEE International Conference (2006):38 – 42.
[6] Smarandache F," Neutrosophy. Neutrosophic Probability, Set, and Logic." ProQuest Information & Learning, Ann Arbor, Michigan, USA, 105 p., 1998; http://fs.gallup.unm.edu/eBook-neutrosophics6.pdf (last edition online)
[7] Smarandache F, "Nidus idearum. Scilogs, III: Viva la Neutrosophia! ." Brussels, 2017,134p. ISBN 978-1-59973-508-5
[8] Smarandache F," Neutrosophic overset, neutrosophic underset, Neutrosophic offset,Similarly for Neutrosophic Over-/Under-/OffLogic, Probability, and Statistic."Pons Editions, Brussels, 2016, 170p.
[9] Smarandache F,"Symbolic Neutrosophic Theory." (Europanova asbl, Brussels, 195 p., Belgium 2015.

[10] Wang H, Smarandache F, Zhang Y, and Sunderraman R,"Single valued Neutrosophic Sets, Multisspace and Multistructure 4, (2010):410-413.
[11] http://fs.gallup.unm.edu/NSS/.
[12] H .Wang, F. Smarandache, Y.Q .Zhang and R. Sunderraman, Interval neutrosophic Sets and Logic: Theory and Applications in Computing." Hexis, Phoenix, AZ, 2005.
[13] Atanassov K, "Intuitionistic fuzzy sets, Fuzzy Sets and Systems, vol. 20 (1986):87-96.
[14] Zadeh L," Fuzzy sets. Inform and Control, 8 (1965):338-353.
[15] Ali M, Smarandache F,"Complex neutrosophic set." Neural Computing and Applications 2015; DOI:10.1007/s00521-015-2154-y.
[16] Ali M, Dat L.Q, Son L. H, Smarandache F,"Interval Complex Neutrosophic Set: Formulationand Applications in Decision-Making."Int. J. Fuzzy Syst, DOI 10.1007/s40815-017-0380-4
[17] Malik M. A, Hassan A, Broumi S and Smarandache F, "Regular Single Valued Neutrosophic Hypergraphs."Neutrosophic Sets and Systems 13, (2016):18-23.
[18] Mullai M, Broumi S, Stephen A, " Shortest path problem by minimal spanning tree algorithm using bipolar neutrosophic numbers." International Journal of Mathematic Trends and Technology, Vol 46, N2 (2017): 80-87.
[19] Akram M , Sitara M, "Interval-Valued Neutrosophic Graph Structures." Punjab University Journal of Mathematics 50(1), (2018):113-137
[20] Sethi N, Das S. K and Panda D.C, "Probablilistic interpretation of complex fuzzy set." International Journal of Computer Science, Engineering and Information Technology (IJCSEIT), Vol.2, No.2 (2012):31-44
[21] Shah N, "Some Studies in Neutrosophic Graphs."Neutrosophic Sets and Systems, Vol. 12 (2016):54-64.
[22] Shah N and Hussain A, "Neutrosophic Soft Graphs." Neutrosophic Sets and Systems 11, (2016):31-44.
[23] Yazdanbakhsh O, Dick S," systematic review of complex fuzzy sets and logic." Fuzzy Sets and Systems
[24] Singh P. K, "Three-way fuzzy concept lattice representation using neutrosophic set." International Journal of Machine Learning and Cybernetics (2016): 1–11.
[25] Singh P.K," Interval-Valued Neutrosophic Graph Representation of Concept Lattice and Its (α,β,γ)-Decomposition." Arab J Sci Eng (2017), page 18 DOI 10.1007/s13369-017-2718-5
[26] Thirunavukarasu P, Sureshand R, Viswanathan K. K, "Energy of a complex fuzzy graph." International J. of Math. Sci. & Engg. Appls. (IJMSEA) 10 No. I(2016): 243-248.
[27] Thirunavukarasu P, Suresh R and Thamilmani P, "Application of complex fuzzy sets." JP Journal of Applied Mathematics 6, Issues 1 & 2 (2013):5-22.
[28] Husban R and Salleh A. R ," Complex vague set." Accepted in: Global Journal of Pure and Applied Mathematics (2015).
[29] Husban R, "Complex vague set." MSc Research Project, Faculty of Science and Technology, University Kebangsaan Malaysia, (2014).
