Update the path integral in quantum mechanics by using the energy pipe streamline

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Abstract

The path integral in quantum mechanics is a very important mathematical tools. It is widely applied in quantum electrodynamics and quantum field theory. But its basic concepts confuse all of us. The first thing is the propagation of the probability. The second is the path can be any paths you can draw. How this can work? In this article, a new definition of energy pipe streamline integral is introduced in which the mutual energy theorem and the mutual energy flow theorem, mutual energy principle, self-energy principle, Huygens principle, and surface integral inner product of the electromagnetic fields are applied to offer a meaningful and upgraded path integral. The mutual energy flow is the energy flow from the emitter to the absorber. This energy flow is built by the retarded wave radiates from the emitter and the advanced wave radiates from the absorber. The mutual energy flow theorem guarantees that the energy go through any surface between the emitter and the absorber are all equal. This allow us to build many slender flow pipes to describe the energy flow. The path integral can be defined on these pipes. This is a updated path integral is referred as the energy pip streamline integral. The Huygens principle allow us to insert virtual current sources on any place of the pipes. Self-energy principle tell us that any particles are consist of 4 waves: the retarded wave, the advanced wave and another two time-reversal waves. All these waves are canceled and hence the waves do not carry or transfer any energy. Energy is only carried and transferred by the mutual energy flow. Hence the mutual energy flow theorem is actually the energy flow theorem. Wave looks like probability wave, but mutual energy flow are real energy flow is not a probability flow. In this article the streamline integral is applied to electromagnetic field or photon or other particle for example electrons.

Keywords: Wave function; Poynting; Maxwell; Schrödinger; Dirac; Self-energy; Mutual energy; Mutual energy flow; Path integral;

Contents

1 Introduction

1.1 The traditional way to introduce the path integral

The traditional way to introduce the path integral in quantum mechanics are Feynman's way and Dirac's way.

In Feynman's way, it said a professor teach the double slits excrement and tell the students there are two paths. The wave can go from the source to the sink. The amplitudes of the two wave can be superposed. A student ask what happens if there are 3 slits? What happens if there is another partition board in which also have a few slits? This way the concept of the path integral is built.

In the Dirac's way, let us divide the time T as N segments each lasting $\delta t = T/N$, Then write,

$$\langle q_F| e^{-iHT} |q_I\rangle = \langle q_F| e^{-iH\delta t} e^{-iH\delta t} \cdots e^{-iH\delta t} |q_I\rangle \tag{1}$$

Considering,

$$\int dq \, |q_n\rangle \langle q_n| = 1 \tag{2}$$

$$\langle q_F| e^{-iHT} |q_I\rangle = \prod_{n=1}^{N-1} (\int dq_n) \langle q_F| e^{-iH\delta t} |q_{N-1}\rangle \langle q_{N-1}| e^{-iH\delta t} |q_{N-2}\rangle \langle q_{N-2}| \cdots$$
$$\cdots \langle q_2| e^{-iH\delta t} |q_1\rangle \langle q_1| e^{-iH\delta t} |q_I\rangle \tag{3}$$

Focus on an individual factor $\langle q_{n+1}| e^{-iH\delta t} |q_n\rangle$, consider $H = \frac{\hat{p}^2}{2m}$. The hat on $\hat{p}$ is a operator. Denote by $|p\rangle$ the eigenstate of $\hat{p}$. Namely $\hat{p}\,|p\rangle = p\,|p\rangle$. Do you remember from your course in quantum mechanics that $\left\langle q \middle| p \right\rangle = e^{ipq}$. Considering $\frac{1}{2\pi}\int dq\,|p\rangle\langle p| = 1$

$$\langle q_{n+1}| e^{-iH\delta t} |q_n\rangle = \frac{1}{2\pi}\int dp \left\langle q_{n+1} \middle| e^{-i\hat{p}^2\delta t/2m} \middle| p \right\rangle \left\langle p \middle| q_n \right\rangle$$
$$= \frac{1}{2\pi}\int dp e^{-ip^2\delta t/2m} \left\langle q_{n+1} \middle| p \right\rangle \left\langle p \middle| q_n \right\rangle$$
$$= \frac{1}{2\pi}\int dp e^{-ip^2\delta t/2m} e^{ip(q_{n+1}-q_n)} \tag{4}$$

Above is Gaussian integral, it can integral out.

$$\langle q_{n+1}| e^{-iH\delta t} |q_n\rangle = (\frac{-im}{2\pi\delta t})^{\frac{1}{2}} e^{[im(q_{n+1}-q_n)^2]/2\delta t}$$
$$= (\frac{-im}{2\pi\delta t})^{\frac{1}{2}} e^{i\delta t(m/2)[(q_{n+1}-q_n)/\delta(t)]^2} \tag{5}$$

Hence,

$$\langle q_F| e^{-iHT} |q_I\rangle = (\frac{-im}{2\pi\delta t})^N (\prod_{n=1}^{N-1}\int dq_n) e^{i\delta t(m/2)\sum_{n=0}^{N-1}[(q_{n+1}-q_n)/\delta(t)]^2} \tag{6}$$

In the above $q_F = q_N$, $q_I = q_0$. Define

$$\int Dq(t) = \lim_{N\to\infty} (\frac{-im}{2\pi\delta t})^N (\prod_{n=1}^{N-1}\int dq_n) \tag{7}$$

Considering

$$\dot{q} = \frac{q_{n+1}-q_n}{\delta t} \tag{8}$$

$$\int\limits_{t=0}^{T} dt = \delta t \sum_{n=0}^{N-1} \tag{9}$$

$$i\int\limits_{t=0}^{T} e^{m\dot{q}/2} dt = i\delta t(m/2)\sum_{n=0}^{N-1}[(q_{n+1}-q_n)/\delta(t)]^2 \tag{10}$$

$$\langle q_F| e^{-iHT} |q_I\rangle = \int Dq(t) e^{i\int_{t=0}^{T} e^{m\dot{q}/2} dt} \tag{11}$$

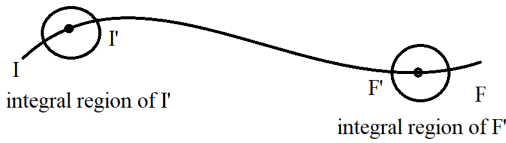

Figure 1: In this example, a integral is done at the place close to I'. Another integral is done in the place close to F'. Are these integral in the same 3D space?

