

# **Representation of gravity field equation and solutions, Hawking Radiation in Data General Relativity theory**

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## **ABSTRACT**

In the general relativity theory, we find the representation of the gravity field equation and solutions. We treat the representation of Schwarzschild solution, Reissner-Nodstrom solution, Kerr-Newman solution, Robertson -Walker solution. Specially, Robertson -Walker solution is an uniqueness. We found new general relativity theory (we call it Data General Relativity Theory;DGRT). We treat the data of Hawking radiation by Data general relativity theory.

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**Robertson-Walker solution**

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## 1. Introduction

In the general relativity theory, our article's aim is that we find the representation of the gravity field equation and solutions. We found new general relativity theory (we can call it Data General relativity theory). We more obtain data of Hawking radiation by Data general relativity theory.

First, the gravity potential  $g_{\mu\nu}$  is

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu \quad (1)$$

In gravity potential  $g_{\mu\nu}$ , we introduce tensor  $f_{\mu\nu}$  and scalar  $K$ .

$$f_{\mu\nu} = Kg_{\mu\nu}, \quad \frac{\partial K}{\partial x^\lambda} = 0$$

$$\begin{aligned} ds'^2 &= f_{\mu\nu} dx^\mu dx^\nu = \bar{g}_{\mu\nu} d\bar{x}^\mu d\bar{x}^\nu = \bar{g}_{\mu\nu} \frac{\partial \bar{x}^\mu}{\partial \bar{x}'^\alpha} \frac{\partial \bar{x}^\nu}{\partial \bar{x}'^\beta} d\bar{x}'^\alpha d\bar{x}'^\beta \\ &= \bar{g}'_{\alpha\beta} d\bar{x}'^\alpha d\bar{x}'^\beta = f'_{\alpha\beta} dx'^\alpha dx'^\beta \end{aligned}$$

$$\bar{g}'_{\alpha\beta} = \bar{g}_{\mu\nu} \frac{\partial \bar{x}^\mu}{\partial \bar{x}'^\alpha} \frac{\partial \bar{x}^\nu}{\partial \bar{x}'^\beta}, \quad f'_{\alpha\beta} = f_{\mu\nu} \frac{\partial x^\mu}{\partial x'^\alpha} \frac{\partial x^\nu}{\partial x'^\beta},$$

$$g_{\mu\nu} = \bar{g}_{\mu\nu} \quad (2)$$

In inverse gravity potential  $g^{\mu\nu}$ ,

$$f^{\mu\nu} f_{\mu\nu} = \delta_\mu^\nu = \left(\frac{1}{K} g^{\mu\nu}\right) (Kg_{\mu\nu}), \quad f^{\mu\nu} = \frac{1}{K} g^{\mu\nu} \quad (3)$$

In Christoffel symbol  $\Gamma^\rho_{\mu\nu}$ ,

$$\begin{aligned} \Gamma^{\rho}_{\mu\nu} &= \frac{1}{2} f^{\rho\lambda} \left( \frac{\partial f_{\mu\lambda}}{\partial x^\nu} + \frac{\partial f_{\nu\lambda}}{\partial x^\mu} - \frac{\partial f_{\mu\nu}}{\partial x^\lambda} \right) \\ &= \frac{1}{2} \left( \frac{1}{K} g^{\rho\lambda} \right) \left( K \frac{\partial g_{\mu\lambda}}{\partial x^\nu} + K \frac{\partial g_{\nu\lambda}}{\partial x^\mu} - K \frac{\partial g_{\mu\nu}}{\partial x^\lambda} \right) = \Gamma^{\rho}_{\mu\nu} \end{aligned}$$

$$\bar{\Gamma}^{\rho}_{\mu\nu} = \frac{1}{2} \bar{g}^{\rho\lambda} \left( \frac{\partial \bar{g}_{\mu\lambda}}{\partial \bar{x}^\nu} + \frac{\partial \bar{g}_{\nu\lambda}}{\partial \bar{x}^\mu} - \frac{\partial \bar{g}_{\mu\nu}}{\partial \bar{x}^\lambda} \right) = \frac{1}{\sqrt{K}} \Gamma^{\rho}_{\mu\nu} \quad (4)$$

