

On the origin of the laws of physics from the properties of algorithms

Alexandre Harvey-Tremblay

Independent researcher (aht@protonmail.ch)

July 23, 2018 — most recent revisions on July 24, 2018

I propose a method to derive the familiar laws of physics from algorithmic information theory (AIT). Specifically, I introduce the notion of a proven computing reserve and I use it to connect AIT to physics.

1 Introduction

We intuitively understand scientific inquiry as a methodology to improve our understanding of the objective world. According to falsifiability, the evidence is to be collected with the intent to falsify hypotheses. Through an iterative process, ever more validated physical theories are produced, tested, and falsified. Confidence in a scientific theory is increased by actively attempting to falsify it (and failing to do so). The end goal of scientific inquiry is a final theory which would presumably explain all known and future evidence.

In practice, the inquiry process is usually divided into an experimental part and a theoretical part. Experimentalists gather experimental data, patterns are noticed within this data, and theoreticians formalize these patterns within the model of mathematics.

Whilst those involved understand the world through the lenses of science and falsifiability, the theories so produced are, however, unaware of the process which created them. Indeed, each such formal theory is defined first and foremost as a set of axioms. Then, its theorems are the indubitable consequence of its axioms. Although falsifying a theory ought to be a scientist's primary motivation, this possibility of future falsification is, however, not derivable from the axioms of the theory alleged to be a correct description of reality.

This typical type of formal construction does not correspond to how the world is scientifically understood to be. First and foremost, scientists understand that the world is and thus its description overwrites and takes priority over any hypothesized set of axioms; scientifically, the axioms which explains the world are implied by the world. Consequently, in the framework of scientific inquiry and in practice, axioms are disposable, mutable, and interchangeable. Their shared general constraint is that each correct set of axioms must produce an accurate description of the world. How can we create a formal model aware of this information?

The model presented here semantically corrects this error. The primary step will be to reverse the usual formalization of a typical

physical theory. Instead of describing the theory (with axioms) and solving for a description of the world (listing the theorems), we will first describe the world, then solve for the theories that explain it. Reversing the problem in this way greatly reduces the difficulty. Let's introduce a few definitions that will allow us to do just that.

1.1 Definitions

We will formulate this work within the formal system of Zermelo-Fraenkel set theory (ZF).

We recall the definition of a language L over an alphabet Σ . The sentences of L are a subset of Σ^* , defined as the set of all sentences over the alphabet Σ . For instance, the alphabet of the binary language is the set $\Sigma := \{0, 1\}$. We define the set Σ^* as an infinite set containing all possible sentences over the alphabet Σ , thus $\Sigma^* := \{\epsilon, 0, 1, 00, 01, 10, 11, 000, \dots\}$ where ϵ is the empty sentence. The language L is a subset of Σ^* , thus $L \subseteq \Sigma^*$. A language can be categorized in many different ways depending on the computational complexity of the rules used to decide if a sentence of Σ^* is or is not a sentence of L . The most common classes are perhaps a) decidable and b) recursively enumerable.

We will now introduce a number of definitions, first in a heuristic sense, then formally. From philosophy, a fact is a statement that is consistent with reality or can be proven with evidence. The usual test for a statement of fact is verifiability — that is, whether it can be demonstrated to correspond to experience. Inspired by this definition, we introduce a formal analog to it as the foundation of the model using a definition for the world as a set of facts.

Definition 1.1.1 (World). *A fact is a sentence s of an alphabet Σ^* such that $s \in W$ and such that W can be constructed via a formal system T by listing its theorems. Thus, a World is:*

$$W := \{s : (s \in \Sigma^*) \wedge (T \vdash s)\} \quad (1.1.2)$$

In this definition, W is a language and T are the rules to validate the sentences. In the case of the real world, we understand T to be a final theory (e.g., the theory of everything). The goal is finding T on the understanding that W is "given".

We will say that the world is describable by a formal theory if it admits this definition, otherwise, it is not. As we will see, algorithmic information theory is well suited to study this definition of the world along with its connection to the theory that explains it.

1.2 *The World implies the theory*

This definition is key as it reverses the usual implication of T with respect to W . To see how this definition resolves the semantic error that we previously mentioned, let's compare it to a typical physical theory.

Typically in theoretical physics, T is hypothesized from experimental data and then tested. The axioms of T are presented as the primary actors of the theory, and they command the most attention. In most cases, the axioms directly represent the laws or symmetries of nature and are inspired from empirical data.

Using mathematics, a theoretical physicist can unpack the axioms of T into theorems. If all theorems of the theory are found to be an element of W , then the theory is validated, and thus, it has survived falsification. However, if a mismatch is found, T is falsified and must be replaced with an alternative.

Due to this formulation, physical theories so produced will erroneously proclaim, on paper, that the world (the theorems) is a consequence of the theory. Indeed, formulated as such, the description of the world W is obtained by unpacking T into its theorems. This typical construction will semantically claim the following:

$$T \implies W \quad (\text{The theory implies the world}) \quad (1.2.1)$$

The knowledge that the origin of T actually lies within W is understood in the minds of those who hypothesized T (as the consequence of scientific inquiry) but is absent from the formal description of T . Constructed as such, T is fundamental, and W is a mere consequence of it.