1.2 The problems for the path integral

The above looks good. But the problem is the definition of the integral $\int Dq(t)$ is a limit of infinite integral that is too complicated. Dirac can define the integral this way, but how we can understand it? How this infinite integral can be converged to some thing? Another thing is that the path integral is define on the top of the concept of the probability. Einstein cannot understand the probability interpretation, how we can understand it? How this probability propagates to produce a particle? Feynman said "no one can understand it". I think it is true!

In the mathematics there is also problem. Why $\int dq \left|q_n\right\rangle \left\langle q_n\right| = 1$ established? This need the inner product $(\cdot, \cdot)$ or the integral $\int dq$ is made in the same place. Even in the integral in the path integral definition is a whole 3D space, but actually the center of integral has shift along the path, see Figure 1.

If the original inner product is at the place I, you can have $\int dq \left|q_n\right\rangle \left\langle q_n\right| = 1$ at I, that is clear, but in the path integral we need to apply $\int dq \left|q_n\right\rangle \left\langle q_n\right| = 1$ to another place for example Γ or F. That is not self-explanatory. In Figure 1 we have show two place I' and F'. I' is a place close to the start point I, F' is a place close to the end point F. In both places the integrals are on 3D infinite space. My argument is that are these two integrals in a same 3D region? I do not think so. Hence even you can make $\int dq \left|q_n\right\rangle \left\langle q_n\right| = 1$ at I' that doesn't guarantee you can do it at F'!

For example I is the place of emitter and F is the place of absorber, assume the distance from emitter to the absorber has a few light years, even you know in the place I' you can have $\int dq \left|q_n\right\rangle \left\langle q_n\right| = 1$, how you can know in the place F' you still can have $\int dq \left|q_n\right\rangle \left\langle q_n\right| = 1$? If the integral $\int dq$ is converge, it should have significant region which close to its centers. Since the two centers are far away, this led the integral at I' and F' are actually in different places. This is a big problem for path integral.

Actually the integral $\int dq$ in path integral can be defined on a serial of surfaces. This way the definition of the path integral become simple, but in that case Dirac has to deal the problem (The surface I' and the surface F' are clear not the same surface) distinctly. Dirac define the integral in a 3D space, try to hide behind the problem.

I think Feynman has been aware of the problem. He doesn't think Dirac's derivation is meaningful. Hence, in his paper[8] he spend a lot of ink on the probability presumptions. He started from this presumptions and when obtained the results, he prove his result to be same with Schrödinger equation. That means in Feynman's paper, he didn't sure the operation of the path integral is correct, but because the result is inspect and verify by Schrödinger equation, he finally believe it. My argument is even you get correct result, but if the definition and derivation has something wrong, we still need to correct it.

We also know in the 3D space the amplitude of the field is decrease, hence, the field cannot be written as

$$\exp(something) \tag{12}$$

However, in the path integral a constant amplitude has been applied without any explanation!

In this article I will simplify the definition of path integral, in order to do this. I will abandon the interpretation of probability or the interpretation of Copenhagen school. The energy flow will be used instead. The concepts of the mutual energy theorem, mutual energy flow theorem, mutual energy principle, self-energy principle, inner product of the electromagnetic fields, Huygens principle will be applied. In the following, I first introduction all these concepts.

By the way Huygens principle has been mentioned in the path integral[8] by Feynman, but he did not offer any details how to apply this concept. It is important to combine the Huygens principle with the mutual energy flow theorem which will be done in this article.

1.3 Review the work on the topic of mutual energy and mutual energy flow

In this article the author will updated the concept path integral with the energy pipe streamline integral. In order to build the pipe streamline integral the theory of the mutual energy flow is involved, which Further related the concept of the retarded wave, advanced wave and the time-reversal waves corresponding to the retarded wave and the advanced waves.

The field theory is first introduced by Faraday and later it is introduced by Maxwell in 1865. The action at a distance which are introduced by Weber 1848 [29]. Maxwell's theory allow existent of the advanced potential with the retarded potential. Advanced potential or is referred as advanced wave. Weber's theory also allow the advanced wave. But these two theories did not say that the advance wave must exist. Hence there are two theories one supported the concept of advanced wave, one denied the exist of the advanced wave.

There is also another action-at-a-distance principle, which was introduced by introduced by Schwarzschild, Tetrode and Fokker [25, 9, 27]. According to this principle, a retarded wave and an advanced wave must be sent by the current source together. Following the action-at-a-distance J.A. Wheeler and R.P. Feynman introduced the absorber theory [1, 2]. In the absorber theory, the absorber is the reason of a emitter can radiate. Based on the absorber theory, John Cramer has introduced the transactional interpretation for quantum physics [5, 6]. In the transaction process the retarded wave and the advanced wave can have a "handshake".

W.J. Welch has introduced a reciprocity theorem in arbitrary time-domain [28] in 1960 (this will be referred as Welch's reciprocity theorem in this article). In 1963 V.H. Rumsey mentioned a method to transform the Lorentz reciprocity theorem to a new formula[24], (this will be referred as Rumsey's reciprocity theorem). In the early of 1987 Shuang-ren Zhao has introduced the concept of mutual energy and the mutual energy theorem [12] (this will be referred as Zhao's mutual energy theorem). In the end of 1987 Adrianus T. de Hoop introduced the time domain cross-correlation reciprocity theorem[7], (this will be referred as Hoop's reciprocity theorem). Welch's reciprocity theorem is a special case of the Hoop's reciprocity theorem. Welch proved his reciprocity theorem by a retarded wave and advanced wave, that means Welch's reciprocity theorem is a theorem between a retarded wave and an advanced wave. Since the reciprocity theorem need to applied to antenna system, this reciprocity theorem tell us the transmitting antenna sends a retarded wave, the receiving antenna sends an advanced wave.