Therefore, in the curvature tensor  $R^\rho_{\mu\nu\lambda}$ ,

$$\begin{aligned} R^{\rho}_{\mu\nu\lambda} &= \frac{\partial \Gamma^{\rho}_{\mu\nu}}{\partial x^\lambda} - \frac{\partial \Gamma^{\rho}_{\mu\lambda}}{\partial x^\nu} + \Gamma^{\sigma}_{\mu\nu} \Gamma^{\rho}_{\sigma\lambda} - \Gamma^{\sigma}_{\mu\lambda} \Gamma^{\rho}_{\sigma\nu} \\ &= \frac{\partial \Gamma^{\rho}_{\mu\nu}}{\partial x^\lambda} - \frac{\partial \Gamma^{\rho}_{\mu\lambda}}{\partial x^\nu} + \Gamma^{\sigma}_{\mu\nu} \Gamma^{\rho}_{\sigma\lambda} - \Gamma^{\sigma}_{\mu\lambda} \Gamma^{\rho}_{\sigma\nu} = R^{\rho}_{\mu\nu\lambda} \end{aligned}$$

$$\begin{aligned}\bar{R}^\rho{}_{\mu\nu\lambda} &= \frac{\partial \bar{\Gamma}^\rho{}_{\mu\nu}}{\partial \bar{X}^\lambda} - \frac{\partial \bar{\Gamma}^\rho{}_{\mu\lambda}}{\partial \bar{X}^\nu} + \bar{\Gamma}^\sigma{}_{\mu\nu} \bar{\Gamma}^\rho{}_{\sigma\lambda} - \bar{\Gamma}^\sigma{}_{\mu\lambda} \bar{\Gamma}^\rho{}_{\sigma\nu} \\ &= \frac{1}{K} \left( \frac{\partial \Gamma^\rho{}_{\mu\nu}}{\partial X^\lambda} - \frac{\partial \Gamma^\rho{}_{\mu\lambda}}{\partial X^\nu} + \Gamma^\sigma{}_{\mu\nu} \Gamma^\rho{}_{\sigma\lambda} - \Gamma^\sigma{}_{\mu\lambda} \Gamma^\rho{}_{\sigma\nu} \right) = \frac{1}{K} R^\rho{}_{\mu\nu\lambda} \quad (5)\end{aligned}$$

In Ricci tensor  $R_{\mu\nu}$ ,

$$R'_{\mu\nu} = R'^\rho{}_{\mu\rho\nu} = R^\rho{}_{\mu\rho\nu} = R_{\mu\nu}, \quad \bar{R}_{\mu\nu} = \bar{R}^\rho{}_{\mu\rho\nu} = \frac{1}{K} R^\rho{}_{\mu\rho\nu} = \frac{1}{K} R_{\mu\nu} \quad (6)$$

In curvature scalar  $R$

$$\begin{aligned}R' &= f^{\mu\nu} R'_{\mu\nu} = \frac{1}{K} g^{\mu\nu} R_{\mu\nu} = \frac{1}{K} R \\ \bar{R} &= \bar{g}^{\mu\nu} \bar{R}_{\mu\nu} = \frac{1}{K} g^{\mu\nu} R_{\mu\nu} = \frac{1}{K} R \quad (7)\end{aligned}$$

Hence, in the gravity field equation of Einstein,

$$\begin{aligned}R'_{\mu\nu} - \frac{1}{2} f_{\mu\nu} R' &= R_{\mu\nu} - \frac{1}{2} K g_{\mu\nu} \left( \frac{1}{K} R \right) \\ &= R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -\frac{8\pi G}{c^4} T_{\mu\nu} \\ \bar{R}_{\mu\nu} - \frac{1}{2} \bar{g}_{\mu\nu} \bar{R} &= \frac{1}{K} \left( R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R \right) \\ &= -\frac{8\pi G}{c^4} \frac{1}{K} T_{\mu\nu} \quad (8)\end{aligned}$$

In Newtonian approximation, Energy-momentum tensor  $T_{\mu\nu}$  is

$$\begin{aligned}\nabla^2 f_{00} = \nabla^2 K g_{00} &\approx -\frac{8\pi G}{c^4} K T_{00} = -\frac{8\pi G}{c^4} T'_{00} \\ \rho c^2 = T_{00}, \quad K \rho c^2 &= T'_{00} \quad (9)\end{aligned}$$

$$\begin{aligned}\bar{\nabla}^2 \bar{g}_{00} = \frac{1}{K} \nabla^2 g_{00} &\approx -\frac{8\pi G}{c^4} \frac{1}{K} T_{00} = -\frac{8\pi G}{c^4} \bar{T}_{00} \\ \rho c^2 = T_{00}, \quad \frac{1}{K} \rho c^2 &= \bar{T}_{00} \quad (10)\end{aligned}$$

Hence,

$$T'_{\mu\nu} = K T_{\mu\nu}, \quad \frac{1}{K} T_{\mu\nu} = \bar{T}_{\mu\nu} \quad (11)$$

