

Bell's inequality refuted on Bell's own terms

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Abstract We use elementary algebra to show that Einstein-Podolsky-Rosen-Bohm expectation values are bound by an inequality that is never false while Bell's related inequality is false more often than not. So Bell's famous inequality is refuted on Bell's own terms and replaced by an inequality that holds for any experiment with binary outcomes.

1. Introduction

1.1. We begin with some hard facts. (i) Bell's 1964 inequality is violated by both quantum theory (QT) and experiment: worse, we show that it is false under elementary algebra. (ii) Bell's fame is due to those who share Bell's dilemma about the physical significance of his inequality: better, we resolve that dilemma. (iii) In brief and to be clear, here's Bell's (1990) specification of that dilemma:

p.5, '... I cannot say that action at a distance [AAD] is required in physics. I can say that you cannot get away with no AAD.' p.6, '... the Einstein program fails. ... it might be that we have to learn to accept not so much AAD, but [the] inadequacy of no AAD.' p.13, 'That is the dilemma. ... I step back from asserting that there is AAD, I say only that you cannot get away with locality. You cannot explain things by events in their neighbourhood.'

1.2. Then, straddling Bell's 1990 dilemma: Stapp (1975:271) describes Bell's (1964) theorem as 'one of the most profound discoveries of science.' Watson (1989), 'Bell's theorem is false.' Van der Merwe *et al.* (1992:v), 'Bell's theorem will be remembered in times to come as one of the few essential discoveries of twentieth century physics.' Peres (1995:164), 'Ironically, Bell's theorem is "the most profound discovery of science" (after Stapp 1975) because it is *not* obeyed by the experimental facts.'

1.3. So, using accepted facts and elementary algebra to ensure that our work is independent of any physical theory (and thus beyond dispute), we reinforce our 1989 claim and show that Bell's inequality is algebraically false. We thereby refute Peres' view (1995:162) that 'Bell's theorem applies to any physical system with dichotomic variables.' We thus show that Bellians are being 'rather silly' as we here deliver one of the four key phrases in Bell's (1990:9) hope. In short with ¶1.1 in mind, they are:

(i) 'This AAD and no AAD business will pass.' (ii) 'If we're lucky it will be to some big new development like the theory of relativity.' (iii) 'Maybe someone will just point out that we were being rather silly.' (iv) 'But I believe the questions will be resolved.'

1.4. Thus this essay—delivering item (iii)—also introduces Watson 2018E (forthcoming Oct. 2018): though all results here are independent of 2018E. For 2018E goes on to resolve Bell's dilemma at its source via new developments in line with Bell's hopes at ¶1.3: (i) There is no need for AAD. (ii) For in 2018E we derive new results in such a way that Einstein's program succeeds, locality prevails, you can explain things by events in their neighbourhood; (iv) with all Bell's questions resolved.

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1.5. So we now show that Bell’s theorizing under the Einstein-Podolsky-Rosen-Bohm experiment (EPRB) is silly. For, via Bell’s first equation—Bell 1964:(1)—expectation values are bound by an always-true inequality (10) that holds for any experiment with binary outcomes, here designated ± 1 .

1.6. Then, en route to resolving Bell’s ¶1.1 dilemma, we use (10) as an elementary benchmark to show that Bell’s related (and famous) inequality (3) is false more often than not. So despite the fact that Bell uses the same data, we go beyond Bell’s inequality being false under QT and experiment: we show that it is first and foremost false under elementary algebra alone; an important first step toward resolving Bell’s dilemma (which we do, in 2018E).

1.7. Our notation is Bell’s, and we accept that LHS his 1964:(2) equals RHS his 1964:(3) under QT. [As it does under 2018E when we refute Bell’s ‘impossibility’ theorem—Bell’s claim of “not possible” beneath Bell-(3)—without QT.] This allows us to here supplement our elementary analysis of EPRB with QT: to thus show that, under QT, our inequality (10) correctly delivers every valid value while Bell’s inequality (3) and Bell’s associated theorizing deliver results that are absurd and thus invalid.