All the above theorems are in touch with Fourier transform and can be seen as a same theorem in both time and frequency domains. Welch's reciprocity theorem and Hoop's reciprocity theorem are in time-domain. Rumsey's reciprocity theorem and Zhao's mutual energy theorem are in Fourier domain. In the following this theorem will be referred as Welch-Rumsey-Zhao-Hoop's theorem.

It should be say that Welch-Rumsey-Zhao-Hoop's theorem is not a subtheorem of Lorentz reciprocity theorem. However the two theorems are link by the conjugate transform[10]. But anyway, the Lorentz reciprocity theorem content some important information of Welch-Rumsey-Zhao-Hoop's theorem. Welch-Rumsey-Zhao-Hoop's theorem is physical theorem, but Lorentz reciprocity is only a mathematical theorem which can be used to do the calculation for the directivity diagram.

Similar to the Lorentz reciprocity theorem, Welch-Rumsey-Zhao-Hoop's theorem is a reciprocity theorem. But Shuang-ren Zhao (this author) has noticed this theorem is not only a reciprocity theorem but also a energy theorem and hence the theorem is referred as the mutual energy theorem[12]. Shuang-ren Zhao also found out the surface integral in this theorem is a good inner product. This inner product can used to produce a inner product space for all kind of electromagnetic fields[12, 31]. Shuang-ren Zhao also applied this inner product for spherical wave expansion and plane wave expansion problems [12, 30]. The concept of this surface inner product will together with the concept of the mutual energy flow be applied in this article to define the energy pipe streamline integral.

After around 30 years working on different topic: medical image processing and numerical calculation, this author decided to go back to the topic of mutual energy again. In the beginning this author first proved that the mutual energy theorem is a energy theorem. This is done by proving that the mutual energy theorem is a sub-theorem of the Poynting theorem. Hence the mutual energy theorem is worth of it's name [18].

The articles [21][22] built a photon model with the mutual energy flow. It also guess the possibility for the self-energy flow. There are two major possibility for the self-energy flow: (1) the self-energy flows are collapsed. (2) it is return to their sources by time-reversal waves. That means the retarded wave return to the emitter, the advanced wave return to the absorber.

The article [23] introduced the mutual energy flow and the mutual energy flow theorem. This theorem tell us there are energy flow go from the emitter to the absorber. The energy go through in any surface between the emitter and the absorber is a constant. The author believe this energy is the energy of the photon. Hence, the photon is nothing else, it is the mutual energy flow. In this article the mutual energy flow theorem is a foundation stone for the definition of the energy pipe streamline integral.

The article [26] discussed the wave and particle duality with the mutual energy flow.

The article [14] found the bug in Poynting theorem. It found that Maxwell equations and Poynting theorem together with superposition principle conflict with the energy conservation condition. This lead to the introduce of the self-energy principle and the mutual energy principle. In the self-energy principle, two time-reversal waves are introduced. Hence, any particles are all built with 4 waves: the retarded wave, the advanced wave and the two time-reversal waves.

The article [16] further introduced the self-energy principle and also the mutual energy principle.

The article [19] discussed the possibility to make a experiment for advanced wave using classical electromagnetic field instead of the method of the quantum mechanics.

The articles [13][20] offer a new interpretation for quantum mechanics which is the mutual energy flow interpretation.

The article [15] widened the concept of self-energy principle and the mutual energy principle to the Schrödinger equations and Dirac equations. It point out that for a quantum system which satisfies the Schrödinger equations and the Dirac equations similar to the electromagnetic field which satisfies Maxwell equations, hence, the mutual energy flow theorem, inner product can also be defined. There is also 4 waves, the retarded wave and the advanced wave and the 2 time-reversal waves. Hence all the concept this author has obtained in electromagnetic field theory can be widened to Schrödinger equations and Dirac equations. This will guarantees the energy pipe streamline integral can also be defined based on the system with Schrödinger equations and Dirac equations.

The article [17] discussed the wave and particle duality. Especially in this article, it is proved that in the wave guide, cone-beam wave guide and the free space with a uniformly distributed absorber on the infinite big sphere, the result by using the mutual energy principle and Poynting theorem are equivalent. That means for this 3 situation, even in the beginning you have assumed the electromagnetic field includes the advanced wave and the retarded wave, you still can obtained same results as Poynting theorem is applied with only the retarded wave. Hence, for most engineering problem the Poynting theorem and Maxwell equation still can be applied. In the situation the absorbers are not uniformly distributes on the infinite sphere, the Poynting theorem cannot be applied. For example, for a two antenna system in which one is the transmitting antenna and another is a receiving antenna or an emitter with a scatter. In this situation, in order to correct the wrong doing of the Poynting theorem, the concept of effective scatter section must be applied. In the case of wire antenna, the effective section area is possible to have 1000 times bigger than the original section area of the wire. If we calculate a effective section area of an absorber (for example a charge) which can be infinite times larger. That means that the Poynting theorem get totally wrong result! In case Poynting theorem is wrong, the mutual energy theorem and the mutual energy flow theorem still can offer correct results. It should be say that in the wave guide situation, we calculated the energy with Poynting theorem or mutual energy theorem. But pleas do not use the mutual energy energy together with self-energy. If we add the energy of self-energy, the transferred energy doubled which violated the energy conservation. This also further tell us the self-energy items do not transfer any energy. This further confirm the self-energy principle which tells us there are 2 time-reversal wave which cancels all self-energy items. Since the electron in a orbit is same as the electromagnetic field in a wave guide, hence, in a orbit a electron can be applied with only retarded wave. We do not need to consider the advanced wave. However the retarded wave and advanced wave both exist. Each of that contributed the half of the field.