Therefore, revised Einstein's gravity field equation is

$$\begin{aligned} R'_{\mu\nu} - \frac{1}{2} f_{\mu\nu} R' &= R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -\frac{8\pi G}{c^4} T_{\mu\nu} = -\frac{8\pi G}{c^4} \frac{T'_{\mu\nu}}{K} \\ \bar{R}_{\mu\nu} - \frac{1}{2} \bar{g}_{\mu\nu} \bar{R} &= \frac{1}{K} (R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R) = -\frac{1}{K} \frac{8\pi G}{c^4} T_{\mu\nu} = -\frac{8\pi G}{c^4} \bar{T}_{\mu\nu} \end{aligned} \quad (12)$$

Therefore, revised gravity field equation of tensor  $\bar{g}_{\mu\nu}$  is able to reduce Einstein's gravity field equation.

Therefore,

$$\begin{aligned} \bar{g}^{\mu\nu} [\bar{R}_{\mu\nu} - \frac{1}{2} \bar{g}_{\mu\nu} \bar{R}] &= \frac{1}{K} g^{\mu\nu} [R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R] = -\frac{8\pi G}{c^4} \frac{1}{K} g^{\mu\nu} T_{\mu\nu} = -\frac{8\pi G}{c^4} \frac{1}{K} T^{\lambda}_{\lambda} \\ &= -\frac{8\pi G}{c^4} \bar{g}^{\mu\nu} \bar{T}_{\mu\nu} = -\frac{8\pi G}{c^4} \bar{T}^{\lambda}_{\lambda} \\ \rightarrow -\bar{R} &= -\frac{1}{K} R = -\frac{8\pi G}{c^4} \frac{1}{K} T^{\lambda}_{\lambda} = -\frac{8\pi G}{c^4} \bar{T}^{\lambda}_{\lambda} \end{aligned} \quad (13)$$

Hence,

$$\frac{1}{K} T^{\lambda}_{\lambda} = \bar{T}^{\lambda}_{\lambda} \quad (14)$$

Ricci tensor is

$$\bar{R}_{\mu\nu} = \frac{1}{K} R_{\mu\nu} = -\frac{8\pi G}{c^4} \frac{1}{K} (T_{\mu\nu} - \frac{1}{2} g_{\mu\nu} T^{\lambda}_{\lambda}) = -\frac{8\pi G}{c^4} (\bar{T}_{\mu\nu} - \frac{1}{2} \bar{g}_{\mu\nu} \bar{T}^{\lambda}_{\lambda}) \quad (15)$$

The proper distance is

$$ds^2 = g_{\mu\nu} dx^{\mu} dx^{\nu}, \quad ds'^2 = f_{\mu\nu} dx'^{\mu} dx'^{\nu} = \bar{g}_{\mu\nu} d\bar{x}^{\mu} d\bar{x}^{\nu} \quad (16)$$

## 2. Weak gravity field approximation.

Weak gravity field approximation is

$$\begin{aligned} g_{\mu\nu} &= \eta_{\mu\nu} + h_{\mu\nu}, \quad |h_{\mu\nu}| \ll 1 \\ R_{\mu\nu} &= -\frac{8\pi G}{c^4} (T_{\mu\nu} - \frac{1}{2} g_{\mu\nu} T^{\lambda}_{\lambda}) \end{aligned} \quad (17)$$

According to Eq(15),

$$\bar{g}_{\mu\nu} = \bar{\eta}_{\mu\nu} + \bar{h}_{\mu\nu}, \quad |\bar{h}_{\mu\nu}| \ll 1$$

$$\bar{R}_{\mu\nu} = -\frac{8\pi G}{c^4} (\bar{T}_{\mu\nu} - \frac{1}{2} \bar{g}_{\mu\nu} \bar{T}^{\lambda}_{\lambda}) \quad (18)$$

The tensor of weak gravity field is

$$\begin{aligned} R_{\mu\nu} &\approx -\frac{8\pi G}{c^4} S_{\mu\nu}, \quad \bar{R}_{\mu\nu} \approx -\frac{8\pi G}{c^4} \bar{S}_{\mu\nu} \\ S_{\mu\nu} &= T_{\mu\nu} - \frac{1}{2} \eta_{\mu\nu} T^{\lambda}_{\lambda}, \\ \bar{S}_{\mu\nu} &= \bar{T}_{\mu\nu} - \frac{1}{2} \bar{\eta}_{\mu\nu} \bar{T}^{\lambda}_{\lambda} \end{aligned} \quad (19)$$

