Bell's inequality refuted irrefutably on Bell's own terms

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Abstract Consistent with claims that we've advanced since 1989, we refute Bell's 1964 inequality on Bell's own terms. In sum, in the context of the EPR-Bohm experiment, Bell's 1964:(14b) $\neq$ (14a).

1. Introduction

(i) Bell's theorem has been described as one of the most profound discoveries of science. (ii) Yet, despite this fame, Bell (1990:7) lived on the horns of a dilemma wrt the significance of his theorem. (iii) Here, continuing to use elementary mathematics as an introduction to Watson (2018e) and its use there, we refute Bell's famous inequality on Bell's own terms and respond to an interesting objection.

2. Analysis

(i) Bell (1964) is available free-online, see References; it will be helpful to have it on hand. (ii) Let Bell-(1) be short for Bell 1964:(1); etc. (iii) Let Bell-(14a), (14b), (14c) identify the three expressions between Bell-(14) and Bell-(15). (iv) Then, via Bell-(1) and LHS Bell-(2), the functions required to analyze Bell's work are: result A, given by $A(\vec{a}, \boldsymbol{\lambda}) = \pm 1$, etc; expectation value $P(\vec{a}, \vec{b})$, the average over the product of paired-results $A(\vec{a}, \boldsymbol{\lambda})B(\vec{b}, \boldsymbol{\lambda})$, etc. (v) Thus, working with $(\vec{b}, \vec{c}) = (\vec{a}, \vec{c}) - (\vec{a}, \vec{b})$ for simplicity—and using clearly-identified facts at one with quantum theory (QT), observation and true local realism (Watson 2018e)—we allow via Bell-(1)-(2):

$A(\vec{a}, \boldsymbol{\lambda}) = \pm 1,\ B(\vec{b}, \boldsymbol{\lambda}) = \pm 1, C(\vec{c}, \boldsymbol{\lambda}) = \pm 1$ represent Bellian result functions, (1)

with $-1 \leq P(\vec{a}, \vec{b}) \leq 1,\ -1 \leq P(\vec{a}, \vec{c}) \leq 1, -1 \leq P(\vec{b}, \vec{c}) \leq 1$ the related expectations. (2)

Now it's a fact that $xy \leq y$ if $x \leq 1$ and $0 \leq y$; ie, for us, it's *a priori* true by definition. (3)

So $P(\vec{a}, \vec{b})[1 + P(\vec{a}, \vec{c})] \leq 1 + P(\vec{a}, \vec{c})$ is also *a priori* true [with others] for all $\vec{a}, \vec{b}, \vec{c}$. (4)

So, reworking (4) – ready for Bell's use of absolute values (tho not needed by us) – (5)

we have a new fact: $0 \geq |P(\vec{a}, \vec{b}) - P(\vec{a}, \vec{c})| - 1 + P(\vec{a}, \vec{b})P(\vec{a}, \vec{c})$ = Bell-(15) done correctly (6)

versus Bell's claim: $0 \geq |P(\vec{a}, \vec{b}) - P(\vec{a}, \vec{c})| - 1 - P(\vec{b}, \vec{c})$ for his inequality, Bell-(15). (7)

And if $(\vec{a}, \vec{b}) = (\vec{a}, \vec{c})$: then $P(\vec{b}, \vec{c}) = P(\vec{b}, \vec{b}) = -1$ per Bell-(13). So (6) & (7) hold and (8)

all's well. However : as Bell (1964) well-knows via QT [but see ¶2(vi)] as at (9)

Bell-(2)-(3): $P(\vec{a}, \vec{b}) = -\cos(\vec{a}, \vec{b})$, etc. [Or, without Bell/QT, see Watson 2018e:(19).] (10)