2 Important theorems

Assume there are two current sources $\boldsymbol{J}_1$ and $\boldsymbol{J}_2$. $\boldsymbol{J}_1$ is the current of a transmitting antenna. $\boldsymbol{J}_2$ is the current of a receiving antenna. The field of $\boldsymbol{J}_1$ is described as $\boldsymbol{E}_1$ and $\boldsymbol{H}_1$. The field of the current $\boldsymbol{J}_2$ is $\boldsymbol{E}_2$ and $\boldsymbol{H}_2$. Assume $\boldsymbol{J}_2$ has a some distance with $\boldsymbol{J}_1$. Some time we will use $\xi = [\boldsymbol{E}, \boldsymbol{H}]$ to describe the field together with electric field and magnetic field.

Hoop's reciprocity theorem can be written as,

$$-\int_{t=-\infty}^{\infty} \iiint_{V_1} \boldsymbol{J}_1(t+\tau) \cdot \boldsymbol{E}_2(t) dV = \int_{t=-\infty}^{\infty} \iiint_{V_2} \boldsymbol{E}_1(t+\tau) \cdot \boldsymbol{J}_2(t) dV \qquad (13)$$

if $\tau = 0$, we have,

$$-\int_{t=-\infty}^{\infty} \iiint_{V_1} \boldsymbol{J}_1(t) \cdot \boldsymbol{E}_2(t) dV = \int_{t=-\infty}^{\infty} \iiint_{V_2} \boldsymbol{E}_1(t) \cdot \boldsymbol{J}_2(t) dV \qquad (14)$$

This is Welch's reciprocity theorem. The Fourier transform of Hoop's reciprocity theorem can be written as,

$$-\int_{t=-\infty}^{\infty}\iiint_{V_1}\boldsymbol{J}_1(\omega)\cdot\boldsymbol{E}_2(\omega)^*dV=\int_{t=-\infty}^{\infty}\iiint_{V_2}\boldsymbol{E}_1(\omega)\cdot\boldsymbol{J}_2(\omega)^*dV \qquad (15)$$

Where "*" is the complex conjugate operator. In this article if the variable t is applied the formula is in time domain. If ω is applied, it is in Fourier domain. For simplification, we do not use $\tilde{A}$ to describe a variable in Fourier domain. Eq.(15) is the Rumsey's reciprocity theorem and also Zhao's mutual energy theorem. Hence this 4 theorems can be seen as one theorem in different domain: time-domain and Fourier domain.

2.1 Conjugate transform

Assume that a field system with its source is ζ,

$$\zeta=[\boldsymbol{E}(t),\boldsymbol{H}(t),\boldsymbol{J}(t),\boldsymbol{K}(t),\boldsymbol{\epsilon}(t),\mu(t)] \qquad (16)$$

where $\boldsymbol{K}(t)$ is magnetic current intensity. The magnetic current intensity $\boldsymbol{K}$ is normally as 0. The conjugate transform [10] can be defined as,

$$\mathbb{C}\zeta=[\boldsymbol{E}(-t),-\boldsymbol{H}(-t),-\boldsymbol{J}(-t),\boldsymbol{K}(-t),\epsilon(-t),\mu(-t)] \qquad (17)$$

In the frequency domain,

$$\zeta=[\boldsymbol{E}(\omega),\boldsymbol{H}(\omega),\boldsymbol{J}(\omega),\boldsymbol{K}(\omega),\boldsymbol{\epsilon}(\omega),\mu(\omega)] \qquad (18)$$

The conjugate transform can be defined as,

$$\mathbb{C}\zeta=[\boldsymbol{E}(\omega)^*,-\boldsymbol{H}(\omega)^*,-\boldsymbol{J}(\omega)^*,\boldsymbol{K}(\omega)^*,\epsilon(\omega)^*,\mu(\omega)^*] \qquad (19)$$

After a conjugate transform a retarded wave become an advanced wave and an advanced wave becomes a retarded wave.

2.2 Lorentz reciprocity theorem

In frequency domain Lorentz reciprocity theorem [3, 4] can be written as,

$$\int_{t=-\infty}^{\infty}\iiint_{V_1}\boldsymbol{J}_1(\omega)\cdot\boldsymbol{E}_2(\omega)dV=\int_{t=-\infty}^{\infty}\iiint_{V_2}\boldsymbol{E}_1(\omega)\cdot\boldsymbol{J}_2(\omega)dV \qquad (20)$$

In the Lorentz reciprocity theorem ζ_1 and ζ_2 are all retarded field. It can be shown that from Lorentz reciprocity theorem applied the conjugate transform to one of the field for example ζ_2, the above formula become the mutual energy theorem Eq.(15). From last subsection we know if ζ_2 is a retarded field, after the conjugate transform, ζ_2 become the advanced field. Hence, inside the the mutual energy formula the two fields one is retarded field and another must be advanced field. We will further prove that ζ_2 is an advanced field in later sections.

2.3 Inner product of electromagnetic fields

Shuang-ren Zhao has define the inner product for electromagnetic fields[12]. Assume $\xi_1 = [\boldsymbol{E}_1, \boldsymbol{H}_1]$, $\xi_2 = [\boldsymbol{E}_2, \boldsymbol{H}_2]$ we have inner product,

$$(\xi_1, \xi_2) = \oiint_{\Gamma} (\boldsymbol{E}_1 \times \boldsymbol{H}_2^* + \boldsymbol{E}_2^* \times \boldsymbol{H}_1) \cdot \hat{n} d\Gamma \tag{21}$$

Γ is closed surface. It should be notice here, the character ξ is electromagnetic field, ζ is the electromagnetic field together with its source. Shuang-ren find that this formula satisfy inner product 3 conditions[12],