1.8. Fig.1 introduces our results via two snapshots. Like an archeological sieve set to screen dry clay and capture historic objects, our mathematical sieve (via screens from <http://www.wolframalpha.com>) is designed to capture historic real numbers that refute Bell’s inequality. That is, based on (12), we capture real numbers $\mathbb{R} < 0$ or $\mathbb{R} > 1$ as black marks, see (B); all other numbers passing cleanly through our sieve and away. So white-space in (A) shows our EPRB-based inequality (10) to be always true, while black areas in (B) show its famous Bellian sibling (3) to be frequently false.

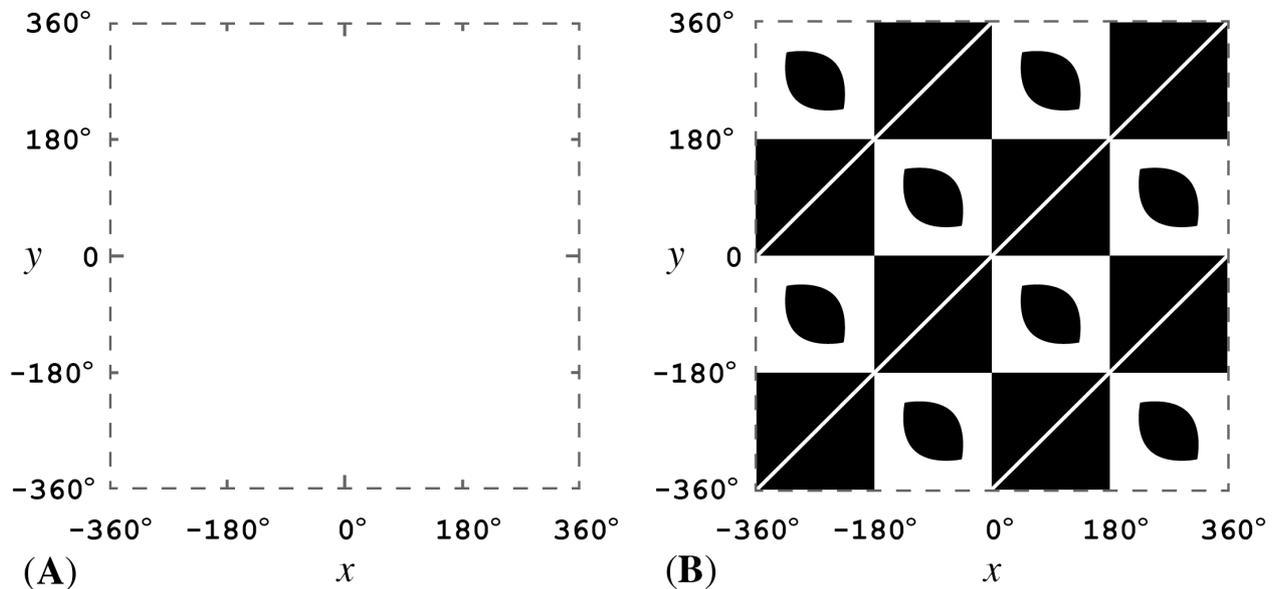


Fig.1. Snapshots of our mathematical sieve under Rule-1: (A) after screening our (10); (B) after screening Bell’s related and famous inequality (3). In keeping with the adequate simplicity of our analysis (sufficient to prove our claims) we allow what we at ¶2.6 call Rule-1: $x = (\vec{a}, \vec{b})$ and $y = (\vec{a}, \vec{c})$ are angles in 3-space such that $(\vec{b}, \vec{c}) = (\vec{a}, \vec{c}) - (\vec{a}, \vec{b})$.

1.9. White space in (A) shows (10) to be always true, never breaching its bounds of 0 and 1. (B) shows the same sieve after screening (3) to (10)'s benchmarks, black areas showing (3) false more often than not. That is: black lenses show (3) breaching (10)'s benchmark lower bound of zero via (13); black elsewhere shows (3) breaching the benchmark upper bound of one via (14).