As per the definition of W (definition 1.1.1), this implication is reversed for the model presented here. Indeed, in this new model, the fundamental actors are now the elements of W , and the formal theory T that explains W is a consequence of W . Thus, the relation is reversed, and the model semantically claims that it is instead the world that implies the theory:

$$W \implies T \quad (\text{The world implies the theory}) \quad (1.2.2)$$

Based on the results presented throughout this work, we would argue that the incorrect implication (i.e., the theory implies the world) is the primary error in the way T is typical constructed. Once the relationship is reversed, solving for T is surprisingly simple. In our model, rather than guessing T via iterative falsifiability, T will be obtained as a solution to the model using the tools of algorithmic information theory.

1.3 *The computational cost to explain a fact*

Due to non-negligible computing costs, one who sub-divides his explanation of the world into multiple theories, each applicable to a certain subset of the world, may be more successful and ultimately obtain a more complete understanding of reality than one who doesn't. Please note that here we distinguish between a more complete understanding and a more complete theory. In this context, a theory neglects the computing costs whereas an understanding doesn't. For example, it could be argued that a person who understands biology plus chemistry plus history possibly understands more about the world than someone only knowns of quantum fields but without access to a supercomputer to solve intractable problems. In the case of quantum fields, the theoretician is in possession of a more complete theory, but the subset of W which he has verified from the theory is smaller than the subset verified by the person who understands many different fields. Quantum fields can, in principle, be unpacked to recover the theorems of chemistry or biology but the computational cost to do so is intractable.

We understand, at least heuristically, that our understanding is computationally bounded. However, we perhaps have yet to fully appreciated its consequence as it pertains to the formalization of T and how it connects to our ability to verify or falsify it. Most facts of the world are computationally intractable with respect to the theory T that explains them. For example, determining if there is milk in the fridge is experimentally easy; by opening the door to the fridge and simply looking inside. However, obtaining the same answer by solving the equations of physics from the initial state of the Big Bang is intractable. Indeed, almost all facts of the universe as they stand today are the results of 13.7 billions years of extremely high-speed physical interactions and would require enormous amounts of computation to solve from first principles.

We are only beginning to encounter computer-assisted proofs in mathematics. The first and possibly most famous is the four-color theorem proven in 1997 by Robertson, Sanders, Seymour, and Thomas. Computer-assisted proofs are not without "mild controversy" in mathematics (i.e., they are considered unaesthetic by some). However, physically, most problems are intractable. Almost any question about the present state of the universe requires solving the equations from first principles; from the initial state of the Big Bang. Each such solution requires the simulation of all molecules which have or could have contributed to, say, the presence or absence of milk in the fridge. Solving this kind of problem for a macroscopic sized system is intractable.

Historically, our understanding of the world was improved by strong insights (i.e., short programs). Thus, there is a hope that accumulating insights is the key to making sense of the world. For example, the discovery of the scientific method (a relatively simple program) has contributed tremendously to our understanding of the world. Insights occur when one sees a new symmetry in nature or a simplified way to solve an equation, or even by finding a new exact equation to a complicated formula. However, insight is in limited supply. At some point, we will run out of insights (there are more long programs than short programs). Our ultimate and final ability to verify our understanding of the world is limited, in the most general of cases, by the computing resources made available to us by the world.

For instance, the use of computational resources to explain facts introduces an opportunity cost that must be paid for each fact that we chose to verify from the set of W . For instance, if a physicist spends 20 years of his career trying to explain one fact, then that is 20 fewer years available to him to explain other facts. The resource of time is consumed.

2 *The proven computing reserve*

Let's investigate this further with a more technical example. Let's say, without loss of generality, that a physicist picks one element s of W (e.g., one fact of the world) and produces an explanation for it. In algorithmic information theory, this explanation is a computer program p that takes as input the element of W it claims to explain. The explanation is verified if the program halts, and invalid if the program never halts. To credibly claim that the program is a valid explanation of s (i.e., $p(s)$ halts), the physicist must have, at a minimum, run the program once in order to verify that it does halt (Otherwise, he is just hoping that he has an explanation).

By carefully studying what the physicist has done, we are able to make claims in regards to his experimental setup from that alone. This will be our path into a description of the physical world. By credibly providing us with an explanation for a fact, we must conclude that the physicist has access, somehow, to computing resources in a quantity necessary to verify the explanation. We will call these resources the proven computing reserve.

The idea that computing resources are limited is often neglected (but never violated) if we understand the world primarily through computationally simple and aesthetically powerful insights. However, once we grow out of this regime, the limits on our ability to verify a theory T of the world W are, at their most general expression, for-

malized in terms of available computing resources.

We can precisely identify these computing resources by looking at the characteristics of the program the physicist has supplied to us. For example, say the program itself has 40 bits, and the element of W it explains has 700 bits. Thus, by verifying this explanation, the physicist has proven that he can access a machine able to read 740 bits of input. Furthermore, he must have run the program at least once. Thus the physicist has further proven that his machine can halt within a certain time (i.e., it has a certain runtime for this program). The claim that these resources must somehow exist (perhaps in a lab) is an unavoidable claim deduced purely from the application of algorithmic information theory as it pertains to the computational verification of facts.