(I) Conjugate symmetry:

$$(\xi_1, \xi_2) = (\xi_2, \xi_1)^* \tag{22}$$

(II) Linearity:

$$(a\xi_1' + b\xi"_1, \xi_2) = a(\xi_1', \xi_2) + b(\xi"_1, \xi_2) \tag{23}$$

(III) Positive-definiteness:

$$(\xi, \xi) > 0 \tag{24}$$

$$(\xi, \xi) = 0 \Rightarrow \xi = 0 \tag{25}$$

"$\Rightarrow$" means "can derive". Shuang-ren Zhao found that the mutual energy theorem can be also written as [12, 31],

$$-(J_1, \xi_2)_{V_1} = (\xi_1, J_2)_{V_2} \tag{26}$$

where

$$(J_1, \xi_2)_{V_1} = \iiint_{V} \boldsymbol{E}_2(\omega)^* \cdot \boldsymbol{J}_1(\omega) dV \tag{27}$$

$$(\xi_1, J_2)_{V_2} = \iiint_{V_2} \boldsymbol{E}_1(\omega) \cdot \boldsymbol{J}_2(\omega)^* dV \tag{28}$$

Shuang-ren Zhao also derived the mutual energy flow theorem[13],

$$-(J_1, \xi_2)_{V_1} = (\xi_1, \xi_2) = (\xi_1, J_2)_{V_2} \tag{29}$$

where,

$$(\xi_1, \xi_2) = (\xi_1, \xi_2)_\Gamma = \oiint_{\Gamma} (\boldsymbol{E}_1 \times \boldsymbol{H}_2^* + \boldsymbol{E}_2^* \times \boldsymbol{H}_1) \cdot \hat{n} d\Gamma \tag{30}$$

It is clear that the integral on V_1 and V_2 Eq.(27,28) are also the inner product. The in product Eq.(30) at surface Γ is not clear, but shuang-ren Zhao has discovered that is a inner product. It is found that Γ does not need to be written, since it can be proved that Γ can be taken at arbitrary surface between the two volumes, V_1 and V_2. Hence, (ξ_1, ξ_2) can be seen as energy flow. This theorem is used by Shuang-ren as the Huygens principle [12, 31]. Since, if we let $J_2 = \delta(\boldsymbol{x} - \boldsymbol{x}')\hat{m}$

$$-(J_1, \xi_2)_{V_1} = (\xi_1, J_2)_{V_2} = \boldsymbol{E}_1 \cdot \hat{m} \tag{31}$$

can tell us the field ξ_1 at the direction $\hat{m}$. $\boldsymbol{E}_1$ is calculated at integral V_2. But the following formula can calculate ξ_1 at the place V_2 at a direction $\hat{m}$ from any surface Γ.

$$(\xi_1, \xi_2)_\Gamma = (\xi_1, J_2)_{V_2} = \boldsymbol{E}_1 \cdot \hat{m} \tag{32}$$

In the above formula $\boldsymbol{E}_1$ is calculated at V_2. This is just the Huygens principle, which tell us the wave can be calculated on the surface Γ instead on its sources. The inner product is also applied to the spherical wave expansions and the plane wave expansions[12, 30].

Since now this author has known the work of Welch and de Hoop, the definition of the inner product can be widen to time-domain,

$$(\xi_1, \xi_2)_\Gamma = \int_{-\infty}^{\infty} \oiint_\Gamma (E_1(t+\tau) \times H_2(t) + E_2(t) \times H_1(t+\tau)) \cdot \hat{n} d\Gamma dt \tag{33}$$

where τ can be taken as any value, it is often just take as 0, hence we have,

$$(\xi_1, \xi_2)_\Gamma = \int_{-\infty}^{\infty} \oiint_\Gamma (E_1(t) \times H_2(t) + E_2(t) \times H_1(t)) \cdot \hat{n} d\Gamma dt \tag{34}$$

In this article, Eq.(34) will be our definition of surface inner product. Since in this article the research is done mostly at the time-domain.

$$q = \oiint_\Gamma (E_1(t) \times H_2(t) + E_2(t) \times H_1(t)) \cdot \hat{n} d\Gamma \tag{35}$$

is the mutual energy flow flux at time t. Hence $(\xi_1, \xi_2)_\Gamma$ is the total energy go through the surface Γ.

$$\boldsymbol{S}_{12} = E_1(t) \times H_2(t) + E_2(t) \times H_1(t) \tag{36}$$

is the mixed Poynting vector or mutual energy Poynting vector. It is also the mutual energy flux intensity vector. It should be noticed that the word "mutual" can be taken a way, since the mutual energy flow is actually the energy flow which carries the energy from the emitter to the absorber. Hence the mutual energy flow is the microscopic energy flow. The the energy flow corresponding to the Poynting vector is an average energy flow corresponding there are infinite more absorbers and the absorbers uniformly distribute on a surface. In the following section we will further prove this. It should be notice that the Fourier transform of Eq.(33) is Eq.(30).

3 The mutual energy flow theorem

First we need to prove the mutual energy theorem[12] is really a energy theorem. To do this we should prove it from the Poynting theorem instead to prove it from Lorentz reciprocity theorem. In the sub-section 2.2 we have mentioned that the mutual energy theorem can be proved from Lorentz reciprocity theorem. It should be noticed even the mutual energy theorem can be proved by using conjugate transform from Lorentz reciprocity theorem, the mutual energy theorem is not a sub-theorem of the Lorentz reciprocity theorem, that is because the conjugate transform is not a mathematical transform like Fourier transform, conjugate transform is a physical transform. This can be seen by notice that the conjugate transform needs the Maxwell equations to prove it. Hence mutual energy theorem is still a unattached theorem.