[30] Şahin R , "An approach to neutrosophic graph theory with applications." Soft Computing, (2017):1–13, doi.org/10.1007/s00500-017-2875-1
[31] Al-Qudah Y and Hassan N,"Operations on complex multi-fuzzy sets."Journal of Intelligent & Fuzzy Systems 33 (2017):1527–1540.
[32] Broumi S, & Smarandache F, "Correlation coefficient of interval neutrosophic set." In Applied Mechanics and Materials, Trans Tech Publications 436, (2013):511-517
[33] Broumi S, Talea M, Bakali A, Smarandache F, "Single Valued Neutrosophic Graphs."Journal of New Theory, N 10, (20.16): 86-101.
[34] Broumi S, Talea M, Smarandache F, and Bakali A, "Single Valued Neutrosophic Graphs: Degree, Order and Size." IEEE International Conference on Fuzzy Systems (FUZZ), (2016): 2444-2451.

[35] Broumi S, Bakali A, Talea M, Smarandache F, "Isolated Single Valued Neutrosophic Graphs." Neutrosophic Sets and Systems 11, (2016): 74-78.
[36] Broumi S, Smarandache F, Talea M, and Bakali A, "Decision-Making Method Based On the Interval Valued Neutrosophic Graph." Future Technologie, IEEE, (2016):44-50.
[37] Broumi S, Bakali A, Talea M, Smarandache F, and Ali M, "Shortest Path Problem under Bipolar Neutrosphic Setting." Applied Mechanics and Materials 859, (2016):59-66.
[38] Broumi S, Bakali A, Talea M, Smarandache F, and Vladareanu L, "Computation of Shortest Path Problem in a Network with SV-Trapezoidal Neutrosophic Numbers." Proceedings of the 2016 International Conference on Advanced Mechatronic Systems, Melbourne, Australia, (2016):417-422.
[39] Broumi S, Bakali A, Talea M, Smarandache F, and Vladareanu L, "Applying Dijkstra Algorithm for Solving Neutrosophic Shortest Path Problem." Proceedings of the 2016 International Conference on Advanced Mechatronic Systems, Melbourne, Australia, November 30 - December 3,(2016):412-416.
[40] Broumi S, M. Talea, Bakali A, Smarandache F, "On Bipolar Single Valued Neutrosophic Graphs."Journal of New Theory 11, (2016): 84-102.
[41] Broumi S, Smarandache F, Talea M, and Bakali A, "An Introduction to Bipolar Single Valued Neutrosophic Graph Theory."Applied Mechanics and Materials 841 (2016):184-191.
[42] Broumi S, Smarandache F, Talea M and Bakali A, "Operations on Interval Valued Neutrosophic Graphs." chapter in book- New Trends in Neutrosophic Theory and Applications- Florentin Smarandache and Surpati Pramanik (Editors) (2016):231-254. ISBN 978-1-59973-498-9
[43] Broumi S, Bakali A, Talea M and Smarandache F andKishore Kumar P.K, "Shortest Path Problem on Single Valued Neutrosophic Graphs."2017 International Symposium on Networks, Computers and Communications (ISNCC) (2017):1- 8
[44] Broumi S, Bakali A, Talea M, Smarandache F, "Complex Neutrosophic Graphs of Type 1, 2017 IEEE International Conference on INnovations in Intelligent SysTems and Applications (INISTA), Gdynia Maritime University, Gdynia, Poland (2017):432-437
[45] Broumi S, Smarandache F, Talea M and Bakali A, "Operations on Interval Valued Neutrosophic Graphs." chapter in book- New Trends in Neutrosophic Theory and Applications- Florentin Smarandache and Surpati Pramanik (Editors) (2016): 231-254. ISBN 978-1-59973-498-9
[46] Broumi S, Bakali A, Talea M, Smarandache F and Hassan A, "generalized single valued neutrosophic graphs of first type." SISOM & ACOUSTICS 2017, Bucharest 18-19 May
[47] Broumi S, Talea M, Bakali A, Smarandache F, "Interval Valued Neutrosophic Graphs, Critical Review, XII(2016): 5-33.
[48] Broumi S, Dey A, Bakali A, Talea M, Smarandache F, Son L. H and Koley D, "Uniform Single Valued Neutrosophic Graphs." Neutrosophic Sets and Systems, Vol. 17,(2017):42-49.