This proven computing reserve define (and constraint) the laboratory of the physicist (here understood primarily as a supercomputer). Under these constraints, the physicist could have chosen a different fact to explain, or a different explanation for the same fact. The only constraint would be that the explanation of any fact he picked would have to be verified by consuming no more than the proven computing reserve. The physicist could, in principle, find additional resources to contribute to this goal (e.g., he could increase his supercomputing budget) and this could increase the potency of his explanations or the total number of facts that he is able to explain while keeping the potency of each explanation constant.

This computational relation, perhaps surprisingly, will be sufficient to recover the laws of physics as a necessary consequence of the computational connection between facts and their explanations. Let's first see in more detail why that would be the case; then we will show it explicitly.

2.1 *An inviolable relation*

A physicist has a lot of freedom in how to build and run experiments (e.g., how to set up his lab). However, whatever the physicist does or ultimately concludes, what he cannot do is violate the relation between the size of the proven computing reserve and the degree to which he can verify a theory T of the world W .

Due to the nature of this argument, our model implies the two following consequences:

- Via the notion of the proven computing reserve, claims are made about the real world from algorithmic information theory. Simplified; if a physicist can supply us with x bits of verified explanation, then the world which embeds him must be able to supply him with $\{n\}$ resources of computation to verify those x bits.

- This relation cannot be violated in any way and under any circumstances. For example, a physicist can explain a small fact of only one bit using only one iteration, and the relation must hold. Or, he can explain all elements of W if he can somehow manage to convert the whole universe into a giant computer, and the relation must still hold.

To see why this relation cannot be violated, let us consider the implications of it failing. If the equation is violated, it would mean a violation of the theory of computation in the universe. This would defeat formal logic as a discipline. For example, it could mean that certain theorems can be verified using less steps than what their proofs would require (i.e. it would be non-sense).

2.2 *Sketch of the derivation*

We hypothesize that a relation which 1) makes claims in regard to the world (i.e., there exists a proven computing reserve); and 2) is inviolable, must somehow encode some laws of physics (and perhaps all?) as this is what the laws of physics ultimately are. We will verify this hypothesis by explicitly deriving the laws of physics from this relation.

We will show that the computational relation between W and T can be formally described as a partition function of statistical physics. We can see that this is the case by considering that the physicist is free to verify the explanation of any subset of W by consuming the proven computing reserve until it runs out. The Lagrange multipliers of the ensemble will be equated with the characteristics of the proven computing reserve, and the entropy of the system will be associated with the quantity of valid and complete theories of everything which explains the facts within the available resources. Here, the theories T are the micro-states of the system and the macroscopic state is described in terms of computing resources.

The equation of states of this system is most interesting as it represents the rules that are common for all verified theories T , which explains the world W . These common rules will be the laws of physics!

In this sense, the relationship is able to import constraints of pure logic (e.g., the theory of computation) into claims about the physical world, via the concept of a proven computing reserve, and then show that the laws of physics are an emergent consequence of these constraints.

3 Technical introduction

3.1 Statistical physics

We will provide a brief recap of statistical physics. In statistical physics, we are interested in the distribution that maximizes entropy,

$$S = -k_B \sum_{x \in X} p(x) \ln p(x) \quad (3.1.1)$$

subject to the fixed macroscopic quantities. The solution for this is the Gibbs ensemble. Typical thermodynamic quantities are:

<i>quantity</i>	<i>name</i>	<i>units</i>	<i>type</i>	
$T = 1/(k_B\beta)$	temperature	K	intensive	(3.1.2)

E	energy	J	extensive	(3.1.3)
-----	--------	---	-----------	---------

$p = \gamma/\beta$	pressure	J/m ³	intensive	(3.1.4)
--------------------	----------	------------------	-----------	---------

V	volume	m ³	extensive	(3.1.5)
-----	--------	----------------	-----------	---------

$\mu = \delta/\beta$	chemical potential	J/kg	intensive	(3.1.6)
----------------------	--------------------	------	-----------	---------

N	number of particles	kg	extensive	(3.1.7)
-----	---------------------	----	-----------	---------

Taking these quantities as examples, the partition function becomes:

Gibbs ensemble

$$Z = \sum_{x \in X} e^{-\beta E(x) - \gamma V(x) - \delta N(x)} \quad (3.1.8)$$

The probability of occupation of a micro-state is:

Gibbs measure

$$p(x) = \frac{1}{Z} e^{-\beta E(x) - \gamma V(x) - \delta N(x)} \quad (3.1.9)$$

The average values and their variance for the quantities are:

$$\bar{E} = \sum_{x \in X} p(x) E(x) \quad \bar{E} = \frac{-\partial \ln Z}{\partial \beta} \quad \overline{(\Delta E)^2} = \frac{\partial^2 \ln Z}{\partial \beta^2} \quad (3.1.10)$$

$$\bar{V} = \sum_{x \in X} p(x) V(x) \quad \bar{V} = \frac{-\partial \ln Z}{\partial \gamma} \quad \overline{(\Delta V)^2} = \frac{\partial^2 \ln Z}{\partial \gamma^2} \quad (3.1.11)$$

$$\bar{N} = \sum_{x \in X} p(x) N(x) \quad \bar{N} = \frac{-\partial \ln Z}{\partial \delta} \quad \overline{(\Delta N)^2} = \frac{\partial^2 \ln Z}{\partial \delta^2} \quad (3.1.12)$$