3.1 Proving the above mentioned theorem is an energy theorem

Assume there are two charges, the superposition of the two fields of the two charges are $\xi_1 = [\boldsymbol{E}_1, \boldsymbol{H}_2]$, $\xi_1 = [\boldsymbol{E}_1, \boldsymbol{H}_2]$, can be written as $\xi = [\boldsymbol{E}, \boldsymbol{H}]$, where,

$$\boldsymbol{E} = \boldsymbol{E}_1 + \boldsymbol{E}_2 \tag{37}$$

$$\boldsymbol{H} = \boldsymbol{H}_1 + \boldsymbol{H}_2 \tag{38}$$

The Poynting theorem[11] can be written as

$$-\oiint_\Gamma \boldsymbol{E} \times \boldsymbol{H}) \cdot \hat{n} d\Gamma = \int_V \boldsymbol{E} \cdot \boldsymbol{J} dV + \int_V (\boldsymbol{E} \cdot \partial \boldsymbol{D} + \boldsymbol{H} \cdot \partial \boldsymbol{B}) dV \tag{39}$$

where $\partial \equiv \frac{\partial}{\partial t}$. This is referred as total energy formula. Similarly there is,

$$-\oiint_\Gamma \boldsymbol{E}_1 \times \boldsymbol{H}_1) \cdot \hat{n} d\Gamma = \int_V \boldsymbol{E}_1 \cdot \boldsymbol{J}_{\mathbf{1}} dV + \int_V (\boldsymbol{E}_1 \cdot \partial \boldsymbol{D}_1 + \boldsymbol{H}_1 \cdot \partial \boldsymbol{B}_{\mathbf{1}}) dV \tag{40}$$

$$-\oiint_\Gamma \boldsymbol{E}_2 \times \boldsymbol{H}_2) \cdot \hat{n} d\Gamma = \int_V \boldsymbol{E}_2 \cdot \boldsymbol{J}_2 dV + \int_V (\boldsymbol{E}_2 \cdot \partial \boldsymbol{D}_2 + \boldsymbol{H}_2 \cdot \partial \boldsymbol{B}_{\mathbf{2}}) dV \tag{41}$$

The above two formula is referred as self-energy formulas. Substitute Eq.(37,38) to Eq.(39) and then subtract Eq.(40) and Eq.(41) we obtains,

$$-\oiint_\Gamma (\boldsymbol{E}_1 \times \boldsymbol{H}_2 + \boldsymbol{E}_2 \times \boldsymbol{H}_1) \cdot \hat{n} d\Gamma$$

$$= \int_V (\boldsymbol{E}_1 \cdot \boldsymbol{J}_2 + \boldsymbol{E}_2 \cdot \boldsymbol{J}_1) dV$$

$$+\int_V (\boldsymbol{E}_1\cdot\partial\boldsymbol{D}_2+\boldsymbol{E}_2\cdot\partial\boldsymbol{D}_1+\boldsymbol{H}_1\cdot\partial\boldsymbol{B}_2+\boldsymbol{H}_2\cdot\partial\boldsymbol{B}_1)dV \tag{42}$$

We can call the above formula as the mutual energy formula. Considering to make a time integral $\int_{t=-\infty}^{\infty}$to the above formula, we have,

$$-\int_{t=-\infty}^{\infty}\oiint_{\Gamma}(\boldsymbol{E}_1\times\boldsymbol{H}_2+\boldsymbol{E}_2\times\boldsymbol{H}_1)\cdot\hat{n}d\Gamma dt$$

$$=\int_{t=-\infty}^{\infty}\int_V(\boldsymbol{E}_1\cdot\boldsymbol{J}_2+\boldsymbol{E}_2\cdot\boldsymbol{J}_1)dVdt$$

$$+\int_{t=-\infty}^{\infty}\int_V(\boldsymbol{E}_1\cdot\partial\boldsymbol{D}_2+\boldsymbol{E}_2\cdot\partial\boldsymbol{D}_1+\boldsymbol{H}_1\cdot\partial\boldsymbol{B}_2+\boldsymbol{H}_2\cdot\partial\boldsymbol{B}_1)dVdt \tag{43}$$

Considering,

$$\int_{t=-\infty}^{\infty}\int_V(\boldsymbol{E}_1\cdot\partial\boldsymbol{D}_2+\boldsymbol{E}_2\cdot\partial\boldsymbol{D}_1+\boldsymbol{H}_1\cdot\partial\boldsymbol{B}_2+\boldsymbol{H}_2\cdot\partial\boldsymbol{B}_1)dVdt$$

$$=\int_{t=-\infty}^{\infty}dU=[U(\infty)-U(-\infty)]$$

$$=0 \tag{44}$$

where

$$U=\int_V(\epsilon\boldsymbol{E}_1\cdot\boldsymbol{E}_2+\mu\boldsymbol{H}_1\cdot\boldsymbol{H}_2)\ dV \tag{45}$$

is the mutual energy intensity in the space. We have assumed that in the time $t=-\infty$, $U(-\infty)=0$, $t=\infty$, $U(\infty)=0$. Substitute Eq.(44) to Eq.(43) We obtain,

$$-\int_{t=-\infty}^{\infty}\oiint_{\Gamma}(\boldsymbol{E}_1\times\boldsymbol{H}_2+\boldsymbol{E}_2\times\boldsymbol{H}_1)\cdot\hat{n}d\Gamma dt=\int_{t=-\infty}^{\infty}\int_V(\boldsymbol{E}_1\cdot\boldsymbol{J}_2+\boldsymbol{E}_2\cdot\boldsymbol{J}_1)dVdt \tag{46}$$

This formula is called the mutual energy theorem with surface integral see Figure 2.

Assume ξ_1 is retarded wave. ξ_2 is advanced wave. Assume Γ is a infinite big sphere. And assume the current $\boldsymbol{J}_1$ and $\boldsymbol{J}_2$ inside the volume V. The volume V is inside of the surface Γ. Since the retarded wave reach the surface Γ at a

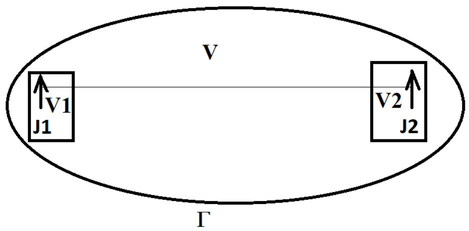

Figure 2: $J_1$is inside V_1. J_2 is inside V_2. V_1 and V_2 are inside V. The boundary surface of V is Γ.

future time. The advanced wave reach the the surface Γ at a past time, hence the two field ξ_1 and ξ_2 do not reach the surface Γ in the same time, that means they are not nonzero at the same time. Hence, there is,

$$\oiint_{\Gamma} (\boldsymbol{E}_1 \times \boldsymbol{H}_2 + \boldsymbol{E}_2 \times \boldsymbol{H}_1) \cdot \hat{n} d\Gamma = 0 \tag{47}$$

The proof of the above formula can be found in [28]. It should be notice if ξ_1 and ξ_2 are all retarded wave, in general, we cannot prove the above equation. It is same if ξ_1 and ξ_2 are all advanced waves. Hence, it is important that here the two waves, one is a retarded wave, the another must be an advanced wave.

