Bell's inequality refuted on Bell's terms

Abstract We refute Bell's inequality by showing quite simply that his terms lead to an algebraic inequality that never fails while his famous inequality fails algebraically more often than not.

1. Introduction

1.1. Bell's theorem is described as one of the most profound discoveries of science (Stapp 1975:271) to be remembered as one of the few essential discoveries of 20th Century physics (van der Merwe *et al.* 1992:v). Yet Bell (1990:5, 6, 13) is confused about the physical significance of his inequality:

> '... I cannot say that action at a distance [AAD] is required in physics. I can say that you cannot get away with no AAD.' '... the Einstein program fails. ... it might be that we have to learn to accept not so much AAD, but [the] inadequacy of no AAD.' 'And that is the dilemma. ... I step back from asserting that there is AAD, I say only that you cannot get away with locality. You cannot explain things by events in their neighbourhood.'

1.2. So against Bell and nipping his dilemma in the bud, we show that Bell's terms lead to an algebraic inequality (10) that never fails while his inequality (11) fails algebraically more often than not.

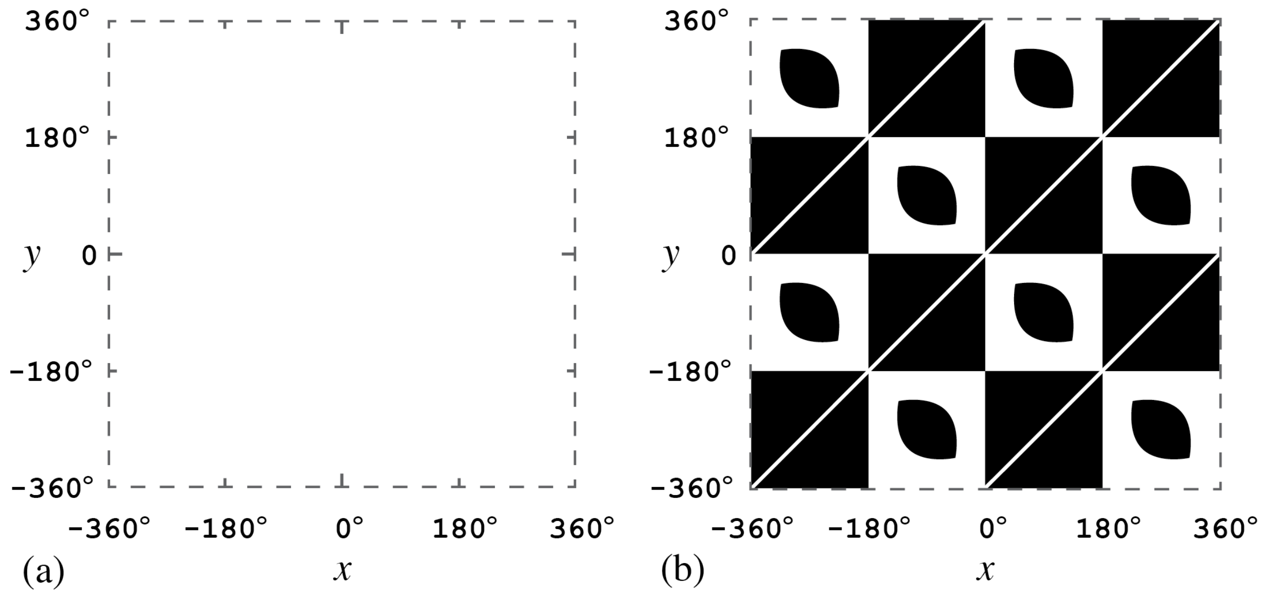

Fig.1. Shots of our algebraic sieve after screening (10) and Bell's inequality (11) taking $x = (\vec{a}, \vec{b})$ and $y = (\vec{a}, \vec{c})$ to be independent angles about $\vec{a}$ in 3-space with $(\vec{b}, \vec{c})$ thereby constrained. White space in (a) shows (10) never-failing by never breaching its bounds of 0 and 1. (b) shows the same sieve after screening (11) to (10)'s irrefutable

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benchmarks, black areas showing (11) failing more often than not. That is: black lenses show where (11) breaches (10)'s benchmark lower bound of 0 via (14); black elsewhere shows where (11) breaches the benchmark upper bound of 1 via (15). White space in (b) thus shows the limited validity of Bell's inequality, with white lines (think zero-width) showing the relation $\cos(\vec{a}, \vec{b}) = \cos(\vec{a}, \vec{c})$ where (11) is also true; though (11) is often false under the same cosine relation at points like $(\vec{a}, \vec{b}) = 270°$, $(\vec{a}, \vec{c}) = 90°$.