So unless $(\vec{a}, \vec{b}) = (\vec{a}, \vec{c})$: Bell needs $P(\vec{a}, \vec{b})P(\vec{a}, \vec{c}) \geq -P(\vec{b}, \vec{c})$ for famous (7) to hold: (11)

otherwise (7) could be $>$ 0, exceeding its claimed upper-bound. But, via (11), see (23): (12)

unless $(\vec{a}, \vec{b}) = (\vec{a}, \vec{c})$: Bell now needs $0 \geq \sin(\vec{a}, \vec{b})\sin(\vec{a}, \vec{c})$ for (7) to hold. And this (13)

leads to a new fact : Bell-(15) is false for $\sin(\vec{a}, \vec{b})\sin(\vec{a}, \vec{c}) > 0$ and $(\vec{a}, \vec{b}) \neq (\vec{a}, \vec{c})$; (14)

see Figure 1. Thus (7) $>$ 0 for some $\vec{a}, \vec{b}, \vec{c}$ and Bell's upper-bound of 0 is breached. (15)

So, as claimed in 1989 : Bell-(15) is false, his theorem refuted at its source: *QED.*■(16)

for Bell-(14b) = (7) $\neq$ (6) = Bell-(14a); ie, in Bell 1964: (14b)$\neq$(14a). *QED.*■(17)

[nb: LHS Bell-(14a) = $P(\vec{a}, \vec{b}) - P(\vec{a}, \vec{c})$ which gives *a priori* true (4), and thus (6).] (18)

So Bell's (1964:196-7) : "not possible" and "contradiction" are unjustified; *QED.*■(19)

so too Bell's conclusion – Bell (1964:199) – see Watson 2018e:¶5 too. *QED.*■(20)

nb: $\forall\, \vec{a}, \vec{b}, \vec{c}$ in (4)-(6): $0 \geq P(\vec{a}, \vec{b}) - P(\vec{a}, \vec{c}) - 1 + P(\vec{a}, \vec{b})P(\vec{a}, \vec{c})$; whereas for Bell in (7) (21)

for some $\vec{a}, \vec{b}, \vec{c}$: $\frac{1}{2} \geq |P(\vec{a}, \vec{b}) - P(\vec{a}, \vec{c})| - 1 - P(\vec{b}, \vec{c})$; as surmised at (12). *QED.*■(22)

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(vi) nb: some physicists object that our use of $P(\vec{a},\vec{b}) = -\cos(\vec{a},\vec{b})$ in (10) is invalid because Bell (against QT) wanted to keep open the possibility that this "quantum correlation" was false. Given Bell's "impossibility" claim below Bell-(3)—with its implication that a standard mathematical formulation for an average won't work in the EPR-Bohm experiment (EPRB): it does, see Watson 2018e:¶4—we find merit in this suggestion: for it helps explain Bell's acceptance of a result that is clearly false under QT. However, as stated in (10), and certainly consistent with our acceptance of QT: at Watson 2018e:(19) we derive this "quantum correlation" without QT. So, while (10) may be unavailable to Bell under his belief-system: (10) is clearly available to us, with QT's endorsement (and via that alone if necessary—see next—to allow our focus on Bell's errors to continue).

(vii) Thus—and still "on Bell's terms"—we have the following Bellian analysis invoking QT:

> '... . Thus $P(\vec{b},\vec{c})$ cannot be stationary at its minimum value (-1 at $\vec{b}=\vec{c}$) and cannot equal the quantum mechanical value $-\cos(\vec{b},\vec{c})$ from Bell-(3),' after Bell (1964:198).

(viii) Against this—and meeting all the quantum mechanical values—we again beat Bell's QT comparison. For our (6) reduces, via our (10) or Bell-(3), to another observationally-significant fact:[2]

$$\forall\, \vec{a},\vec{b},\vec{c}: \cos(\vec{a},\vec{c}) - \cos(\vec{a},\vec{b}) - 1 + \cos(\vec{a},\vec{b})\cos(\vec{a},\vec{c}) = -4\sin^2\frac{(\vec{a},\vec{b})}{2}\cos^2\frac{(\vec{a},\vec{c})}{2} \le 0.\ \textit{QED.}\blacksquare \qquad (23)$$

3. Conclusions

(i) Whatever the justification for Bell's (14b), it is mathematically false; see Figure 1. (ii) Further, in Bell's (1964) context—EPRB—it is unphysical: akin, in fact—see Watson 2018e:¶7—to the presumption that naive-realism might succeed in Bohm's brilliant quantum setting: a setting, alas, with a higher degree of correlation than naive-realism can deliver.