1.10. White space in (B) thus shows the limited validity of Bell's inequality; with white lines (think zero-width) showing the relation $\cos(\vec{a}, \vec{b}) = \cos(\vec{a}, \vec{c})$ where (3) is also true. [Note that (3) is false under similar cosine relations at points in the lenses when it is the benchmark lower bound of zero that is breached; eg, when $(\vec{a}, \vec{b}) = 90^\circ$, $(\vec{a}, \vec{c}) = 270^\circ$, $(\vec{b}, \vec{c}) = 180^\circ$, and (3) = -1.]

2. Analysis

2.1. Bell 1964 (freely available, see References) provides definitions (see ¶2.2 next) of the key terms used in (1), (2) and (6). Let Bell-(1) be short for Bell 1964:(1), etc. Let (14a)-(14c) identify the unlabelled relations between Bell-(14)-(15); the remainder being (15a), (21a)-(21e), (23).

2.2. We begin by observing three key relations in Bell 1964: (i) At Bell-(1), a result A is given by a function $A(\vec{a}, \lambda) = \pm 1$. (ii) At LHS Bell-(2), an expectation value $P(\vec{a}, \vec{b})$ is given by the average over the product of paired-results $A(\vec{a}, \lambda)$ and $B(\vec{b}, \lambda)$: obtained via paired-tests on pristine (ex-source) particle-pairs. (iii) Linking LHS Bell-(2) to RHS-Bell-(3) via QT, $P(\vec{a}, \vec{b}) = -\cos(\vec{a}, \vec{b})$, etc.

2.3. So, from Bell-(1) and its preamble, and sharing Bell's indifference re the nature of λ for now,

$$A(\vec{a}, \lambda) = \pm 1, B(\vec{b}, \lambda) = \pm 1; \text{ etc.} \quad (1)$$

2.4. The related expectation values are then in the closed interval $[-1, 1]$:

$$-1 \leq P(\vec{a}, \vec{b}) \leq 1, -1 \leq P(\vec{a}, \vec{c}) \leq 1, -1 \leq P(\vec{b}, \vec{c}) \leq 1. \quad (2)$$

2.5. So Bell's inequality Bell-(15) may be written

$$\left| P(\vec{a}, \vec{b}) - P(\vec{a}, \vec{c}) \right| - P(\vec{b}, \vec{c}) \leq 1 \quad (3)$$

with an immediate difficulty for Bell: (3) is algebraically absurd. For, from (2), given

$$P(\vec{a}, \vec{b}) = P(\vec{b}, \vec{c}) = -\frac{1}{2}, P(\vec{a}, \vec{c}) = \frac{1}{2}, \quad (4)$$

Bell's inequality—Bell-(15), our (3)—delivers the absurd result that LHS (3) is greater than RHS (3):

$$\frac{3}{2} \leq 1 \blacktriangle \quad (5)$$

so that Bell's inequality is refuted via (4)—and the consequent absurdity (5)—with Bell's causative assumption shown at ¶2.16 below. Thus, as in ¶1.3, (1)-(2) represent the accepted facts and (4)-(5) the elementary algebra that deliver our result independent of any underlying physical theory.

2.6. Note in passing that (5) also holds under QT. For (4) is an allowable combination under EPRB (Bell's context) and QT (Bell's chosen comparison-theory): see Bell 1964, pp.195,199, and below Bell-(2), Bell-(15), Bell-(23); with Bell in the line below Bell-(2) providing all that we need from QT for now. That is, via QT,

$$P(\vec{a}, \vec{b}) = -\cos(\vec{a}, \vec{b}), P(\vec{a}, \vec{c}) = -\cos(\vec{a}, \vec{c}), P(\vec{b}, \vec{c}) = -\cos(\vec{b}, \vec{c}), \quad (6)$$

so (4) holds for $(\vec{a}, \vec{b}) = (\vec{b}, \vec{c}) = 60^\circ, (\vec{a}, \vec{c}) = 120^\circ$: and (3)'s absurdity holds via (4)-(5) under QT. That is, in this essay under the convenient simplification and ample coverage that we (for convenience) call Rule-1: the wide-ranging detector-settings are such that $(\vec{b}, \vec{c}) = (\vec{a}, \vec{c}) - (\vec{a}, \vec{b})$ in 3-space.

