Proof of the Fourth Landau's Problem

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***Abstract*—This article solves the problem for the third time using the Formula of Disjoint Sets of Odd Numbers numbers that I proposed**

***Index Terms*—algorithm**

I. Definition of the Fourth Landau's Problem

Is the set of primes of the form (n^2+1) infinite, where $n \in \mathbb{N}^*$ and $\mathbb{N}^*$ be natural numbers without zero?

II. Algorithm for Proof of the Fourth Landau's Problem

Let the odd number be y.
Half of the numbers (n^2+1), following odd $n^2 = y^2$ are even and correspondingly composite numbers.
The other half (n^2+1) follows an even number of $n^2 = (y+1)^2$.
Then:

$$n^2+1 = (y+1)^2+1 = y^2+2y+2 = y(y+2)+2. \quad (1)$$

Let's represent (1) with respect to the odd number y_k following the given y:

$$n^2+1 = y_k(y_k-2)+2. \quad (2)$$

Consider the sets:

$$\{y_n \mid y_k(y_k-2) < y_n < y_k^2,\ y_k \geq 3\}. \quad (3)$$

The number of terms in each set (3):

$$N_{y_n} = y_k - 1. \quad (4)$$

But the segments between $y_k(y_k-2)$ and y_k^2 are segments in a sequence of odd numbers for which the Formula of Disjoint Sets of Odd Numbers is valid. For the entire sequence of odd numbers y, it has the form of the following expression:

$$\begin{aligned} Z_y = \Bigg(& 0{,}0\ldots01\%(1) + 33{,}3\ldots3\%(\{3y\}) + \\ & + \sum_{n=3}^{n\to\infty} Z_{y_{on}} \Big(\{ y_{on} y_n \mid \frac{y_n}{3} \notin \mathbb{N}^*, \ldots \\ & \ldots, \frac{y_n}{y_{o(n-1)}} \notin \mathbb{N}^* \} \Big) \Bigg) \to 100\%, \end{aligned} \quad (5)$$

where:
Z_y is of appearance of all odd numbers y;
the number of digits represented by $(\ldots)$ in the first two terms $\to \infty$;
$Z_{y_{on}}$ is the frequency of appearance of the given set (in %) in the sequence $\{y\}$;
n is the number of a member of a sequence of odd primes;
y_n is a sequence of odd numbers with the conditions given in the formula;
$y_{o(n-1)}$ is the prime number in sequence of primes just before y_{on}.

For the segments (4) of (3) Formula of Disjoint Sets of Odd Numbers (5) takes the following form, where the percentage of the sets remains unchanged:

$$\begin{aligned} & Z_{y_{comp}}(\{y_{comp}\}) = \\ & = 33{,}3\ldots3\%(\{3y \mid y \geq 3,\ 3y = y_n\}) + \\ & + \sum_{m=3} Z_{y_{om}} \Big(\{ y_{om} y_m \mid y_m \geq y_{om},\ y_{om} y_m = y_n, \\ & y_{om} < N_{y_n}, \frac{y_m}{3} \notin \mathbb{N}^*, \ldots \frac{y_m}{y_{o(m-1)}} \notin \mathbb{N}^* \} \Big), \end{aligned} \quad (6)$$

where:
$Z_{y_{comp}}$ is the frequency of the appearance of composite numbers (in %) in a given segment of the sequence {y};
y_{comp} is a composite odd number in a given segment of a sequence of odd numbers y;
the number of digits represented by $(\ldots)$ in the first term, $\to \infty$;
m is the number of a member of a sequence of odd primes;
$Z_{y_{om}}$ is the frequency of appearance of the given set (in %) in the sequence $\{y\}$;
y_m is a sequence of odd numbers with the conditions given in the formula;
N_{y_n} is the number of terms in (3) (see (4)).

But since in the whole sequence of odd numbers y the frequency of appearance of known sets according to (5) only tends to 100%, then in the case of (6):

$$Z_{y_{comp}}(\{y_{comp}\}) < 100\%. \quad (7)$$

But the number represented by (2) is the first in (3).
Therefore, although with increasing y_k the probability of the appearance of a composite number in the first term of the set (3) increases, it never reaches 100%.

That is, the set of primes of the form (n^2+1) is infinite.

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REFERENCES