The solution is

$$\begin{aligned} h_{\mu\nu}(t, \vec{x}) &= \frac{4G}{c^2} \int d^4 x' \frac{S_{\mu\nu}(t - |\vec{x} - \vec{x}'|, \vec{x}')}{|\vec{x} - \vec{x}'|}, \\ \int d^3 x T_{00} &= M \end{aligned} \quad (20)$$

$$\begin{aligned} \bar{h}_{\mu\nu}(\bar{t}, \vec{\bar{x}}) &= \frac{4G}{c^2} \int d^4 \bar{x}' \frac{\bar{S}_{\mu\nu}(\bar{t} - |\vec{\bar{x}} - \vec{\bar{x}}'|, \vec{\bar{x}}')}{|\vec{\bar{x}} - \vec{\bar{x}}'|}, \\ \int d^3 \bar{x} \bar{T}_{00} &= \int K \sqrt{K} d^3 x \frac{1}{K} T_{00} = \sqrt{K} M = \bar{M}, \quad T_{00} = K \bar{T}_{00} \end{aligned} \quad (21)$$

As

$$\begin{aligned} \bar{h}_{00}(\vec{\bar{x}}) &\approx \frac{4G}{\bar{r}c^2} \int d^3 \bar{x}' [\bar{T}_{00} - \frac{1}{2} \bar{T}_{00}] = \frac{2\sqrt{K}GM}{\bar{r}c^2}, \\ \bar{h}_{ij}(\vec{\bar{x}}) &\approx \frac{4G}{\bar{r}c^2} \int d^3 \bar{x}' [\frac{1}{2} \delta_{ij} \bar{T}_{00}] = \frac{2\sqrt{K}GM}{\bar{r}c^2} \delta_{ij}, \end{aligned} \quad (22)$$

The proper distance is

$$-ds^2 = c^2 d\tau^2 = -g_{\mu\nu} dx^\mu dx^\nu \approx (1 - \frac{2GM}{rc^2}) c^2 dt^2 - (1 + \frac{2GM}{rc^2}) \delta_{ij} dx^i dx^j \quad (23)$$

The proper distance is in this theory

$$\begin{aligned} -ds'^2 &= -K ds^2 = K c^2 d\tau^2 = -K g_{\mu\nu} dx^\mu dx^\nu \\ &\approx K (1 - \frac{2GM}{rc^2}) c^2 dt^2 - K (1 + \frac{2GM}{rc^2}) \delta_{ij} dx^i dx^j \\ &= (1 - \frac{2\sqrt{K}GM}{\bar{r}c^2}) c^2 d\bar{t}^2 - (1 + \frac{2\sqrt{K}GM}{\bar{r}c^2}) \delta_{ij} d\bar{x}^i d\bar{x}^j \end{aligned}$$

$$\begin{aligned}
&= \left(1 - \frac{2G\bar{M}}{\bar{r}c^2}\right)c^2 d\bar{t}^2 - \left(1 + \frac{2G\bar{M}}{\bar{r}c^2}\right)\delta_{ij}d\bar{x}^i d\bar{x}^j \\
&\approx \bar{g}_{\mu\nu}d\bar{x}^\mu d\bar{x}^\nu
\end{aligned}$$

$$\sqrt{K}t = \bar{t}, \sqrt{K}x^i = \bar{x}^i, \sqrt{K}x^j = \bar{x}^j, \sqrt{K}r = \bar{r}, \sqrt{KM} = \bar{M} \quad (24)$$

### 3. The other representation in Schwarzschild solution, Reissner-Nodstrom solution, Kerr-Newman solution and Robertson-Walker solution

Schwarzschild solution (vacuum solution) is

$$\begin{aligned}
R_{\mu\nu} &= 0 \\
ds^2 &= -c^2\left(1 - \frac{2GM}{rc^2}\right)dt^2 + \frac{dr^2}{1 - \frac{2GM}{rc^2}} + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \quad (25)
\end{aligned}$$

