Refutation of conditional events in quantum logic

We assume the method and apparatus of Meth8/VŁ4 with `T`autology as the designated *proof* value, `F` as contradiction, `N` as truthity (non-contingency), and `C` as falsity (contingency). Results are a 16-valued truth table in row-major and horizontal, or repeating fragments of 128-tables for more variables.

LET p, q, r, s: a, b, c, d; ~ Not, " ' "; + Or, ∨, ∪; & And, ∧, ∩; \ Not And, |;
> Imply, greater than, ⊃; = Equivalent; @ Not Equivalent;
(p=p) `T`; (p@p) `F`, ordinal zero; ~(p>p) p≤p.

Section 1.

From: Calabrese, P.G. (2003). Toward a more natural expression of quantum logic with Boolean fractions. arxiv.org/abs/quant-ph/0305009 ; arxiv.org/ftp/quant-ph/papers/0305/0305009.pdf

LET p, q, r, s, t:
a, a1, c, c1, e;

3.1.1 Definition of Simultaneous Verifiability. Two members "a" and "c" of a logic *L* are *simultaneously verifiable*, a↔c, if there are members a_1, c_1, and e of *L* such that $a_1 \vee c_1=0$ and $a_1 \vee$=0 and $e \vee c_1=0$, and with $a=a_1 \vee e$ and $c=e \vee c_1$.

(((((q&s)=(p@p))&((q&t)=(p@p)))&((t&s)=(p@p))) & ((p=(q+t))&(r=(t+s))))>(p=r) ;
`TTTF TTTT TTTT FTTT,   TTTT TTTT TTTT TTTT` (3.1.2)

3.2.1 Theorem on Simultaneous Verifiability. In the conditional event algebra two conditionals (a|b) and (c|d) are *simultaneously verifiable*, (a|b)↔(c|d), if and only if ab≤d and cd≤b.

(~((p&q)>s)&~((r&s)>q))>((p\q)=(r\s)) ; `TTTT TTTT TTTT TTTT` (3.2.2)

3.3.1 Corollary on Simultaneous Verifiability. (a|b)↔(c|d) iff (a|b)∧(c|d)=(abcd|b∨d).

(((p\q)&(r\s))=(((p&q)&(r&s))\(q+s)))>((p\q)=(r\s)) ;
`TTTT TTTT TTTT TTTT` (3.3.2)

3.4.1 Definition (Simultaneous Falsifiability). Two conditionals (a|b) [,] (c|d) are *simultaneously falsifiable* if and only if their negations, (a'|b) [,] (c'|d), are simultaneously verifiable. That is, (a'|b)↔(c'|d).

((~p\q)=(~r\s))>((p\q)@(r\s)) ; `FFTT FFTT TTFT TTTF` (3.4.2)

3.5.1 Corollary [sic] on Simultaneous Falsifiability. In the conditional event logic two conditionals (a|b) [,] (c|d) are simultaneously falsifiable if and only if a'b ≤ d and c'd ≤ b, that is, is the falsity of one conditional implies the applicability of the other conditional.

(~((~p&q)>s)&~((~r&s)>q))>((p\q)@(r\s)); TTTT TTTT TTTT TTTT (3.5.2)

3.6.1 Corollary on Simultaneous Verifiability and Falsifiability. Two conditionals (a|b) [,] (c|d) are simultaneously verifiable and simultaneously falsifiable if and only if b=d.

(q=s)>(((p\q)=(r\s))&((p\q)@(r\s))) ; FFTT FFTT TTFF TTFF (3.6.2)

3.9.1 Theorem on Uniqueness of Relative Negation. Let (a|b) [,] (c|d) be two conditionals. Then (a|b)∧(c|d)=(0|b∨d) and (a|b)∨(c|d)=(1|b∨d) if and only if b=d and (c|d)=(a|b)’.

((q=s)&((r\s)&((q\s)=~(p\q))))>((((p\q)&(r\s))=((p@p)\(q+s)))&(((p\q)+(r\s))
=((p=p)\(q+s)))) ; TTTT TTTT TTFT TTTT (3.9.2)

For Section 1, Eqs. 3.2.2, 3.3.2, and 3.5.2 as rendered are tautologous. However, the other Eqs. are *not* tautologous. This means the following conjectures are refuted:

3.1.1 Definition of simultaneous verifiability;
3.4.1 Definition of simultaneous falsifiability;
3.6.1 Corollary on simultaneous verifiability and falsifiability; and
3.9.1 Theorem on uniqueness of relative negation.