The laws of thermodynamics can be recovered by taking the following derivatives

$$\left. \frac{\partial S}{\partial \bar{E}} \right|_{V,N} = \frac{1}{T} \quad \left. \frac{\partial S}{\partial \bar{V}} \right|_{E,N} = \frac{p}{T} \quad \left. \frac{\partial S}{\partial \bar{N}} \right|_{E,V} = -\frac{\mu}{T} \quad (3.1.13)$$

, which can be summarized as

$$d\bar{E} = TdS - pd\bar{V} + \mu d\bar{N} \quad (3.1.14)$$

This is known as the equation of states of the thermodynamic system. The entropy can be recovered from the partition function and is given by:

$$S = k_B (\ln Z + \beta \bar{E} + \gamma \bar{V} + \delta \bar{N}) \quad (3.1.15)$$

3.2 Algorithmic thermodynamics

Many authors (Bennett et al., 1998, Chaitin, 1975, Fredkin and Toffoli, 1982, Kolmogorov, 1965, Zvonkin and Levin, 1970, Solomonoff, 1964, Szilard, 1964, Tadaki, 2002, 2008) have discussed the similarity between physical entropy $S = -k_B \sum p_i \ln p_i$ and the entropy in information theory $S = -\sum p_i \log_2 p_i$. Furthermore, the similarity between the halting probability Ω and the Gibbs ensemble of statistical physics has also been studied¹. Tadaki suggests to augment Ω with a multiplication constant D , which acts as a decompression term on Ω .

$$\text{Chaitin construction} \qquad \qquad \text{Tadaki ensemble} \quad (3.2.1)$$

$$\Omega = \sum_{q \in \text{halts}} 2^{-|q|} \qquad \rightarrow \qquad \Omega_D = \sum_{q \in \text{halts}} 2^{-D|q|} \quad (3.2.2)$$

With this change, the Gibbs ensemble compares to the Tadaki ensemble as follows;

$$\begin{array}{ll} \text{Gibbs ensemble} & \text{Tadaki ensemble} \\ Z = \sum_{x \in X} e^{-\beta E(x)} & \Omega_D = \sum_{q \in \text{halts}} 2^{-D|q|} \end{array} \quad (3.2.3)$$

Interpreted as a Gibbs ensemble, the Tadaki construction forms a statistical ensemble where each program corresponds to one of its micro-state. The Tadaki ensemble admits a single quantity; the prefix code length $|q|$ conjugated with D . As a result, it describes the partition function of a system, which maximizes the entropy subject to the constraint that the average length of the codes is some constant $\overline{|q|}$;

¹ Ming Li and Paul M.B. Vitanyi. An Introduction to Kolmogorov Complexity and Its Applications. Springer Publishing Company, Incorporated, 3 edition, 2008. ISBN 0387339981, 9780387339986; Cristian S. Calude and Michael A. Stay. Natural halting probabilities, partial randomness, and zeta functions. Inf. Comput., 204(11): 1718–1739, November 2006. ISSN 0890-5401. DOI: 10.1016/j.jc.2006.07.003. URL <http://dx.doi.org/10.1016/j.jc.2006.07.003>; John Baez and Mike Stay. Algorithmic thermodynamics. Mathematical Structures in Comp. Sci., 22(5):771–787, September 2012. ISSN 0960-1295. DOI: 10.1017/S0960129511000521. URL <http://dx.doi.org/10.1017/S0960129511000521>; and Kohtaro Tadaki. A generalization of chaitin's halting probability omega and halting self-similar sets. Hokkaido Math. J., 31(1):219–253, 02 2002. DOI: 10.14492/hokmj/1350911778. URL <http://dx.doi.org/10.14492/hokmj/1350911778>

$$\overline{|q|} = \sum_{q \in \text{halts}} |q| 2^{-|q|} \quad \text{from 3.1.10} \quad (3.2.4)$$

The entropy of the Tadaki ensemble corresponds to the average length of prefix-free codes available to encode programs.

$$S = k_B (\ln \Omega + D \overline{|q|} \ln 2) \quad \text{from 3.1.15} \quad (3.2.5)$$

The constant $\ln 2$ comes from the base 2 of the halting probability function instead of base e of the Gibbs ensemble.

John C. Baez and Mike Stay² take the analogy further by suggesting an interpretation of algorithmic information theory based on thermodynamics, where the characteristics of programs are considered to be thermodynamic quantities. Starting from Gregory Chaitin's Ω number, the Chaitin construction

² John Baez and Mike Stay. Algorithmic thermodynamics. *Mathematical Structures in Comp. Sci.*, 22(5):771–787, September 2012. ISSN 0960-1295. DOI: 10.1017/S0960129511000521. URL <http://dx.doi.org/10.1017/S0960129511000521>

$$\Omega = \sum_{q \in \text{halts}} 2^{-|q|} \quad (3.2.6)$$

is extended with algorithmic quantities to obtain

$$\text{Gibbs ensemble} \quad \text{Baez-Stay ensemble} \quad (3.2.7)$$

$$Z = \sum_{x \in X} e^{-\beta E(x) - \gamma V(x) - \mu N(x)} \quad \Omega' = \sum_{q \in \text{halts}} 2^{-\beta E(q) - \gamma V(q) - \delta N(q)} \quad (3.2.8)$$

Noting the similarity between the Gibbs ensemble of statistical physics (3.1.8) and (3.2.8), these authors suggest an interpretation where E is the expected value of the logarithm of the program's runtime, V is the expected value of the length of the program, and N is the expected value of the program's output. Furthermore, they interpret the conjugate variables as (quoted verbatim from their paper);

"

1. $T = 1/\beta$ is the *algorithmic temperature* (analogous to temperature). Roughly speaking, this counts how many times you must double the runtime in order to double the number of programs in the ensemble while holding their mean length and output fixed.
2. $p = \gamma/\beta$ is the *algorithmic pressure* (analogous to pressure). This measures the trade-off between runtime and length. Roughly speaking, it counts how much you need to decrease the mean length to increase the mean log runtime by a specified amount while holding the number of programs in the ensemble and their mean output fixed.

