Union of two arithmetic sequences
Basic calculation formula
(2)

Waldemar Zieliński

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Abstract. This paper is a supplement to the previous "Union of two arithmetic sequences - Basic calculation formula (1)" (viXra:1712.0636). We will derive a simpler version of formula for the union of two arithmetics progressions.

1 Notation.

- In this paper we will keep the entire notation from the last version of the previous paper [1].
- If we will refer to the formula from [1], we will use the syntax: [1]($formula\ number$), eg: [1](12).
- If we will refer to the section from [1], we will use the syntax: [1, *section number*], eg: [1, 3.5].
- If we will refer to the section from [1], we will use the syntax: [1];*section number*, eg: [1];3.5.

2 Simpler version of formula $u_N=f(N)$.

We will simplify final formula [1](14) from previous work and we will show that $u_N = \max(ai(N), bj(N))$, where $j(N)$, $i(N)$ are results of formulas [1](15) and [1](16). This will avoid $C(N)$ calculation in [1];3.4, used in [1](14) and simplify the formula a bit.

Please be careful: i, j are indexes of sequences A, B, but $i(N)$, $j(N)$ are results of formulas. $i(N)$ corresponds to i for C=1 only. For C=0 a_i don't exist, hence i don't exist, but $i(N)$ can be calculated as a result of the formula [1](16). Respectively for $j(N)$ and C=1.

For the above dependence to be true, for each N must be (see [1](4)):
1. $ai(N) \geq bj(N)$ for C=1 (c>0)
2. $bj(N) \geq ai(N)$ for C=0 (c=0)

Ad. 1: C=1
The inequation is true directly from Conditions 2.1 in [1].

Ad. 2: C=0
Two options should be considered:
i) r=0
ii) r>0
where r is relative row number [1, 2.3].

Ad. i) c=0, r=0

In this case n=0. Common terms of A, B are in the begining of each group (Definition 2.1 in [1]), hence $ai(n)$=$bj(n)$ and $ai(N)$=$bj(N)$ for N=$g|U^G|$ (see [1](1)).

Ad. ii) c=0, r>0
Please see Table 1 in [1, 2.1]. Let's denote:
i^R_{max} - the largest term index from sequence A in row R
N^R_{max} - the largest union index in row R
$i^{R,c=1}$ - the smallest term index from sequence A in row R (in column 1)
$N^{R,c=0}$ - the smallest union index in row R (in column 0)

We have obvious relationship for N:

$$N^{R,c=0}{=}N^{R-1}_{max}+1 \tag{1}$$

For c>0 is also (see [1, (12)]):

$$i{=}N{-}R{+}g \tag{2}$$

This relationship can not occur for c=0 because in [1] Table 1 column 0 there are no terms from A. Ignoring this, we can calculate $i(N)^{R,c=0}$ (from [1](16)) and fictitious term $ai(N)^{R,c=0}$. From (2):

$$i(N)^{R,c=0}{=}N^{R,c=0}{-}R{+}g \tag{3}$$

This is correct, because (2) does not depend on the column number.

Now we will use (3), (1) and (2):

$$i(N)^{R,c=0}{=}N^{R,c=0}{-}R{+}g{=}N^{R-1}_{max}{+}1{-}R{+}g{=}N^{R-1}_{max}{-}(R{-}1){+}g{=}i^{R-1}_{max}$$

where i^{R-1}_{max} is existing and corect index of A. Cause $ai^{R-1}_{max}{<}bj(N)^{R,c=0}$, hence:

$$ai(N)^{R,c=0}{<}bj(N)^{R,c=0}$$

Summarizing:

from	1:	c>0		→	$ai(N)>bj(N)$
from	2.i):	c=0	r=0	→	$ai(N)=bj(N)$
from	2.ii):	c=0	r>0	→	$ai(N)<bj(N)$

It means, we can write formula [1](14) in an equivalent form:

$$u_N{=}\max(ai(N),bj(N)) \tag{4}$$

or

$$u_N{=}\max\left(a\left\lceil\frac{b}{a{+}b}\left(\left\lfloor\frac{N(a{+}b)}{a{+}b{-}\Theta}\right\rfloor{-}\frac{a}{b}\right)\right\rceil, b\left\lfloor\frac{a}{a{+}b}\left\lfloor\frac{N(a{+}b)}{a{+}b{-}\Theta}{+}1\right\rfloor\right\rfloor\right) \tag{5}$$

Using the dependency: $\max(x,y){=}\frac{1}{2}(x{+}y{+}|x{-}y|)$ we can write (4) in form:

$$u_N{=}\frac{1}{2}\left(ai(N){+}bj(N){+}|ai(N){-}bj(N)|\right) \tag{6}$$

References

[1] ZIELIŃSKI, W.: *Union of two arithmetic sequences - Basic calculation formula (1)*, viXra:1712.0636.