1.3. Fig.1 introduces our results via two snapshots. Like an archeological sieve set to screen dry clay and capture historic objects, our algebraic sieve (built from screens at http://www.wolframalpha.com) is designed to capture historic real numbers that refute Bell's inequality. That is, based on (12) for any $\vec{a}, \vec{b}, \vec{c}$, we capture real numbers $\mathbb{R} < 0$ or $\mathbb{R} > 1$ as black marks, see Fig.1b, all other numbers passing cleanly through our sieve and out of sight. So white-space in (a) shows our (10) always true while black areas in (b) show (11) to be more often false than not.

1.4. In brief, consistent with claims we've made since Watson 1989, (10) delivers all valid values of (11). But (11) regularly exceeds the relevant irrefutable bounds of (10) and is thus refuted.

1.5. In this way the elementary mathematics used here serves as an introduction to its further use in forthcoming Watson 2018E which delivers further new results via similar analysis. In particular, a classical quantum theory emerges from our study of the Einstein Podolsky Rosen Bohm experiment (EPRB) without reference to any modern quantum theory: a result that Bell might have liked and is perhaps what he wanted. However, please note, all results here are independent of 2018E.

2. Analysis

2.1. Bell 1964 (freely available, see References) provides definitions of the key terms that we use in (1) & (2). Let Bell-(1) be short for Bell 1964:(1), etc. And for future reference, let (14a)-(14c) identify the unlabelled relations between Bell-(14)-(15), the rest being (15a), (21a)-(21e), (23).

2.2. We begin by observing two key relations in Bell 1964: (i) at Bell-(1), a result A given by a function $A(\vec{a}, \boldsymbol{\lambda}) = \pm 1$; (ii) at LHS Bell-(2), an expectation value $P(\vec{a}, \vec{b})$ given by the average over the product of paired-results $A(\vec{a}, \boldsymbol{\lambda})$ and $B(\vec{b}, \boldsymbol{\lambda})$, each obtained via a paired-test on a fresh particle-pair.

2.3. So from Bell-(1) [Bell taking the variable $\boldsymbol{\lambda}$ to be a single continuous parameter] we can write

$$A(\vec{a}, \boldsymbol{\lambda}) = \pm 1, B(\vec{b}, \boldsymbol{\lambda}) = \pm 1, C(\vec{c}, \boldsymbol{\lambda}) = \pm 1. \tag{1}$$

2.4. The related expectation values (the related averages) for all $\vec{a}, \vec{b}, \vec{c}$ in 3-space are then

$$-1 \leq P(\vec{a}, \vec{b}) \leq 1, -1 \leq P(\vec{a}, \vec{c}) \leq 1, -1 \leq P(\vec{b}, \vec{c}) \leq 1 \tag{2}$$

and we take $(\vec{a}, \vec{b})$ & $(\vec{a}, \vec{c})$ to be the independent vector-pairings about $\vec{a}$ which constrain $(\vec{b}, \vec{c})$.

2.5. For, taking any 2 vector-pairs from a set of 3 vector-pairs $\{(\vec{a},\vec{b}),(\vec{a},\vec{c}),(\vec{b},\vec{c})\}$ the 3rd is constrained by default: eg, if $(\vec{a},\vec{b}) = 120°, (\vec{a},\vec{c}) = 60°$, then (interestingly) $60° \leq (\vec{b},\vec{c}) \leq 180°$. Thus, recognizing such constraints and under Bell's terms as we insist: Bell's inequality—in the format of Bell-(15)—is

$$1 + P(\vec{b},\vec{c}) \geq \Big|P(\vec{a},\vec{b}) - P(\vec{a},\vec{c})\Big| \tag{3}$$

with an immediate possible difficulty for Bell. For if, from (2)

$$P(\vec{a},\vec{b}) = -\tfrac{1}{2}, P(\vec{a},\vec{c}) = \tfrac{1}{2}, P(\vec{b},\vec{c}) = -\tfrac{1}{2} \tag{4}$$

then (3), which is Bell's inequality in its original form, would be algebraically false and absurd with

$$\tfrac{1}{2} \geq 1 \blacktriangle \tag{5}$$

and by which Bell-(15) would be refuted and Bell's dilemma would be resolved at its source at once.