3. $\mu = -\delta/\beta$ is the *algorithmic potential* (analogous to chemical potential). Roughly speaking, this counts how much the mean log runtime increases when you increase the mean output while holding the number of programs in the ensemble and their mean length fixed.

"

–John C. Baez and Mike Stay

From equation (3.2.8), they derive analogs of Maxwell's relations and consider thermodynamic cycles, such as the Carnot cycle or Stoddard cycle. For this, they introduce the concepts of *algorithmic heat* and *algorithmic work*.

Other authors have suggested other alternative mappings in other but related contexts³.

3.3 Populating the set W

W can be recursively enumerated. First, we pose the assumption that the world can embed a Universal Turing machine. Then, under this assumption, W is the set of all halting computer programs, and it can be recursively enumerated as follows:

For all sentences $s \in \Sigma^*$, we input s to a universal Turing machine (UTM). If the UTM halts on s , then $s \in W$.

In this case, the universal Turing machine is responsible for delimiting the program-part and the data-part of the input. The program-part represents the explanation of the data-part. The explanation can be verified for a computational cost paid to execute the UTM to completion.

In this construction, all elements of W represent a statement (data-part) that is computationally connected to some arbitrary explanation (program-part). Thus, in this encoding, the universe would contain no "brute facts". As we will see in the following section, the explanations being arbitrary suggests an entropy over explanations of equivalent computing consumption.

4 The laws of physics

Let us now explicitly derive the equation governing the computational relation between T and W . As stated, the physicist is free to pick any subset of facts to explain. Thus, we will seek a probability distribution $\rho(s)$ that maximizes the entropy over the set of facts:

$$S = - \sum_{s \in W} \rho(s) \ln \rho(s) \quad (4.0.1)$$

³ Ming Li and Paul M.B. Vitanyi. An Introduction to Kolmogorov Complexity and Its Applications. Springer Publishing Company, Incorporated, 3 edition, 2008. ISBN 0387339981, 9780387339986; and Kohtaro Tadaki. A statistical mechanical interpretation of algorithmic information theory. In Local Proceedings of the Computability in Europe 2008 (CiE 2008), pages 425–434. University of Athens, Greece, Jun 2008. URL <http://arxiv.org/abs/0801.4194>

Consistent with the proven computational reserve associated with a set of facts and their explanations, we must further constrain the entropy with an average program runtime \bar{t} , and an average program length \bar{x} . These are the computational constraints limiting the physicist's inquiry, and it characterizes his available computing resources.

$$\bar{t} = \sum_{s \in W} \rho(s)t(s) \quad (4.0.2)$$

$$\bar{x} = \sum_{s \in W} \rho(s)x(s) \quad (4.0.3)$$

and by the usual unitary condition that the probabilities sum to 1.

$$1 = \sum_{s \in W} \rho(s) \quad (4.0.4)$$

Here, the functions $t(s)$ and $x(s)$ map a fact of W to a real number.

$$t : s \rightarrow \mathbb{R} \quad (4.0.5)$$

$$x : s \rightarrow \mathbb{R} \quad (4.0.6)$$

where $s \in W$. Specifically, $t(s)$ is the running time of s (in iterations), and $x(s)$ is the length of s (in bits).

We maximize the entropy using the method of the Lagrange multipliers.

$$\begin{aligned} \mathcal{L} = & \left(- \sum_{s \in W} \rho(s) \ln \rho(s) \right) + \lambda_1 \left(\sum_{s \in W} \rho(s) - 1 \right) \\ & + \lambda_2 \left(\sum_{s \in W} \rho(s)t(s) - \bar{t} \right) + \lambda_3 \left(\sum_{s \in W} \rho(s)x(s) - \bar{x} \right) \end{aligned} \quad (4.0.7)$$

Maximizing \mathcal{L} with respect to $\rho(s)$ is done by taking its derivative and posing it equal to zero:

$$0 = \frac{\partial \mathcal{L}}{\partial \rho(s)} = -\ln \rho(s) - 1 + \lambda_1 + \lambda_2 t(s) + \lambda_3 x(s) \quad (4.0.8)$$

Solving for $\rho(s)$ we obtain:

$$\rho(s) = e^{-1+\lambda_1+\lambda_2 t(s)+\lambda_3 x(s)} \quad (4.0.9)$$

From the constraint $1 = \sum_{s \in W} \rho(s)$, we can find the value for λ_1 :

$$1 = \sum_{s \in W} \rho(s) \quad (4.0.10)$$

$$1 = \sum_{s \in W} e^{-1 + \lambda_1 + \lambda_2 t(s) + \lambda_3 x(s)} \quad (4.0.11)$$

$$1 = e^{-1 + \lambda_1} \sum_{s \in W} e^{\lambda_2 t(s) + \lambda_3 x(s)} \quad (4.0.12)$$

Posing $Z := \sum_{s \in W} e^{\lambda_2 t(s) + \lambda_3 x(s)}$,

$$1 = e^{-1 + \lambda_1} Z \quad (4.0.13)$$

Then we rewrite $\rho(s)$ using Z , we obtain the Gibbs measure:

$$\rho(s) = \frac{1}{Z} e^{\lambda_2 t(s) + \lambda_3 x(s)} \quad (4.0.14)$$

where λ_2 and λ_3 are the Lagrange multipliers.