2.7. So now, before getting to the source of Bell's difficulties at ¶2.16, we first identify his problem algebraically. That is, influenced by (3)'s form and the breach in (5), we seek the related bounds associated with (2). To that end we begin with a trivial relation for the real numbers $p, q \in \mathbb{R}$:

$$\text{if } p \leq 1 \text{ and } 0 \leq q \text{ then } pq \leq q. \quad (7)$$

2.8. Thus, via (2) and (7) with $P(\vec{a}, \vec{b})$ equivalent to p , and $1 + P(\vec{a}, \vec{c})$ equivalent to q , we have

$$P(\vec{a}, \vec{b})[1 + P(\vec{a}, \vec{c})] \leq 1 + P(\vec{a}, \vec{c}). \quad (8)$$

$$\therefore P(\vec{a}, \vec{b}) - P(\vec{a}, \vec{c}) + P(\vec{a}, \vec{b})P(\vec{a}, \vec{c}) \leq 1. \quad (9)$$

2.9. Then, adding abstract brackets to (9), we have a relation to test Bell-(15) as in (3):

$$0 \leq \left| P(\vec{a}, \vec{b}) - P(\vec{a}, \vec{c}) \right| + P(\vec{a}, \vec{b})P(\vec{a}, \vec{c}) \leq 1, \quad (10)$$

where, using (2) and proof by exhaustion independent of any physical theory: (10)'s lower bound is zero when $P(\vec{a}, \vec{b}) = P(\vec{a}, \vec{c}) = 0$; with its upper bound of one confirmed via $P(\vec{a}, \vec{b}) = 0, P(\vec{a}, \vec{c}) = 1$, etc.

2.10. Thus, with (4)-(5) in mind and using (4) with (10), we find

$$0 \leq \frac{3}{4} \leq 1 \quad (11)$$

with (10)'s bounds intact. Then, comparing Bell's inequality Bell-(15) as in (3) with our inequality (10), we see that Bell's difficulties begin with that $-P(\vec{b}, \vec{c})$ in (3). For the test-angles used in (10)-(11) are unchanged from those used in (3) under QT at ¶2.6.

2.11. We therefore establish (10)'s irrefutable bounds as the benchmarks that (3) must meet; which, see Fig.2, is in Bell's favor. That is: via $-P(\vec{b}, \vec{c})$, (3) is certainly false for any related $\vec{a}, \vec{b}, \vec{c}$ if

$$(3) < 0 \text{ or } (3) > 1 \quad (12)$$

to the point that Bell's theorizing that delivered his inequality (3) would be refuted in general.

2.15. So, seeking to maximize generality and sampling diversity, let's randomly distribute $2n$ particle-pairs from $2n$ sources over $2n$ detector-pairs with randomly paired detector-settings equating to (\vec{a}, \vec{b}) and (\vec{a}, \vec{c}) as above. And to correlate particle-pairs with paired-outcomes, let's allow each particle-pair to be uniquely indexed— $[i = 1, 2, \dots, n$ over the settings (\vec{a}, \vec{b}) ; $j = n+i$ over the settings (\vec{a}, \vec{c})]—and reasonably unique since the probability of λ_i being replicated by λ_j is negligible: each being a random orientation in 3-space (¶2.14). We then allow n to be such that, to an adequate accuracy (satisfying serious critics) and for convenience in presentation:

$$\text{LHS Bell-(14a)} = P(\vec{a}, \vec{b}) - P(\vec{a}, \vec{c}) \quad (16)$$

$$= \frac{1}{n} \sum_{i=1}^n [A(\vec{a}, \lambda_{n+i})A(\vec{c}, \lambda_{n+i}) - A(\vec{a}, \lambda_i)A(\vec{b}, \lambda_i)], \text{ using our (15)}. \quad (17)$$