The other representation of Schwarzschild solution is

$$\begin{aligned}
ds'^2 &= f_{\mu\nu}dx^\mu dx^\nu = Kg_{\mu\nu}dx^\mu dx^\nu = K \cdot ds^2 \\
&= -c^2K\left(1 - \frac{2GM}{rc^2}\right)dt^2 + \frac{Kdr^2}{1 - \frac{2GM}{rc^2}} + Kr^2 d\theta^2 + Kr^2 \sin^2 \theta d\phi^2 \\
&= -c^2\left(1 - \frac{2\sqrt{K}GM}{\bar{r}c^2}\right)d\bar{t}^2 + \frac{d\bar{r}^2}{1 - \frac{2\sqrt{K}GM}{\bar{r}c^2}} + \bar{r}^2 d\bar{\theta}^2 + \bar{r}^2 \sin^2 \bar{\theta} d\bar{\phi}^2 \\
&= -c^2\left(1 - \frac{2G\bar{M}}{\bar{r}c^2}\right)d\bar{t}^2 + \frac{d\bar{r}^2}{1 - \frac{2G\bar{M}}{\bar{r}c^2}} + \bar{r}^2 d\bar{\theta}^2 + \bar{r}^2 \sin^2 \bar{\theta} d\bar{\phi}^2 = \bar{g}_{\mu\nu}d\bar{x}^\mu d\bar{x}^\nu
\end{aligned}$$

$$\sqrt{K}t = \bar{t}, \sqrt{K}r = \bar{r}, \theta = \bar{\theta}, \phi = \bar{\phi}, \sqrt{KM} = \bar{M} \quad (26)$$

Reissner-Nodstrom solution is

$$\begin{aligned}
ds^2 &= g_{\mu\nu}dx^\mu dx^\nu \\
&= -c^2\left(1 - \frac{2GM}{rc^2} + \frac{kGQ^2}{r^2c^4}\right)dt^2 + \frac{dr^2}{1 - \frac{2GM}{rc^2} + \frac{kGQ^2}{r^2c^4}} + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \quad (27)
\end{aligned}$$

The other representation of Reissner-Nodstrom solution is

$$\begin{aligned}
ds^2 &= f_{\mu\nu} dx^\mu dx^\nu = Kg_{\mu\nu} dx^\mu dx^\nu = K \cdot ds^2 \\
&= -Kc^2 \left(1 - \frac{2GM}{rc^2} + \frac{kGQ^2}{r^2 c^4}\right) dt^2 + \frac{Kdr^2}{1 - \frac{2GM}{rc^2} + \frac{kGQ^2}{r^2 c^4}} + Kr^2 d\theta^2 + Kr^2 \sin^2 \theta d\phi^2 \\
&= -c^2 \left(1 - \frac{2\sqrt{K}GM}{\bar{r}c^2} + \frac{KkGQ^2}{\bar{r}^2 c^4}\right) d\bar{t}^2 + \frac{d\bar{r}^2}{1 - \frac{2\sqrt{K}GM}{\bar{r}c^2} + \frac{KkGQ^2}{\bar{r}^2 c^4}} + \bar{r}^2 d\bar{\theta}^2 + \bar{r}^2 \sin^2 \bar{\theta} d\bar{\phi}^2 \\
&= -c^2 \left(1 - \frac{2G\bar{M}}{\bar{r}c^2} + \frac{kG\bar{Q}^2}{\bar{r}^2 c^4}\right) d\bar{t}^2 + \frac{d\bar{r}^2}{1 - \frac{2G\bar{M}}{\bar{r}c^2} + \frac{kG\bar{Q}^2}{\bar{r}^2 c^4}} + \bar{r}^2 d\bar{\theta}^2 + \bar{r}^2 \sin^2 \bar{\theta} d\bar{\phi}^2 \\
&= \bar{g}_{\mu\nu} d\bar{x}^\mu d\bar{x}^\nu
\end{aligned}$$

$$\sqrt{K}t = \bar{t}, \sqrt{K}r = \bar{r}, \theta = \bar{\theta}, \phi = \bar{\phi}, \sqrt{KM} = \bar{M}, KQ^2 = \bar{Q}^2 \quad (28)$$

Kerr-Newman solution is

$$\begin{aligned}
ds^2 &= g_{\mu\nu} dx^\mu dx^\nu \\
&= -c^2 \left(1 - \frac{2c^2 GMr - kGQ^2}{c^4 \Sigma}\right) dt^2 + 2(2c^2 MGr - kGQ^2) \frac{a \sin^2 \theta}{c^4 \Sigma} c dt d\phi \\
&\quad - \frac{c^4 \Sigma}{r^2 - c^2 2GMr + a^2 + kGQ^2} dr^2 - \Sigma d\theta^2 \\
&\quad - \sin^2 \theta [r^2 + a^2 + (2c^2 GMr - kGQ^2) \frac{a^2 \sin^2 \theta}{c^4 \Sigma}] d\phi^2
\end{aligned}$$