4.1 The equation of states of W

Finally, we obtain the partition function Z .

$$Z = \sum_{s \in W} e^{\lambda_2 t(s) + \lambda_3 x(s)} \quad (4.1.1)$$

along with its equation of states:

$$dS = \lambda_2 d\bar{t} + \lambda_3 d\bar{x} \quad (4.1.2)$$

This partition function and its equation of states describe the entropy of all possible theory T that a physicist can produce to explain W consuming the proven computing reserve —itself characterized by an average program runtime \bar{t} and an average program size \bar{x} .

Recall that there can be no violation of this equation in the World, or we have deeper problems with logic itself. We then ask, what physical system is described by the partition function? We can use it to model the case of a physicist attempting to explain just a few facts, or we will see in the next section, can use it to model the universe on the largest of scales.

4.2 The connection to physics

Logically speaking, if two theories T would give different laws of physics, then it follows that at least one of them must be invalid. Therefore, and perhaps as intuitively expected, the laws of physics

would be the group of rules that are invariant with respect to the choice of a valid theory T for the set W . Furthermore, to exclusively study this group of laws, we need simply to focus on the equation of states of the system since the macroscopic laws that it describes are the same for all valid T .

So what does the equation of states represent? In another paper, I have shown that the equation of Z is a valid model of the cosmos. It connects the large-scale structure of the universe (the macroscopic state of the system) to an ensemble of computationally verified facts (the microscopic states). In other words, the equation for the proven computing reserve can be rewritten as a model of the cosmos (expressed over the usual time and space quantities). We will now give a sketch of the process.

Specifically, the model is able to explain the origins of special relativity, general relativity, dark energy, the law of inertia, the cosmological horizons, and the arrow of time from an arbitrary ensemble of computationally verified facts.

From the equation of states Z , we obtain the laws of physics by solving specific thermodynamic regimes and by a perturbation expansion over the \bar{x} variable.

$$dS = \lambda_2 d\bar{t} + \lambda_3 \left(\bar{x}'(0) d\bar{x} + \bar{x}''(0) \bar{x} d\bar{x} + \frac{1}{2} \bar{x}'''(0) \bar{x}^2 d\bar{x} + \dots \right) \quad (4.2.1)$$

for convenience, we rewrite the equation to

$$TdS = -Pd\bar{t} + Fd\bar{x} + kd\bar{A} + pd\bar{V} + \dots \quad (4.2.2)$$

where $\lambda_2 := -P/T$, $\lambda_3 \bar{x}'(0) := F/T$, $\lambda_3 \bar{x}''(0) := k/T$, $\bar{x} d\bar{x} := d\bar{A}$, $(1/2) \bar{x}'''(0) := p/T$ and $\bar{x}^2 d\bar{x} := d\bar{V}$.

4.3 Mapping to the space-time background

Z describes a physical system by the properties of its proven computing reserve. In the common nomenclature of physics, however, it is usually preferred to use terms such as time and space to describe a physical system. Therefore, it would be interesting to convert the current description into one based on common physical quantities such as time and space.

Our strategy to do so will be borrowed from introductory statistical physics. We will adopt the same line of reasoning which allows the Lagrange multiplier β of statistical physics to be connected to the notion of a physical temperature. As you may recall, in introductory statistical physics:

1. The Gibbs ensemble is first derived from statistical arguments as the ensemble which maximizes the entropy subject to fixed quantities. The process introduces a multiplication constant known as the Lagrange multiplier and is designated by β .
2. From the Gibbs ensemble, a relation between β , energy and entropy is obtained: $\beta dE = dS$.
3. Then, it is shown that this relation recovers a well-known and empirically-uncontested law; such that the two are exact replicas if and only if β is defined using the temperature T . In this case, $S = \ln \Omega$ is used to connect β to T via $\beta = 1/(k_B T)$.
4. Thus, we conclude that the Gibbs ensemble is a description of a physical system involving energy, entropy, and temperature.

We adopt the same line of reasoning for the derivation of the space-time background from Z . Our goal is to derive as many laws of physics as we can from Z so as to show the extent of the physical connection. Specifically, we will show that some quantity of Z corresponds to the time in the equations for Special relativity, general relativity, the law of inertia, etc., and that some other quantity of Z corresponds to space in those same equations.

The validity of the mapping between the quantities of Z and the physical notion of space-time is ultimately a conclusion of this work and rests on deriving overwhelmingly many known laws of physics from Z and to a degree such that it exceeds that which would be expected from a mere coincidence.