$$\therefore \left| P(\vec{a}, \vec{b}) - P(\vec{a}, \vec{c}) \right| = \left| \frac{1}{n} \sum_{i=1}^n A(\vec{a}, \lambda_i)A(\vec{b}, \lambda_i)[A(\vec{a}, \lambda_i)A(\vec{b}, \lambda_i)A(\vec{a}, \lambda_{n+i})A(\vec{c}, \lambda_{n+i}) - 1] \right| \quad (18)$$

$$\leq \frac{1}{n} \sum_{i=1}^n [1 - A(\vec{a}, \lambda_i)A(\vec{b}, \lambda_i)A(\vec{a}, \lambda_{n+i})A(\vec{c}, \lambda_{n+i})], \quad (19)$$

$$\therefore \left| \frac{1}{n} \sum_{i=1}^n A(\vec{a}, \lambda_i)A(\vec{b}, \lambda_i) \right| \leq 1. \quad (20)$$

$$\therefore 0 \leq \left| P(\vec{a}, \vec{b}) - P(\vec{a}, \vec{c}) \right| + P(\vec{a}, \vec{b})P(\vec{a}, \vec{c}) \leq 1, \quad (21)$$

(19) reducing via (15) to (21); which is (10), with the same related bounds via the same exhaustion of (2). Elementary engineering thus delivers the algebraically irrefutable (10) independently.

2.16. And we now see the use that Bell makes of his cryptic remark “using Bell-(1)” below Bell-(14b). For reverse-engineering requires—and exposes—that Bell naively uses $\lambda_i = \lambda_{n+i}$ in (19) to allow

$$\left| P(\vec{a}, \vec{b}) - P(\vec{a}, \vec{c}) \right| \leq \frac{1}{n} \sum_{i=1}^n [1 - A(\vec{a}, \lambda_i)A(\vec{b}, \lambda_i)A(\vec{a}, \lambda_i)A(\vec{c}, \lambda_i)]. \quad (22)$$

$$\therefore \left| P(\vec{a}, \vec{b}) - P(\vec{a}, \vec{c}) \right| \leq 1 + P(\vec{b}, \vec{c}) \quad (23)$$

which is that algebraically false Bell-(15), (3): Bell reducing (22) to (23) via

$$A(\vec{a}, \lambda_i)A(\vec{a}, \lambda_i) = 1 \text{ and } -\frac{1}{n} \sum_{i=1}^n [A(\vec{b}, \lambda_i)A(\vec{c}, \lambda_i)] = P(\vec{b}, \vec{c}). \quad (24)$$

2.17. So Bell-(15), see (3) or (23), is based on an unphysical assumption under EPRB and is refuted as above. For the source of Bell's error is this: under EPRB, the context for Bell (1964), Bell's use of Bell-(1) via $\lambda_i = \lambda_{n+i}$ at ¶2.16 implies that our extreme randomizing in ¶2.15 has somehow produced two identical cohorts of ordered particle-pairs. To the contrary, our fair randomizing has produced two representative cohorts of dissimilar particle-pairs which, if exchanged, would leave the results unchanged to an adequate accuracy: a true measure of fair sampling. The result is that Bell-(14b) does not equal Bell-(14a). See forthcoming 2018E for further analysis and discussion.

2.18. Now some suggest privately that our use of (6) to move (12) to (13)-(14) is invalid because Bell (against QT) wants to keep open the possibility that the *quantum correlations* in (6) are false. And we find some merit in this objection: to us it's a possible explanation for Bell's strange claim of "not possible" below Bell-(3) with its false implication—shown to be false at Watson 2018E:(31), consistent with QT—that a standard formula for an average in Bell-(2) doesn't work here under EPRB.

2.19. However, as shown via our new approach to the physics in EPRB via Watson 2018E, we can derive (6) without QT. So though Bell and private others may have reservations re (6), it is clearly available to us: with QT's endorsement; and via QT alone if necessary today.