$$\Sigma = r^2 + a^2 \cos^2 \theta \quad (29)$$

The other representation of Kerr-Newman solution is

$$\begin{aligned}
ds^2 &= f_{\mu\nu} dx^\mu dx^\nu = Kg_{\mu\nu} dx^\mu dx^\nu = K \cdot ds^2 \\
&= -Kc^2 \left(1 - \frac{2c^2 GMr - kGQ^2}{c^4 \Sigma}\right) dt^2 + 2K(2c^2 MGr - kGQ^2) \frac{a \sin^2 \theta}{c^4 \Sigma} c dt d\phi \\
&\quad - \frac{K\Sigma c^4}{r^2 - 2c^2 GMr + a^2 + kGQ^2} dr^2 - K\Sigma d\theta^2 \\
&\quad - K \sin^2 \theta [r^2 + a^2 + (2c^2 GMr - kGQ^2) \frac{a^2 \sin^2 \theta}{c^4 \Sigma}] d\phi^2
\end{aligned}$$

$$\begin{aligned}
&= -c^2 \left(1 - \frac{2c^2 G \sqrt{KM} \sqrt{Kr} - kGKQ^2}{K\Sigma c^4}\right) dt^2 + 2(2c^2 \sqrt{KM} G \sqrt{Kr} - kGKQ^2) \frac{\sqrt{Ka} \sin^2 \theta}{K\Sigma c^4} cd\sqrt{K}td\phi \\
&\quad - \frac{K\Sigma c^4}{Kr^2 - 2c^2 G \sqrt{KM} \sqrt{Kr} + Ka^2 + kGKQ^2} d(\sqrt{Kr})^2 - K\Sigma d\theta^2 \\
&\quad - \sin^2 \theta [Kr^2 + Ka^2 + (2c^2 G \sqrt{KM} \sqrt{Kr} - kGKQ^2) \frac{Ka^2 \sin^2 \theta}{K\Sigma c^4}] d\phi^2 \\
&= -c^2 \left(1 - \frac{2c^2 G \bar{M} \bar{r} - kG\bar{Q}^2}{\bar{\Sigma} c^4}\right) d\bar{t}^2 + 2(2c^2 \bar{M} G \bar{r} - kG\bar{Q}^2) \frac{\bar{a} \sin^2 \bar{\theta}}{\bar{\Sigma} c^4} cd\bar{t}d\bar{\phi} \\
&\quad - \frac{\bar{\Sigma} c^4}{\bar{r}^2 - 2c^2 G \bar{M} \bar{r} + \bar{a}^2 + kG\bar{Q}^2} d\bar{r}^2 - \bar{\Sigma} d\bar{\theta}^2 \\
&\quad - \sin^2 \bar{\theta} [\bar{r}^2 + \bar{a}^2 + (2c^2 G \bar{M} \bar{r} - kG\bar{Q}^2) \frac{\bar{a}^2 \sin^2 \bar{\theta}}{c^4 \bar{\Sigma}}] d\bar{\phi}^2 \\
&= \bar{g}_{\mu\nu} d\bar{x}^\mu d\bar{x}^\nu
\end{aligned}$$

$$\bar{\Sigma} = K\Sigma = Kr^2 + Ka^2 \cos^2 \theta = \bar{r}^2 + \bar{a}^2 \cos^2 \bar{\theta}$$

$$\sqrt{K}t = \bar{t}, \sqrt{K}r = \bar{r}, \theta = \bar{\theta}, \phi = \bar{\phi}, \sqrt{KM} = \bar{M}, KQ^2 = \bar{Q}^2, \sqrt{Ka} = \bar{a},$$

$$\bar{J} = c\bar{M}\bar{a} = KcMa = KJ \quad (30)$$

In this time, we obtain the data of the time  $t$ , the distance  $r$ , the mass  $M$ , the charge  $Q$  and the angular momentum  $J$ .

Robertson-Walker solution is

$$\begin{aligned}
ds^2 &= g_{\mu\nu} dx^\mu dx^\nu \\
&= -c^2 dt^2 + \Omega^2(t) \left[ \frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \right] \quad (31)
\end{aligned}$$

The other representation of Robertson-Walker solution is by the other scalar  $K'$ ,

$$\begin{aligned}
ds'^2 &= f_{\mu\nu} dx^\mu dx^\nu = K' g_{\mu\nu} dx^\mu dx^\nu = K' ds^2 \\
&= -K' c^2 dt^2 + \Omega^2(t) \left[ \frac{K' dr^2}{1 - k \frac{K' r^2}{K'}} + K' r^2 d\theta^2 + K' r^2 \sin^2 \theta d\phi^2 \right] \\
&= -c^2 d\bar{t}^2 + \bar{\Omega}^2(\bar{t}) \left[ \frac{d\bar{r}^2}{1 - \frac{k\bar{r}^2}{K'}} + \bar{r}^2 d\bar{\theta}^2 + \bar{r}^2 \sin^2 \bar{\theta} d\bar{\phi}^2 \right]
\end{aligned}$$