4.4 *The laws of physics*

The permissible thermodynamic regimes and their associated laws are obtained by the usual method of posing some derivatives to zero in the equation of states and then studying how the remaining quantities behave. In the referenced paper, we derive the following laws associated with each regime. The results are summarized in the following table:

The World	$TdS =$	$- Pdt$	$+ Fdx$	$+ kdA$	$+ pdV$	$+ \dots$	(4.4.1)
⋮	⋮	⋮	⋮	⋮	⋮		
Maximum speed	$TdS =$	$- Pdt$	$+ Fdx$				(4.4.2)
Maximum viscosity	$TdS =$	$- Pdt$		$+ kdA$			(4.4.3)
Maximum vol. flow rate	$TdS =$	$- Pdt$			$+ pdV$		(4.4.4)
⋮	⋮	⋮	⋮	⋮	⋮		
Special relativity	$0 =$	$- Pdt$	$+ Fdx$				(4.4.5)
⋮	⋮	⋮	⋮	⋮	⋮		
Arrow of time	$TdS =$	$- Pdt$					(4.4.6)
Law of Inertia	$TdS =$		$+ Fdx$				(4.4.7)
General relativity	$TdS =$			$+ kdA$			(4.4.8)
Dark energy	$TdS =$				$+ pdV$		(4.4.9)
⋮	⋮	⋮	⋮	⋮	⋮		

For example, let us take the regime $0 = -Pd\bar{t} + Fd\bar{x}$. We obtain the relation $P/Fd\bar{t} = d\bar{x}$. The ratio P/F involves the Lagrange multiplier P and F , both of which are constant throughout the system as per the rules of statistical physics. The method allows us to find all the constant quantities of the system and each implies a certain law of physics.

The laws so-obtained are emergent from the entropy of the set of computationally verified facts. They have the same mathematical form as the familiar laws of physics. More precisely, computational analogs to the following are obtained: the law of inertia as an entropic force, general relativity as an entropic surface tension, dark energy as an entropic negative pressure, special relativity as an entropic speed, and the arrow of time is as an negative entropic power. Finally, the cosmological horizons (particle horizon, event horizon, and Hubble horizon) are obtained as the boundaries beyond which the arrow of time is reversed in the model.

The method is very powerful in the sense that all correct numerical factors for these analogous laws are recovered. For example, the multiplication constant in the Einstein field equations $8\pi G/c^4$ is exactly recovered without introduction new assumption. The same goes for all laws obtained from the method including the law of inertia, and dark energy from an holographic boundary on the Einstein-Hilbert action.

4.5 Completing the mapping

All rules derived from Z have the same mathematical structure as their physical counterparts. Based on this correspondence, we now explicitly state the mapping. Each rule mathematically equivalent to a law of physics that we have derived from Z adds weight to the thesis of the mapping.

Property	Variable	Mapping	Units
<i>program-runtime</i>	$t(q)$	$t(q) \rightarrow \text{physical-time}$	<i>seconds</i> (4.5.1)

<i>program-length</i>	$x(q)$	$x(q) \rightarrow \text{physical-length}$	<i>meters</i> (4.5.2)
-----------------------	--------	---	-----------------------

The dimensional units will be introduced as definitions. First, the units are mathematically introduced as follows. We introduce the quotient $\frac{1s}{1s} = 1$ where s is the dimensional unit of the second.

$$\left(\frac{1s}{1s}\right) (\lambda_2 d\bar{t}) \quad \text{multiplication by 1} \quad (4.5.3)$$

Then, the variable λ_2 absorbs the denominator and \bar{t} absorbs the numerator, as

$$\left(\frac{\lambda_2}{1s}\right) (d(1s\bar{t})) \rightarrow \lambda'_2 d\bar{t}' \quad (4.5.4)$$

where λ'_2 has the units of s^{-1} and \bar{t}' has the units of s . The same procedure is done for the length variables by injecting $\frac{1m}{1m} = 1$ into $\lambda_3 d\bar{x}$.

Extending the mapping to the conjugated quantities, the units for all variables of equation 4.2.2 are:

<i>quantity</i>	<i>symbol</i>	<i>unit</i>	
temperature	T	K	(4.5.5)

power	P	J/s	(4.5.6)
-------	-----	-------	---------

time	t	s	(4.5.7)
------	-----	-----	---------

force	F	J/m	(4.5.8)
-------	-----	-------	---------

length	x	m	(4.5.9)
--------	-----	-----	---------

surface tension	k	J/m^2	(4.5.10)
-----------------	-----	---------	----------

area	A	m^2	(4.5.11)
------	-----	-------	----------

pressure	p	J/m^3	(4.5.12)
----------	-----	---------	----------

volume	V	m^3	(4.5.13)
--------	-----	-------	----------

5 *Suggested continued reading*

The author has written this paper under the assumption that the reader who is interested in the explicit derivations of the laws of physics from Z as well as more details about the physical interpretation of Z would read the following paper: ⁴. For this reason and to minimize duplication of the same information, these derivations are omitted from this paper.

⁴ Alexandre Harvey-Tremblay. On the entropic origin of the cosmos. https://www.academia.edu/36633782/On_the_entropic_origin_of_the_cosmos, 2018

6 *Conclusion*

The equation of states of Z , as it cannot be violated in the world, naturally reveals the rules that cannot be violated in the universe.