Section 2.

From: Calabrese, P.G. (2018). Conditional events and quantum logic. Journal of Applied Mathematics and Physics. 6:.1278-89. file.scirp.org/pdf/JAMP_2018062714461720.pdf

... a conditional (a|b) can have any one of 3 truth-values:

(a|b) is *true* if a is true and b is true; (3.1.1)

((p=(p=p))&(q=(p=p)))>(p\q) ; TTTF TTTF TTTF TTTF (3.1.2)

(a|b) is *false* if a is false and b is true ; (3.2.1)

((p=(p@p))&(q=(p=p)))>(p\q) ; TTTT TTTT TTTT TTTT (3.2.2)

(a|b) is *inapplicable* [neither true nor false] if b is false. (3.3.1)

((p=(p@p))&(q=(p@p)))>(p\q) ; TTTT TTTT TTTT TTTT (3.3.2)

The relative negation of “a given b” is the “negation of a, given b”. That is,
(a|b)'=(~a|b)′ (4.1)
~(p\q)=~(~p\q) ; TTFF TTFF TTFF TTFF (4.2)

The disjunction ... (a|b)∨(c|d)=((ab∨cd)|(b∨d))=(ab∨cd|b∨d) (5.1)
(((p\q)+(r\s))=(((p&q)+(r&s))\(q+s)))=(((p&q)+(r&s))\(q+s)) ;
TTTT TTTT TTTT TTTF (5.2)

Conjunction ... $(a|b)\wedge(c|d)=[ab(c\vee d')\vee(a\vee b')cd]|(b\vee d)$ (6.1)

((p\q)&(r\s))=((((p&q)&(r+~s))+((p+~q)&(r&s)))\(q+s)) ;

`TTTT TTTT TTTF TTFT` (6.2)

$= (abd'\vee abcd\vee b'cd|b\vee d)$ (7.1)

(((p&q)&~s)+((p&q)&(r&s)))+((~q&(r&s))\(q+s)) ;

`TTTT TTTT TTTT FFTT` (7.2)

[Eqs 6.2≠7.2.]

Conditional conditionals $((a|b)|c=(a|bc)$ (8.1)

(((p\q)\r)=(p\(q&r)) ; `TTTT FFFF TTTT FFFF` (8.2)

[Iterated] conditional $((a|b)|(c|d))=(a|b\wedge(c|d))=(a|b(c\vee d'))$ (9.1)

(((p\q)\(r\s))=((p\q)&(r\s)))=((p\q)&(r+~s)) ;

`FFFT FFFT TTTT FFFT` (9.2)

The so-called “superposition” of a quantum particle in two possibly mutually inconsistent quantum states B, D can be represented in the new extension algebra of conditional events as the event A given condition B or event C given the alternate condition D. This non-Boolean object $((A|B)\cup(C|D))$ is a member of the new algebra, namely $((A\cap B)\cup(C\cap D)|(B\cup D))$, $[(((A\cap B)\cup(C\cap D))|(B\cup D))\supset((A|B)\cup (C|D))]$. (C1.1)

(((A&B)+(C&D))\(B+D))>((A\B)+(C\D)) ; `TTTT TTTT TTTT TTTT` (C1.2)

For Section 2, Eqs. 3.1.2 and 4.2-9.2 as rendered are *not* tautologous. Eqs. 3.2.2, 3.3.2, and C1.2 are tautologous.

Remark: As the concluding conjecture, Eq. C1.1 is to map superposition or entanglement. However the positions are co-existent until measured. Therefore, we deem a more accurate mapping of the superposition to be for the consequent to read in words as "the event A given condition B *and* event C given alternate condition D": $((A|B)\cap(C|D))$.

For the antecedent to map the presumed intention, it reflects the changed consequent as: $((A\cap B)\cup(C\cap D))|(B\cap D)$. The conjecture to be tested then becomes $(((A\cap B)\cup(C\cap D))|(B\cap D))\supset((A|B)\cap (C|D))$, as (((A&B)+(C&D))\(B&D))>((A\B)&(C\D)) ;

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TTTT TNTN TTCC TNCF, TTTT TTTT TTCC TTCC, NNNN TTTT NNFF TTCC,
TTTT TNTN TTTT TNTN, CCCC CFCF TTTT TNTN, TTTT TTTT TTTT TTTT,
NNNN TTTT NNNN TTTT, CCCC CCCC TTTT TTTT, FFFF CCCC NNNN TTTT
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