2.20. So, with other Bellian claims refuted in Watson 2018E, we conclude here for now by making one final point: wanting to be clear and minimize a common misunderstanding. Though our theorizing can be independent of QT, QT is of course (given its remarkable history) an adequate theory in our terms. But in Watson 2018E we bring a new approach and additional understanding to physics in line with Einstein's ideas, Bell's expectation in ¶1.3, and Born's (1954:266) view of QT:

“The lesson to be learned is that probable refinements of mathematical methods will not suffice to produce a satisfactory theory, but that somewhere in our doctrine is hidden a concept, unjustified by experience, which we must eliminate to open up the road.”

2.21. Hence the related finding in 2018E: Born's hidden-concept is *naive-realism*, for it hides in plain sight here and in physics generally: Bellian realism, Bell's inequality, Bell's theorem, Bell's dilemma, local realism, locally realistic, realism; and via invalid counterfactual reasoning wherein—neglecting the fact that beables may change interactively—pre-measurement properties are taken to be the same as post-measurement properties, etc.

3. Conclusions

3.1. The basis for Bell's original inequality is widely discussed up to the present day. But now, whatever that basis, those captured black marks in (B) prove the following on Bell's own terms: Bell's famous Bell-(15) is algebraically false. And, taking mathematics to be the best logic (as we do), Bell-(15) is also logically false. Moreover, point-by-point these truths about Bell-(15) can be confirmed by auditing Fig.1 with a portable calculator, or via [Web2.0Calc](#) online, with the relevant relation, (3) or (10): (3) being Bell's famous inequality that we refute; (10) on the other hand being an elementary theorem that replaces (3) as a consequence of (2) and (8), each being an irrefutable consequence of Bell-(1), our (1).

3.2. Not only that, but Bell-(15) is also unphysical—see Watson 2018E:¶7—akin to inferring that a naive-realism might succeed in a highly-correlated quantum-setting (EPRB, the setting for Bell 1964 and thus for us here) and *not* then dropping such naivety when it fails so badly.

3.3. Further, whatever triggered Bell’s naive inference, Bell-(15) is also unwarranted. For, against Bell here, Watson 2018E delivers (via true local realism, and without QT) the same results as QT and observation. This shows independently that Bell-(2) equals Bell-(3): at the same time refuting Bell’s theorem—that is, Bell’s “not possible” below Bell-(3)—and thus Bell’s EPRB theorizing generally.

3.4. Thus, to be clear re our long-held position: for us, true local realism is the union of true locality and true realism. *True locality* insists that no influence propagates superluminally, after Einstein. *True realism* insists that some beables *may* change interactively, after Bohr. *Naive-realism* is then any brand of *realism* that negates or neglects that ‘*may*’ when relevant; see ¶2.21.

3.5. Then, re further consequences of this work: please note that the Einstein program does not fail with us; for against Bell in ¶1.1, Watson 2018E does get away with locality and local explanations—that is, without AAD—to refute claims like these:

‘Einstein maintained that quantum metaphysics entails spooky actions at a distance; experiments show that what bothered Einstein is not a debatable point but the observed behaviour of the real world,’ after Mermin (1985:38). ‘Our world is non-local,’ after Davies (1984:48), Goldstein *et al.* (2011:1), Maudlin (2014:25), Brimont (2016:112). ‘... the predictions of quantum theory cannot be accounted for by any local theory,’ after Brunner *et al.* (2014:1), Norsen (2015:1); ‘the search for such models is hopeless,’ Aspect (2002: 9).

3.6. Finally, we conclude: Bell’s inequality, algebraically false independent of its failings under QT and repeated experiments, represents an interesting phase in the history of science. We suggest that 2018E resolves this phase and Bell’s dilemma in a way that Einstein, Born and Bell might like:

“... you make a very thorough analysis of EPR-Bell. As you still remain a ‘realist’ and refer to Bell’s beables when you resolve Bell’s dilemma, Bell might have liked your approach, who knows.” (R. Bertlmann 2017, pers. comm. 26 June); see Watson (2017d).

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