

Union of two arithmetic sequences

Basic calculation formula

(2)

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Abstract. This paper is a supplement to the previous "Union of two arithmetic sequences - Basic calculation formula (1)" (viXra:1712.0636). We will derive a simpler version of formula for the union of two arithmetics progressions.

1 Notation.

- In this paper we will keep the entire notation from the last version of the previous paper [1].
- If we will refer to the formula from [1], we will use the syntax: [1](*formula number*), eg: [1](12).
- If we will refer to the section from [1], we will use the syntax: [1];*section number*, eg: [1];3.5.

2 Simpler version of formula $u_N = f(N)$.

We will simplify final formula [1](14) from previous work and we will show that $u_N = \max(ai(N), bj(N))$, where $j(N)$, $i(N)$ are results of formulas [1](15) and [1](16). This will avoid $C(N)$ calculation in [1];3.4, used in [1](14) and simplify the formula a bit.

Please be careful: i , j are indexes of sequences A , B , but $i(N)$, $j(N)$ are results of formulas. $i(N)$ corresponds to i for $C=1$ only. For $C=0$ a_i don't exist, hence i don't exist, but $i(N)$ can be calculated as a result of the formula [1](16). Respectively for $j(N)$ and $C=1$.

For the above dependence to be true, for each N must be (see [1](4)):

1. $ai(N) \geq bj(N)$ for $C=1$ ($c>0$)
2. $bj(N) \geq ai(N)$ for $C=0$ ($c=0$)

Ad. 1: $C=1$

The inequation is true directly from Conditions 2.1 in [1].

Ad. 2: $C=0$

Two options should be considered:

i) $r=0$

ii) $r>0$

where r is relative row number [1];2.4.

Ad. i) $c=0$, $r=0$

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In this case $n=0$. Common terms of A, B are in the begining of each group (Definition 2.1 in [1]), hence $ai(n)=bj(n)$ and $ai(N)=bj(N)$ for $N=g|U^G|$ (see 1).

Ad. ii) $c=0, r>0$

Please see Table 1 in [1];2.1. Let's denote:

- i_{max}^R - the largest term index from sequence A in row R
- N_{max}^R - the largest union index in row R
- $i^{R,c=1}$ - the smallest term index from sequence A in row R (in column 1)
- $N^{R,c=0}$ - the smallest union index in row R (in column 0)

We have obvious relationship for N :

$$N^{R,c=0} = N_{max}^{R-1} + 1 \quad (1)$$

For $c>0$ is also (see [1](12)):

$$i = N - R + g \quad (2)$$

This relationship can not occur for $c=0$ because in [1] Table 1 column 0 there are no terms from A . Ignoring this, we can calculate $i(N)^{R,c=0}$ (from [1](16)) and fictitious term $ai(N)^{R,c=0}$. From (2):

$$i(N)^{R,c=0} = N^{R,c=0} - R + g \quad (3)$$

This is correct, because (2) does not depend on the column number.

Now we will use (3), (1) and (2):

$$i(N)^{R,c=0} = N^{R,c=0} - R + g = N_{max}^{R-1} + 1 - R + g = N_{max}^{R-1} - (R-1) + g = i_{max}^{R-1}$$

where i_{max}^{R-1} is corect index of existing term from A in [1] Table 1. Cause $ai_{max}^{R-1} < bj(N)^{R,c=0}$, hence:

$$ai(N)^{R,c=0} < bj(N)^{R,c=0}$$

Summarizing:

- from 1: $c>0 \rightarrow ai(N) > bj(N)$
- from 2.i): $c=0 \quad r=0 \rightarrow ai(N) = bj(N)$
- from 2.ii): $c=0 \quad r>0 \rightarrow ai(N) < bj(N)$

It means, we can write formula [1](14) in an equivalent form:

$$u_N = \max(ai(N), bj(N)) \quad (4)$$

or in full version:

$$u_N = \max \left(a \left[\frac{b}{a+b} \left(\left\lfloor \frac{N(a+b)}{a+b-\Theta} \right\rfloor - \frac{a}{b} \right) \right], b \left[\frac{a}{a+b} \left\lfloor \frac{N(a+b)}{a+b-\Theta} + 1 \right\rfloor \right] \right) \quad (5)$$

Using the dependency: $\max(x, y) = \frac{1}{2}(x+y+|x-y|)$ we can write (4) in alternative form:

$$u_N = \frac{1}{2} (ai(N) + bj(N) + |ai(N) - bj(N)|) \quad (6)$$

References

- [1] ZIELIŃSKI, W.: *Union of two arithmetic sequences - Basic calculation formula (1)*, viXra:1712.0636.