Considering Eq.(47,44), Eq.(42) can be written as,

$$\int_{t=-\infty}^{\infty} \int_{V} (\boldsymbol{E}_1 \cdot \boldsymbol{J}_2 + \boldsymbol{E}_2 \cdot \boldsymbol{J}_1) dV dt = 0 \tag{48}$$

Considering that the current $\boldsymbol{J}_1$ is inside V_1 and the current $\boldsymbol{J}_2$ is inside V_2, $V_1 \subset V$ and $V_2 \subset V$. We have,

$$-\int_{t=-\infty}^{\infty} \int_{V_1} \boldsymbol{J}_1(t) \cdot \boldsymbol{E}_2(t) dV = \int_{t=-\infty}^{\infty} \int_{V_2} \boldsymbol{E}_1(t) \cdot \boldsymbol{J}_2(t) dV dt \tag{49}$$

The proof of this formula can be found in[28]. The above formula is the Welch's reciprocity theorem[28]. The proving process of this formula in the above is in principles nearly same as that of Welch. The only difference is that Welch started form Maxwell equations, this article started from the Poynting theorem. Our propose of proof is not to prove this formula satisfy Maxwell equations, but to prove it is a sub-theorem of the Poynting theorem and hence, it is a energy theorem. This proof can also be easily widened to the Hoop's reciprocity theorem, which is,

$$-\int_{t=-\infty}^{\infty} \int_{V_1} \boldsymbol{J}_1(t) \cdot \boldsymbol{E}_2(t+\tau) dV = \int_{t=-\infty}^{\infty} \int_{V_2} \boldsymbol{E}_1(t) \cdot \boldsymbol{J}_2(t+\tau) dV \tag{50}$$

after the Fourier transform it becomes,

$$-\int_{V_1} \boldsymbol{J}_1(\omega)\cdot\boldsymbol{E}_2(\omega)^* dV = \int_{V_2} \boldsymbol{E}_1(\omega)\cdot\boldsymbol{J}_2(\omega)^* dV \tag{51}$$

This is Rumsey's reciprocity theorem or Zhao's mutual energy theorem[12]. From this derivation, it is clear the above formulas Eq.(49,50,51) are energy theorems. Since the formula Eq.(42) is corresponding to the mutual energy part of the Poynting theorem, Shuang-ren Zhao call the formula Eq.(51) as the mutual energy theorem that is correct. It is worth to this name. The formula is a energy conservation formula. Here $\boldsymbol{J}_1$ is the current of a transmitting antenna. $\boldsymbol{E}_1(\omega)$ is the field of the transmitting antenna which is the retarded field. $\boldsymbol{J}_2$ is the current of a receiving antenna. $\boldsymbol{E}_2(\omega)$ is the filed of the receiving antenna which is advanced wave. The mutual energy theorem tell us that the energy sucked by the advanced wave $\boldsymbol{E}_2(\omega)$ from the current of the transmitting antenna $\boldsymbol{J}_1$ is equal to the energy of the retarded field $\boldsymbol{E}_1(\omega)$ applied to the current of the receiving antenna $\boldsymbol{J}_2$. The negative sign in the left of the above formula tell us that the left can offer some energy. Hence, $\boldsymbol{J}_1$ is electric power source. The right side has a positive sign that means it consumes energy and hence $\boldsymbol{J}_2$ is a electric load or sink. The reader perhaps has some confusion with the advanced waves. But think that if $\boldsymbol{E}_2(\omega)$ is also a retarded wave, then the two antenna all radiate retarded waves and they are all transmitting antenna. That is conflict the assumption that the current $\boldsymbol{J}_2$ is a current of a receiving antenna. That is also wrong! Hence $\boldsymbol{E}_2(\omega)$ cannot be retarded wave. About the advanced wave please see the reference of Welch[28] or the Wheeler and Feynman's absorber theory [1, 2].

By the way, the above theorem Welch, Rumsey and de Hoop call it as some reciprocity theorem that is also correct, it is true a reciprocity similar to the Lorentz reciprocity theorem. But it is also important to notice that it is not only a reciprocity theorem, but also a energy theorem.

3.2 The mutual energy flow theorem

In the Eq.(46) the surface can be chosen in any place. If it is chosen as Γ_1 which is the boundary surface of the volume V_1 the results become:

$$-\int_{t=-\infty}^{\infty} \oiint_{\Gamma_1} (\boldsymbol{E}_1\times\boldsymbol{H}_2+\boldsymbol{E}_2\times\boldsymbol{H}_1)\cdot\hat{n}\,d\Gamma dt = \int_{t=-\infty}^{\infty}\int_{V_1} \boldsymbol{J}_1\cdot\boldsymbol{E}_2 dV dt \tag{52}$$

See Figure 3. Here the surface norm vector $\hat{n}$ is direct from volume V_1 to V_2.

If the surface is chosen as Γ_2 which is the boundary surface of volume V_2.

$$-\int_{t=-\infty}^{\infty} \oiint_{\Gamma_2} (\boldsymbol{E}_1\times\boldsymbol{H}_2+\boldsymbol{E}_2\times\boldsymbol{H}_1)\cdot\hat{n}\,d\Gamma dt = \int_{t=-\infty}^{\infty}\int_{V_2} \boldsymbol{E}_1\cdot\boldsymbol{J}_2 dV dt \tag{53}$$

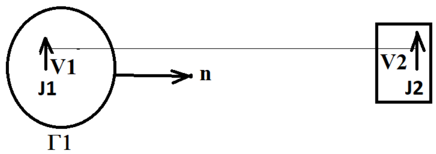

Figure 3: $J_1$is inside V_1. J_2 is inside V_2. V_1 and V_2 are inside V. The boundary surface of V_1 is Γ_1. The surface norm unit vector is $\hat{n}_1$.