$$= -c^2 d\bar{t}^2 + \bar{\Omega}^2(\bar{t}) \left[ \frac{d\bar{r}^2}{1 - k' \bar{r}^2} + \bar{r}^2 d\bar{\theta}^2 + \bar{r}^2 \sin^2 \bar{\theta} d\bar{\phi}^2 \right] = \bar{g}_{\mu\nu} d\bar{x}^\mu d\bar{x}^\nu$$

$$\sqrt{K'} t = \bar{t}, \Omega(t) = \bar{\Omega}(\bar{t}),$$

$$\sqrt{K'} r = \bar{r}, \theta = \bar{\theta}, \phi = \bar{\phi}$$

$$k = (0, 1, -1), \quad k' = \frac{k}{K'} = \left(0, \frac{1}{K'}, -\frac{1}{K'}\right) \quad (32)$$

Hence,  $K' = 1$ , In this time,  $ds^2$  is an uniqueness. This theory didn't apply the cosmology.

#### 4. Obtaining process information of Hawking radiation

Stephen Hawking found black-hole's thermodynamics. By Hawking Radiation, we obtain the new data from formulas of black-hole's thermodynamics. We start the obtaining process informations of Hawking Radiation. In Wikipedia (Hawking Radiation), we know formulas of Hawking Radiation.

The near horizon temperature of Schwarzschild black-hole is

$$T = \frac{\hbar c^3}{4\pi G k_B \sqrt{2M \left( \frac{c^2}{G} r - 2M \right)}}$$

(33)

The near horizon temperature of black-hole is in Data General Relativity theory

$$\begin{aligned} \bar{T} &= \frac{\hbar c^3}{4\pi G k_B \sqrt{2\bar{M} \left( \frac{c^2}{G} \bar{r} - 2\bar{M} \right)}} = \frac{\hbar c^3}{4\pi G k_B \sqrt{2\sqrt{KM} \left( \frac{c^2}{G} \sqrt{K} r - 2\sqrt{KM} \right)}} \\ &= \frac{\hbar c^3}{4\pi G k_B \sqrt{K} \sqrt{2M \left( \frac{c^2}{G} r - 2M \right)}} \\ &= \frac{T}{\sqrt{K}} \end{aligned} \quad (34)$$

The radiation temperature  $T_H$  of Schwarzschild black hole

$$T_H = \frac{\hbar c^3}{8\pi G M k_B} \quad (35)$$

The radiation temperature  $\bar{T}_H$  is in Data General Relativity theory.

$$\bar{T}_H = \frac{\hbar c^3}{8\pi G \bar{M} k_B} = \frac{\hbar c^3}{8\pi G \sqrt{KM} k_B} = \frac{T_H}{\sqrt{K}} \quad (36)$$

The black hole's entropy

$$dS = 8\pi Gk_B M dM / \hbar c = d(4\pi M^2) Gk_B / \hbar c = \frac{dQ}{T} \quad (37)$$

The black-hole's entropy  $\bar{S}$  is in Data General Relativity theory.

$$\begin{aligned} d\bar{S} &= 8\pi \bar{M} d\bar{M} Gk_B / \hbar c = d(4\pi \bar{M}^2) Gk_B / \hbar c = d(4\pi K M^2) Gk_B / \hbar c \\ &= K dS = \frac{d\bar{Q}}{\bar{T}} = \frac{d(\sqrt{K}Q)}{\sqrt{K}T} \end{aligned}$$

$$\bar{S} = KS, \quad \bar{Q} = \sqrt{K}Q \quad (38)$$

Black-hole radiation's power  $P_{ev}$  is

$$P_{ev} = A_s \varepsilon \sigma T_H^4, \quad A_s = 4\pi r_s^2, \quad r_s = \frac{2GM}{c^2}, \quad \varepsilon, \sigma \text{ is constant} \quad (39)$$

Black-hole radiation's power  $\bar{P}_{ev}$  is in Data General Relativity theory.

$$\begin{aligned} \bar{A}_s &= 4\pi \bar{r}_s^2 = K 4\pi r_s^2 = K A_s, \quad \bar{r}_s = \frac{2G\bar{M}}{c^2} = \frac{2G\sqrt{K}M}{c^2} = \sqrt{K} r_s \\ \bar{P}_{ev} &= \bar{A}_s \varepsilon \sigma \bar{T}_H^4 = K A_s \varepsilon \sigma \frac{T^4}{K^2} = \frac{P_{ev}}{K} \end{aligned}$$

$$\text{Stefan-Boltzmann constant } \sigma = \frac{\pi^2 k_B^4}{60 \hbar^3 c^2},$$

