Fermat's last theorem proof by A.I.C.E

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Abstract

Fermat's last theorem states that there are not (a, b, c, x) positive integers that satisfy the equation $a^x + b^x = c^x$ if $x > 2$. Fermat conjetured this in 1637 and Andrew Wiles proved it in 1995 using math techniques not available to Fermat, in the next paper we will see a simple proof using math techniques available to Fermat.

1 Introduction

Fermat's last theorem states that there are not (a, b, c, x) positive integers that satisfy the equation $a^x + b^x = c^x$ if $x > 2$. This theorem has two cases.
Case 1, when $x = 4$ and multiples of 4.
Case 2, when $x = 2n + 1$ where $n \geq 1$, and multiples of $x = 2n + 1$.
$x = 2n + 1$ is all odd numbers. Therefore we can say all odd numbers greater than 1 and their multiples. Fermat proved case 1 ($x = 4$) then let us prove case 2 ($x = 2n + 1, n \geq 1$).

2 Properties of the equation

There are infinite solutions of $a^x + b^x = c^x$ solutions when $x = 1$ and $x = 2$, $x = 2$ is a pythagorean triple and we know there are pythagorean triples primitives and not primitives. A primitive triple is when a, b, c are coprimes, a not primitive triple is when a, b, c have a common factor. We can find infinite solutions from a primitive triple and we can find a primitive triple from a not primitive triple.

2.1 Property one

Let $a^x + b^x = c^x$, (a, b, c, x) positive integers and $x \geq 1$ then $(na)^x + (nb)^x = (nc)^x$ where $n \geq 1$.
Proof
$(na)^x + (nb)^x = (nc)^x$
$n^x a^x + n^x b^x = n^x c^x$
$n^x (a^x + b^x) = n^x c^x$
$\frac{n^x (a^x + b^x)}{n^x} = \frac{n^x c^x}{n^x}$
$a^x + b^x = c^x$
Therefore $a^x + b^x = c^x \Leftrightarrow (na)^x + (nb)^x = (nc)^x$. This is property one.

2.2 Property two

Let $a^x + b^x = c^x$, (a, b, c, x) positive integers and $x = 2n + 1$,$n \geq 1$ then $a^x + b^x = c^x = (a + b)(P(a, b))$ where $P(a, b)$ is polynomial of a and b that depends of x.
Then $a^{2n+1} + b^{2n+1} = c^{2n+1} = (a + b)(P(a, b))$. This is property two.

2.3 Factorization of powers

All n^x can be written as $(n_1^x)(n_2^x)$ where n_1 and n_2 are factors of n, examples.
$5^3 = (1)^3(5)^3$, $n_1 = 1$, $n_2 = 5$
$6^3 = (2)^3(3)^3$, $n_1 = 2$, $n_2 = 3$

3 Proof

3.1 Sets of powers

Let us define some sets ($K1$, $K2$, $K3$, $K4$, $K5$....), Kn is the set of powers with the maximum expressible power of m^y equal to n of Kn. Then $m^y \in Kn$ if $\max(y) = n$. Examples.
$K1 = (1, 2, 3, 5, 6, 7, 10, 11, 12, 13, 14, 15, 17, 18, 19....)$
$K2 = (4, 9, 25, 36, 49, 100, 121, 144, 169, 196, 225....)$
$K3 = (8, 27, 125, 216, 343, 1000, 1331, 1728, 2197....)$
$K4 = (16, 81, 625, 1296, 2401, 10000, 14641, 20736....)$
$Kn = ...$

3.2 Implications

Due the properties of the equation $a^x + b^x = c^x$ with(a, b, c, x) positive integers, $x = 2n + 1, n \geq 1$, the equation has some implications.

Implication one
Let $a^x + b^x = c^x = (a + b)(P(a, b))$, $(a + b) \in K1$ then $a + b = c_1$ where c_1 is a factor of c.
Proof
$a^x + b^x = c^x = (a + b)(P(a, b)) = c_1^x c_2^x$
$a^x + b^x = c^x = (a + b)^1 c_2^{x-1} = c_1^x c_2^x$
$a^x + b^x = c^x = (a + b)^x c_2^x = c_1^x c_2^x$
$(a + b) = c_1$
Then $a^x + b^x = c^x = (a + b)^x c_2^x$, but $(a^x + b^x) < (a + b)^x < (a + b)^x c_2^x$.
Proof
$(a + b)^x = a^x + b^x + P(a, b)$ where $P(a, b)$ is another polynomial that depends of x.
Then $(a^x + b^x) < (a^x + b^x + P(a, b)) < (a^x + b^x + P(a, b))(c_2^x)$
Therefore $a^x + b^x \neq (a + b)^x c_2^x$ then there are not solutions when $(a + b) \in K1$.
Then $a^x + b^x \neq c^x$ if $(a + b) \in K1, x$ an odd number greater than 1.

Implication two
We know there are not solutions when $(a + b) \in K1$ and if $a^x + b^x = c^x$ then $(na)^x + (nb)^x = (nc)^x$, for all n.

Let $a^x + b^x = c^x = (a+b)(P(a,b))$, $(a+b) \in Km, m > 1$ then $(na+nb) \in Km, m > 1$, but it is false, because if $(a+b) \in Km, m > 1$, then $(na+nb) \in K1$, $n \in K1$ and n coprime of $(a+b)$.
Proof
Let $(a+b) \in Km, m > 1$ Then $(na+nb) \in K1$ where $n \in K1$ and coprime of $(a+b)$.
$(a+b) = y^m, y^m \in Km, m > 1$
$(na+nb) = n^1 y^m, n^1 \in K1$
We know if n^1 coprime of y^m then $n^1 y^m \in K1$, Therefore $(na+nb) \in K1$.Then $a^x + b^x \neq c^x$ if $(a+b) \in Km, m > 1$, x an odd number greater than 1.

4 Conclusion

There are not integer solutions when $(a+b) \in K1$ and $(a+b) \in Km, m > 1$.
Therefore there are not(a,b,c,x) positive integers that satisfy the equation
$a^x + b^x = c^x, x = 2n+1, n > 1$. It proves the case two of Fermat's last theorem.