The primary steps in the argument are summarized as follows:

1. We correct the semantic error in the formalization of a typical physical theory. To do so, a model is constructed such that the world implies the theory (instead of the reverse).
2. In the model, the world is described as a set of sentences which are true in it (i.e., they are the facts of the world). The set of facts can be repackaged (compressed) into a theory T for a computing cost.
3. We then ask a physicist to explain elements of W . Each successful explanation produced by the physicist is a claim on the existence of the physical resources available to him to verify such an explanation. In this case, the physical resources are exclusively expressed in terms of computing resources. Together they form what we call the proven computing reserve.
4. Using algorithmic thermodynamics, we obtain the equation of state of the system. This relates the entropy of the group of theories T computationally verified to explain W to the size of proven computing reserve required for the verification to be done.
5. The partition function of the system describes the system as an ensemble of computationally verified facts. All elements of the system are some fact that is computationally connected to some fundamental theory T . Thus, Z describes the properties necessary for a world to be explainable with a formal theory.
6. The equation of states describes the macroscopic laws that are common for all T compatible with the existence of a proven computing reserve necessary to explain the system from a fundamental theory T . They correspond to the familiar laws of physics.

Under this construction, we find that the equation of states is a valid model of the cosmos. Furthermore, many of the familiar laws of physics have been shown to be emergent from the entropy of the system; special relativity, general relativity, the law of inertia, the arrow of time, the second law of thermodynamics, and the cosmological horizons. It would thus appear that the world is best understood as an ensemble of arbitrary computationally verified facts from which the common laws of physics are unavoidably emergent.

References

- John Baez and Mike Stay. Algorithmic thermodynamics. Mathematical Structures in Comp. Sci., 22(5):771–787, September 2012. ISSN 0960-1295. DOI: 10.1017/S0960129511000521. URL <http://dx.doi.org/10.1017/S0960129511000521>.
- C. H. Bennett, P. Gacs, Ming Li, P. M. B. Vitanyi, and W. H. Zurek. Information distance. IEEE Transactions on Information Theory, 44(4):1407–1423, July 1998. ISSN 0018-9448. DOI: 10.1109/18.681318.
- Cristian S. Calude and Michael A. Stay. Natural halting probabilities, partial randomness, and zeta functions. Inf. Comput., 204(11):1718–1739, November 2006. ISSN 0890-5401. DOI: 10.1016/j.jc.2006.07.003. URL <http://dx.doi.org/10.1016/j.jc.2006.07.003>.
- Gregory J. Chaitin. A theory of program size formally identical to information theory. J. ACM, 22(3):329–340, July 1975. ISSN 0004-5411. DOI: 10.1145/321892.321894. URL <http://doi.acm.org/10.1145/321892.321894>.
- Edward Fredkin and Tommaso Toffoli. Conservative logic. International Journal of Theoretical Physics, 21(3):219–253, Apr 1982. ISSN 1572-9575. DOI: 10.1007/BF01857727. URL <https://doi.org/10.1007/BF01857727>.
- Alexandre Harvey-Tremblay. On the entropic origin of the cosmos. https://www.academia.edu/36633782/On_the_entropic_origin_of_the_cosmos, 2018.
- Andrei Nikolaevich Kolmogorov. Three approaches to the definition of the concept “quantity of information”. Problemy peredachi informatsii, 1(1):3–11, 1965.
- Ming Li and Paul M.B. Vitanyi. An Introduction to Kolmogorov Complexity and Its Applications. Springer Publishing Company, Incorporated, 3 edition, 2008. ISBN 0387339981, 9780387339986.

- R.J. Solomonoff. A formal theory of inductive inference. part
i. Information and Control, 7(1):1 – 22, 1964. ISSN 0019-
9958. DOI: [http://dx.doi.org/10.1016/S0019-9958\(64\)90223-2](http://dx.doi.org/10.1016/S0019-9958(64)90223-2).
URL [http://www.sciencedirect.com/science/article/pii/
S0019995864902232](http://www.sciencedirect.com/science/article/pii/S0019995864902232).
- Leo Szilard. On the decrease of entropy in a thermodynamic system
by the intervention of intelligent beings. Behavioral Science, 9(4):
301–310, 1964. ISSN 1099-1743. DOI: 10.1002/bs.3830090402. URL
<http://dx.doi.org/10.1002/bs.3830090402>.
- Kohtaro Tadaki. A generalization of chaitin’s halting probability
omega and halting self-similar sets. Hokkaido Math. J., 31(1):
219–253, 02 2002. DOI: 10.14492/hokmj/1350911778. URL [http:
//dx.doi.org/10.14492/hokmj/1350911778](http://dx.doi.org/10.14492/hokmj/1350911778).
- Kohtaro Tadaki. A statistical mechanical interpretation of algorithmic
information theory. In Local Proceedings of the Computability
in Europe 2008 (CiE 2008), pages 425–434. University of Athens,
Greece, Jun 2008. URL <http://arxiv.org/abs/0801.4194>.
- A K Zvonkin and L A Levin. The complexity of finite objects and the
development of the concepts of information and randomness by
means of the theory of algorithms. Russian Mathematical Surveys,
25(6):83, 1970. URL [http://stacks.iop.org/0036-0279/25/i=6/a=
R05](http://stacks.iop.org/0036-0279/25/i=6/a=R05).