Partition into triangles revisited

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Abstract

We show that if one has ever loved reading Prasolov's books, then one can move on reading our recent article [3] and several words following to deduce that partitioning a graph into triangles is not an easy problem.

I. Proposition

A classical result claims that *three-dimensional matching* (3DM) is NP-complete. This was proved in [1, Chap. 3, pp.50-52].

Proposition: 3DM $\cong_p$ Partition into Triangles

Proof: At the end of this article, we capture a concise picture scanned from G&J book. Given a 3DM instance, we construct our graph as follows. The vertex set is the same as in the hypergraph of the given instance. For each triple $\{\ a,\ b,\ c\}$ in the given instance, we put three edges $(a, b), (b, c), (a, c)$. By a careful scrutiny the contents in the picture, we can conclude that if one can pick any triangle in our newly constructed graph, it must also be a triple in the given 3DM instance. (*Hint*: Consider three cases of *ab*, *ss*, *gg* in the picture) **Q.E.D.**

II. Conclusion

As long as we do the research on a well-known conjecture, we should recall our mathematical nature from Kvant, Prasolov-style of doing mathematics, similar to mathematics of [2] back to those beautiful days.

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References

[1] Michael R. Garey, David S. Johnson, **Computers and Intractability: A Guide to the Theory of NP-Completeness**

[2] Phan Dinh Dieu, Le Cong Thanh, Le Tuan Hoa, **Average Polynomial Time Complexity of Some NP-Complete Problems**. Theor. Comput. Sci. 46(3): 219-237 (1986)

[3] Thinh D. Nguyen, **Exact Weight Perfect Matching of Bipartite Graph Problem Simplified**, viXra:1806.0179

eral, the truth-setting and fan-out component for a variable u_i involves "internal" elements $a_i[j] \in X$ and $b_i[j] \in Y$, $1 \leqslant j \leqslant m$, which will not occur in any triples outside of this component, and "external" elements $u_i[j], \bar{u}_i[j] \in W$, $1 \leqslant j \leqslant m$, which *will* occur in other triples. The triples making up this component can be divided into two sets:

$$T_i^t = \{(\bar{u}_i[j], a_i[j], b_i[j]) : 1 \leqslant j \leqslant m\}$$
$$T_i^f = \{(u_i[j], a_i[j+1], b_i[j]) : 1 \leqslant j < m\} \cup \{(u_i[m], a_i[1], b_i[m])\}$$

Since none of the internal elements $\{a_i[j], b_i[j] : 1 \leqslant j \leqslant m\}$ will appear in any

literals occur in clause c_j. The set of triples making up this component is defined as follows:

$$C_j = \{(u_i[j], s_1[j], s_2[j]) : u_i \in c_j\} \cup \{(\bar{u}_i[j], s_1[j], s_2[j]) : \bar{u}_i \in c_j\}$$

Thus any matching $M' \subseteq M$ will have to contain exactly one triple from C_j. This can only be done, however, if some $u_i[j]$ (or $\bar{u}_i[j]$) for a literal $u_i \in c_j$ ($\bar{u}_i \in c_j$) does not occur in the triples in $T_i \cap M'$, which will be the case if and only if the truth setting determined by M' satisfies clause c_j.

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The construction is completed by means of one large "garbage collection" component G, involving internal elements $g_1[k] \in X$ and $g_2[k] \in Y$, $1 \leqslant k \leqslant m(n-1)$, and external elements of the form $u_i[j]$ and $\bar{u}_i[j]$ from W. It consists of the following set of triples:

$$G = \{(u_i[j], g_1[k], g_2[k]), (\bar{u}_i[j], g_1[k], g_2[k]) : 1 \leqslant k \leqslant m(n-1), 1 \leqslant i \leqslant n, 1 \leqslant j \leqslant m\}$$

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From the comments made during the description of M, it follows immediately that M cannot contain a matching unless C is satisfiable. We now must show that the existence of a satisfying truth assignment for C implies that M contains a matching.

Let $t: U \rightarrow \{T, F\}$ be any satisfying truth assignment for C. We construct a matching $M' \subseteq M$ as follows: For each clause $c_j \in C$, let $z_j \in \{u_i, \bar{u}_i : 1 \leqslant i \leqslant n\} \cap c_j$ be a literal that is set true by t (one must exist since t satisfies c_j). We then set