Black-hole is a perfect blackbody ( $\varepsilon = 1$ ) (40)

The power of evaporation energy of the black-hole is

$$P_{ev} = -\frac{dE_{ev}}{dt} \quad (41)$$

The power of evaporation energy of the black-hole is in Data General Relativity theory.

$$\bar{P}_{ev} = -\frac{d\bar{E}_{ev}}{d\bar{t}} = -\frac{d(E_{ev} / \sqrt{K})}{\sqrt{K} dt} = \frac{P_{ev}}{K},$$

$$\bar{M}_{ev} c^2 = \bar{E}_{ev} = \frac{E_{ev}}{\sqrt{K}} = \frac{M_{ev}}{\sqrt{K}} c^2 \quad (42)$$

The evaporation time  $t_{ev}$  of a black hole is

$$t_{ev} = \frac{5120 \pi G^2 M^3}{\hbar c^4} \quad (43)$$

The evaporation time  $\bar{t}_{ev}$  of a black hole is in Data General Relativity theory.

$$\bar{t}_{ev} = \frac{5120 \pi G^2 \bar{M}^3}{\hbar c^4} = \frac{5120 \pi G^2 K \sqrt{K} M^3}{\hbar c^4} = K \sqrt{K} t_{ev} \quad (44)$$

## 5. Conclusion

We find the other representation of solutions in the General relativity theory. In this time, Robertson-Walker solution is an uniqueness. We more obtain the information of black-hole thermodynamics in Data General Relativity theory.

If we use variable  $\bar{A}$  instead of  $A$ , Data General Relativity theory is reduced to normal general relativity theory. This theory's remarkable thing is if  $\sqrt{K} = 2$  and sun's mass  $M$  is  $M\sqrt{K} = 2M$ , sun's distant of gravitation  $r$  is  $r\sqrt{K} = 2r$ , sun's proper time  $\tau$  is  $\tau\sqrt{K} = 2\tau$ . If rotating black hole's mass  $M$  is to be  $M\sqrt{K} = 2M$ , we predict the angular momentum  $J$  of the black-hole is to be  $KJ = 4J$ .

In this time, we have to apply only black-holes. BH is Black hole.

## Appendix A

In this theory, we have to apply only black-holes.

According to Paul H. Frampton, Physical Letter B(2017), if the mass of sun is  $M_{\odot}$ , observational data is

Astrophysical Object	Mass solar masses	Period $\tau$ seconds	Angular momentum
BH1	$20M_{\odot}$	0.013s	$3.0 \times 10^{37}$
BH2	$100 M_{\odot}$	0.063s	$7.2 \times 10^{38}$
BH3	$1000 M_{\odot}$	0.63s	$7.2 \times 10^{40}$
BH4	$10^4 M_{\odot}$	6.3s	$7.2 \times 10^{42}$
BH5	$10^5 M_{\odot}$	63s	$7.2 \times 10^{44}$

BH6(M87)                       $6 \times 10^9 M_{\odot}$                        $3.8 \times 10^6 s$                        $2.6 \times 10^{54}$

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According to this theory, BH's mass  $M$  is to be  $\sqrt{KM}$ , time  $\tau$  is to be  $\tau\sqrt{K}$ , Angular momentum  $J$  is to be  $KJ$ . Hence, BH2's (from BH1)  $\sqrt{K}$  is 5, BH3's (from BH 2)  $\sqrt{K}$  is 10, BH4's (from BH3)  $\sqrt{K}$  is 10, BH5's (from BH4)  $\sqrt{K}$  is 10, BH6's (from BH5)  $\sqrt{K}$  is  $6 \times 10^4$ .

Therefore, calculated data is in this theory,

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Astrophysical Object	Mass solar masses	Period $\tau$ seconds	Angular momentum
BH1	$20 M_{\odot}$	0.013s	$3.0 \times 10^{37}$
BH2	$100 M_{\odot}$	0.062s	$7.5 \times 10^{38}$
BH3	$1000 M_{\odot}$	0.62s	$7.5 \times 10^{40}$
BH4	$10^4 M_{\odot}$	6.2s	$7.5 \times 10^{42}$
BH5	$10^5 M_{\odot}$	62s	$7.5 \times 10^{44}$
BH6(M87)	$6 \times 10^9 M_{\odot}$	$3.72 \times 10^6 s$	$2.7 \times 10^{54}$

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