2.6. So now, with (5) in mind and as a basis for proving the truth or falsity of (3) whatever its form, we seek a true relation adequately close to (3) with upper and lower bounds determined by (2).

2.7. To that end, and from (3)'s form, we begin with a trivial relation for real numbers $p, q \in \mathbb{R}$

$$\text{if } p \leq 1 \text{ and } 0 \leq q \text{ then } pq \leq q. \tag{6}$$

2.8. Thus via (2) & (6) with $P(\vec{a},\vec{b})$ equivalent to p and $1 + P(\vec{a},\vec{c})$ equivalent to q we have

$$P(\vec{a},\vec{b})[1 + P(\vec{a},\vec{c})] \leq 1 + P(\vec{a},\vec{c}) \tag{7}$$

$$\therefore P(\vec{a},\vec{b}) - P(\vec{a},\vec{c}) + P(\vec{a},\vec{b})P(\vec{a},\vec{c}) \leq 1 \tag{8}$$

via (7)'s expansion and thereby revealing an upper bound of 1.

2.9. So now, testing allowed values of $P(\vec{a},\vec{c})$ & $P(\vec{a},\vec{b})$ from (2) in (8), we find (8)'s lower bound to be -3 when $P(\vec{a},\vec{b}) = -1$ and $P(\vec{a},\vec{c}) = 1$; its upper bound confirmed when $P(\vec{a},\vec{c}) = P(\vec{a},\vec{b}) = 1$, etc.

$$\therefore -3 \leq P(\vec{a},\vec{b}) - P(\vec{a},\vec{c}) + P(\vec{a},\vec{b})P(\vec{a},\vec{c}) \leq 1. \tag{9}$$

2.10. Thus, by now adding abstract brackets to (9) as in (3), we have a relation to test (3)

$$0 \leq \Big|P(\vec{a},\vec{b}) - P(\vec{a},\vec{c})\Big| + P(\vec{a},\vec{b})P(\vec{a},\vec{c}) \leq 1 \tag{10}$$

its lower bound 0 when $P(\vec{a},\vec{b}) = P(\vec{a},\vec{c}) = 0$ from (2), its upper 1 when $P(\vec{a},\vec{b}) = P(\vec{a},\vec{c}) = 1$, etc.

2.11. So: converting (3)—Bell-(15) in its original form—to an equivalent form matching (10)

$$\Big|P(\vec{a},\vec{b}) - P(\vec{a},\vec{c})\Big| - P(\vec{b},\vec{c}) \leq 1 \tag{11}$$

which via that mandatory equivalence means that (11) may also be identified as Bell's inequality. Moreover, we see now that (11)'s upper bound of 1 would be breached, reaching $\frac{3}{2}$, under the possible conditions in (4) that led to the possible absurdity in (5).

2.12. Or more generally and thus more clearly and universally: comparing Bell's inequality as in (11) with (10), we see that (11) is false for any $\vec{a}, \vec{b}, \vec{c}$ if

$$(11) < 0 \text{ or } (11) > 1 \tag{12}$$

to the point of Bell's inequality and Bell-(15) being refuted. For, given (12), (11) would then breach the bounds of (10) as determined via irrefutable (2). Further, if $P(\vec{a}, \vec{b}) = \frac{1}{2}$ and $P(\vec{a}, \vec{c}) = -\frac{1}{2}$, then (10) is true for all $P(\vec{b}, \vec{c})$, *but then* (11) would fail for $P(\vec{b}, \vec{c}) < 0$. And if $P(\vec{a}, \vec{b}) = P(\vec{a}, \vec{c}) = 0$, then (10) is true for all $P(\vec{b}, \vec{c})$, *but then* (11) would fail for $P(\vec{b}, \vec{c}) > 0$.