(iii) Moreover, whatever the triggering assumption, Bell-(14b) is also unwarranted. For, against Bell, Watson (2018e) delivers—via true local realism—the same results as QT and observation; thus showing, independently, that Bell-(2) does equal Bell-(3) as we allowed in (10)-(11). (iv) So Bell's theorem is refuted irrefutably here, on Bell's own terms. (v) And we conclude, as Bell (1990:9) half-expected: Bellians are being rather silly.

4. Appendix

(i) The opaque white plane underlying all of Figure 1 represents the upper-bound [0] of our relation (6)—valid for all $\vec{a},\vec{b},\vec{c}$—with $x=(\vec{a},\vec{b})$, $y=(\vec{a},\vec{c})$; in radians. (ii) So all positive numbers lie behind the plane, hidden from view. (iii) The blue plot is then (7), when true: so, overlaying the white plane in blue, we see Bell's famous inequality when its value does not exceed its claimed upper-bound of 0. (iv) Against our (6) with its 100% validity, Bell's inequality rates 50%. (v) In effect, we cut Bell's Gordian Square (of whatever dimension) into subordinate squares: the remnant white-spaces—wherein (7) is false—being as defined in (14). (vi) nb, wrt (12): via (10-(11) and $(\vec{b},\vec{c}) = (\vec{a},\vec{c}) - (\vec{a},\vec{b})$ from ¶2(v),

$$\text{unless } (\vec{a},\vec{b}) = (\vec{a},\vec{c}),\ \text{Bell needs: } 0 \ge \cos[(\vec{a},\vec{c})-(\vec{a},\vec{b})] - \cos(\vec{a},\vec{b})\cos(\vec{a},\vec{c}) = \sin(\vec{a},\vec{b})\sin(\vec{a},\vec{c}). \qquad (24)$$

[2] nb: the absolute brackets (required by Bell, optional for us) are not required in our valid and more-general analysis.

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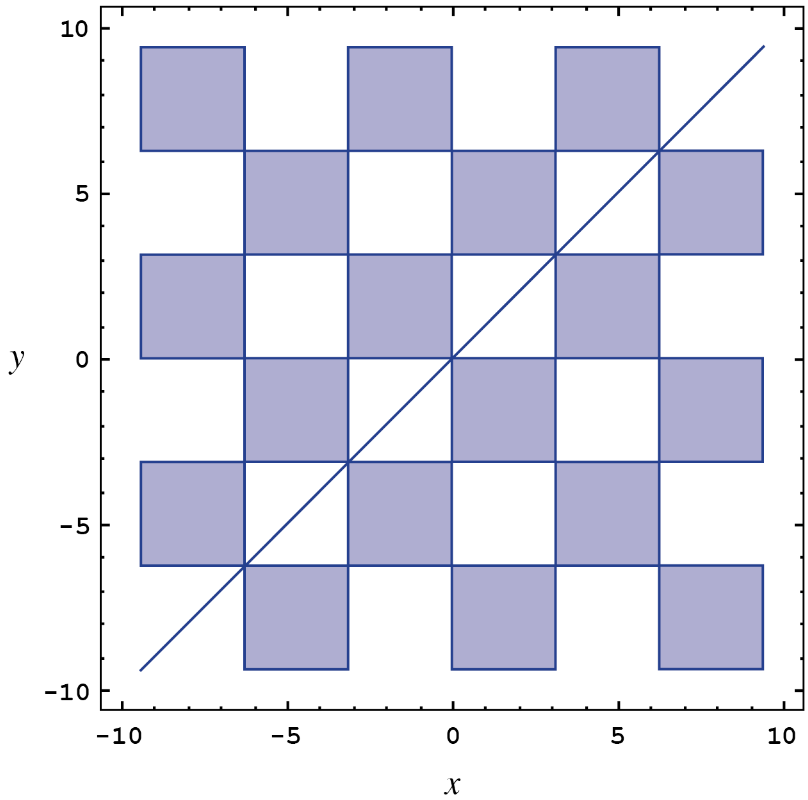

Figure 1: Bell's famous (1964) inequality in blue when true—otherwise false—see our (7). Note that the line $y = x$ — ie, $(\vec{a}, \vec{c}) = (\vec{a}, \vec{b})$ — is a set of points of zero probability.