[49] Broumi S, Bakali A, Talea M, Smarandache F, and Verma R, "Computing Minimum Spanning Tree in Interval Valued Bipolar Neutrosophic Environment." International Journal of Modeling and Optimization, Vol. 7, No. 5 (2017):300-304
[50] Broumi S, Bakali A, Talea M, Smarandache F, "Generalized Bipolar Neutrosophic Graphs of Type 1." 20th International Conference on Information Fusion, Xi'an,(2017):1714-1720
[51] Broumi S, Bakali A, Talea M and Smarandache F and Hassan A, "Generalized Interval Valued Neutrosophic Graphs of First Type 1." 2017 IEEE International Conference on Innovations in Intelligent SysTems and Applications (INISTA), Gdynia Maritime University, Gdynia, Poland (2017):413-419.
[52] Broumi S, Bakali A, Talea M and Smarandache F, "Complex Neutrosophic Graphs of Type 1." 2017 IEEE International Conference on Innovations in Intelligent SysTems and Applications (INISTA), Gdynia Maritime University, Gdynia, Poland (2017):432-437.

[53] Samanta S, Sarkar B, Shin D and Pal M, “Completeness and regularity of generalized fuzzy graphs.” Springer Plus, 2016, DOI 10.1186/s40064-016-3558-6.
[54] Mehra S and Singh M, “Single valued neutrosophic signedgarphs.”International Journal of Computer Applications, Vol 157, N.9, (2017):31-34.
[55] Ashraf S, Naz S, Rashmanlou H, and Malik M. A, “Regularity of graphs in single valued neutrosophic environment.” Journal of Intelligent & Fuzzy Systems,(2017):1-14
[56] Fathi S, Elchawalby H and Salama A. A, “A neutrosophic graph similarity measures.” chapter in book- New Trends in Neutrosophic Theory and Applications- Florentin Smarandache and Surpati Pramanik (Editors)(2016): 223-230.
[57] Vasantha Kandasamy W. B, Ilanthenral K and Smarandache F,” Neutrosophic Graphs: A New Dimension to Graph Theory Kindle Edition, 2015.
[58] Abdel-Basset, Mohamed, et al. "A novel group decision-making model based on triangular neutrosophic numbers." Soft Computing (2017): 1-15. DOI: https://doi.org/10.1007/s00500-017-2758-5
[59] Hussian, Abdel-Nasser, et al. "Neutrosophic Linear Programming Problems." Peer Reviewers: 15.
[60] Abdel-Basset, Mohamed, Mai Mohamed, and Arun Kumar Sangaiah. "Neutrosophic AHP-Delphi Group decision making model based on trapezoidal neutrosophic numbers." Journal of Ambient Intelligence and Humanized Computing (2017): 1-17. DOI: https://doi.org/10.1007/s12652-017-0548-7
[61] Mohamed, Mai, et al. "A Critical Path Problem in Neutrosophic Environment." Peer Reviewers: 167.
[62] Mohamed, Mai, et al. "A Critical Path Problem Using Triangular Neutrosophic Number." Peer Reviewers: 155.
[63] Mohamed, Mai, et al. "Using Neutrosophic Sets to Obtain PERT Three-Times Estimates in Project Management." Peer Reviewers: 143.
[64] Mohamed, Mai, et al. "Neutrosophic Integer Programming Problem." Neutrosophic Sets & Systems 15 (2017).
[65] Abdel-Baset, Mohamed, Ibrahim M. Hezam, and Florentin Smarandache. "Neutrosophic goal programming." Neutrosophic Sets Syst 11 (2016): 112-118.
[66] Hezam, Ibrahim M., Mohamed Abdel-Baset, and Florentin Smarandache. "Taylor series approximation to solve neutrosophic multiobjective programming problem." Neutrosophic Sets and Systems 10 (2015): 39-46.
[67] El-Hefenawy, Nancy, et al. "A review on the applications of neutrosophic sets." Journal of Computational and Theoretical Nanoscience 13.1 (2016): 936-944.
[68] Abdel-Basset, M., Mohamed, M., Zhou, Y., & Hezam, I. (2017). Multi-criteria group decision making based on neutrosophic analytic hierarchy process. Journal of Intelligent & Fuzzy Systems, 33(6), 4055-4066.