2.13. So now, with mounting evidence that the possible absurdity in (5) is almost a certainty—and which from Watson 2018E is independently proven to be absurd in the context of EPRB and without quantum theory (QT)—let's seek closure and adopt Bell's chosen comparison-theory: which is QT. See Bell 1964, pp.195, 199, and below Bell-(2), Bell-(15), Bell-(23). And let's test the bounds of (11) against QT, Bell from below Bell-(2) leaving us a clue to all that we need from QT for now

$$P(\vec{a}, \vec{b}) = -\cos(\vec{a}, \vec{b}),\ P(\vec{a}, \vec{c}) = -\cos(\vec{a}, \vec{c}),\ P(\vec{b}, \vec{c}) = -\cos(\vec{b}, \vec{c}) \tag{13}$$

noting that (though fun) there's no need to test (10) since it is irrefutable, independent of any theory.

2.14. So, using (13) to bring (12) into QT's domain, the failure zones for Bell's theorem (11) are

$$|\cos(\vec{a}, \vec{b}) - \cos(\vec{a}, \vec{c})| - \cos(\vec{b}, \vec{c}) \quad < \quad 0 \tag{14}$$

$$|\cos(\vec{a}, \vec{b}) - \cos(\vec{a}, \vec{c})| - \cos(\vec{b}, \vec{c}) \quad > \quad 1 \tag{15}$$

and now, screening (14) & (15) with our sieve, we see Fig.1b emerge progressively as black marks are captured there: black lenses showing the failure zones for (14), the remaining black showing the failure zones for (15); with each *'but then'* in ¶2.12 now a definite fail for (11) as well.

2.15. Now some suggest privately that our use of (13) to move (12) to (14)-(15) is invalid because Bell (against QT) wants to keep open the possibility that the *quantum correlations* in (13) are false. And we find some merit in this objection: to us it's a possible explanation for Bell's strange claim of *'not possible'* below Bell-(3) with its false implication—shown to be false at Watson 2018E:(31), consistent with QT—that a standard formula for an average in Bell-(2) doesn't work here.

2.16. However, as shown via our new approach to QT in Watson 2018E, we can derive (13) without QT. So though Bell and private others may have reservations re (13), it is clearly available to us: with QT's endorsement; and via QT alone if necessary today.

2.17. So, with other Bellian claims refuted in Watson 2018E, we conclude here for now by making one final point: wanting to be clear and minimize a common misunderstanding. Though our theorizing is independent of QT, QT is of course (given its remarkable history) an adequate theory in our terms.

3. Conclusions

3.1. The basis for Bell's original inequality is widely discussed up to the present day. But now, whatever that basis, those captured black marks in Fig.1b prove this: Bell-(15) is both algebraically and logically false in our terms: for we take mathematics to be the best logic. Moreover, point-by-point this truth about Bell-(15) can be confirmed experimentally by using a portable calculator, or via Web2.0Calc online, with the relevant relation, (11) or (10); (11) being Bell's famous inequality that we refute, (10) an elementary theorem to replace it.

3.2. Not only that, but Bell-(15) is also unphysical—see Watson 2018E:¶7—akin to inferring that a naive-realism might succeed in a highly-correlated quantum-setting (EPRB, the setting for Bell 1964) and *not* then dropping such naivety when it fails so badly.

3.3. Further, whatever triggered Bell's naive inference, Bell-(15) is also unwarranted. For, against Bell here, Watson 2018E delivers (via true local realism, and without QT) the same results as QT and observation. This shows independently that Bell-(2) equals Bell-(3): at the same time refuting Bell's theorem; that is, Bell's *'not possible'* below Bell-(3).

3.4. Thus, to be clear re our long-held position: for us, true local realism is the union of true locality and true realism. *True locality* insists that no influence propagates superluminally, after Einstein. *True realism* insists that some beables *may* change interactively, after Bohr. *Naive-realism* is then any brand of *realism* that negates or neglects that *'may'* when relevant.

3.5. Finally, re further consequences of this work: please note that the Einstein program does not fail with us; for against Bell in ¶1.1, Watson 2018E does get away with locality and without AAD to refute claims like these:

> 'Einstein maintained that quantum metaphysics entails spooky actions at a distance; experiments show that what bothered Einstein is not a debatable point but the observed behaviour of the real world,' after Mermin (1985:38). 'Our world is non-local,' after Davies (1984:48), Goldstein *et al.* (2011:1), Maudlin (2014:25), Bricmont (2016:112). '... the predictions of quantum theory cannot be accounted for by any local theory,' after Brunner *et al.* (2014:1), Norsen (2015:1).

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