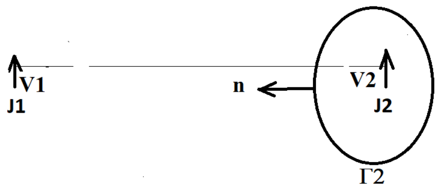

Figure 4: J_1 is inside V_1. J_2 is inside V_2. V is chosen as V_2. The boundary surface of V_2 is Γ_2. The surface norm unit vector is $\hat{n}_2$ is at the direction form V_2 to V_1.

Here the surface norm vector n is direct to the outside of V_2 which is directed from V_2 to V_1, see Figure 4.

We can adjusted the surface normal vector from going to outside to going inside. After correct the direction of the surface norm vector, a negative sign should be added, and hence we have

$$\int_{t=-\infty}^{\infty} \oiint_{\Gamma_2} (\boldsymbol{E}_1 \times \boldsymbol{H}_2 + \boldsymbol{E}_2 \times \boldsymbol{H}_1) \cdot \hat{n} d\Gamma dt = \int_{t=-\infty}^{\infty} \int_{V_2} \boldsymbol{E}_1 \cdot \boldsymbol{J}_2 dV dt \tag{54}$$

Substitute Eq.(52) and Eq.(54) to the mutual energy theorem Eq.(49), we obtain,

$$-\int_{t=-\infty}^{\infty} \int_{V_1} \boldsymbol{J}_1(t) \cdot \boldsymbol{E}_2(t) dV$$

$$= \int_{t=-\infty}^{\infty} \oiint_{\Gamma_1} (\boldsymbol{E}_1 \times \boldsymbol{H}_2 + \boldsymbol{E}_2 \times \boldsymbol{H}_1) \cdot \hat{n} d\Gamma dt$$

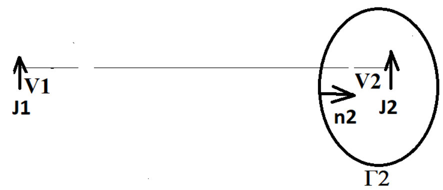

Figure 5: J_1 is inside V_1. J_2 is inside V_2. V is chosen as V_2. The surface norm unit vector is $\hat{n}_2$ is changed the direction. Now it is from V_1 to V_2.

$$= \int_{t=-\infty}^{\infty} \oiint_{\Gamma_2} (\boldsymbol{E}_1 \times \boldsymbol{H}_2 + \boldsymbol{E}_2 \times \boldsymbol{H}_1) \cdot \hat{n} d\Gamma dt$$

$$= \int_{t=-\infty}^{\infty} \int_{V_2} \boldsymbol{E}_1(t) \cdot \boldsymbol{J}_2(t) dV dt \tag{55}$$

This can be written as,

$$- \int_{t=-\infty}^{\infty} \int_{V_1} \boldsymbol{J}_1(t) \cdot \boldsymbol{E}_2(t) dV$$

$$= (\xi_1, \xi_2)_{\Gamma_1} = (\xi_1, \xi_2)_{\Gamma} = (\xi_1, \xi_2)_{\Gamma_2}$$

$$= \int_{t=-\infty}^{\infty} \int_{V_2} \boldsymbol{E}_1(t) \cdot \boldsymbol{J}_2(t) dV dt \tag{56}$$

This is referred as the mutual energy flow theorem[20]. See Figure 6. In the formula, there is,

$$Q = \oiint_{\Gamma} (\boldsymbol{E}_1 \times \boldsymbol{H}_2 + \boldsymbol{E}_2 \times \boldsymbol{H}_1) \cdot \hat{n} d\Gamma \tag{57}$$

as the mutual energy flow,

$$Energy_{\Gamma} = \int_{t=-\infty}^{\infty} Q dt \tag{58}$$

is the energy go though the the surface Γ. The mutual energy flow theorem tell us that for any surface Γ which is between volume V_1 and V_2, the mutual energy flow go through the surface Γ (integral with time) is a constant. The

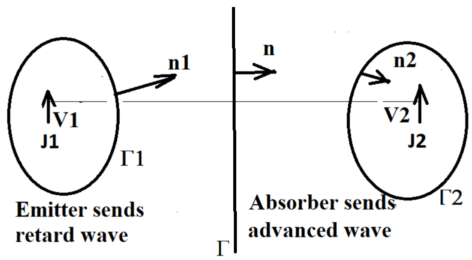

Figure 6: $J_1$is inside V_1. J_2 is inside V_2. V_1 and V_2 are inside V. The boundary surface of V_1 is Γ_1. The boundary surface of V_1 is Γ_1 The surface norm unit vector is $\hat{n}_1$. boundary surface of V_2 is Γ_2. The surface norm unit vector is $\hat{n}_2$. the surface Γ are at the middle of V_1 and V_2. All the surface norm vector $\hat{n}_1$, $\hat{n}_n$, $\hat{n}_2$ all are at the same direction.

Γ can be close surface for example a boundary of volume V_1 or a infinite open surface for example any infinite plane separated the volume V_1 and V_2. Here the field $\xi_1 = [\boldsymbol{E}_1, \boldsymbol{H}_1]$ is the retarded wave, $\xi_2 = [\boldsymbol{E}_2, \boldsymbol{H}_2]$ is the advanced wave. The requirement that one field is retarded and another is advanced is because of Eq.(47). Eq.(47) is established only when the two fields one is retarded field and another is advanced field.

In this article we work at very short time field or short time signal. If $\xi_1 = [\boldsymbol{E}_1, \boldsymbol{H}_1]$ and $\xi_2 = [\boldsymbol{E}_2, \boldsymbol{H}_2]$ are all retarded field, there is,

$$Energy_\Gamma = \int_{t=-\infty}^{\infty} Q dt = (\xi_1, \xi_2)_\Gamma = 0 \tag{59}$$

This is because when the wave ξ_1 reached to the V_2, the current $\boldsymbol{J}_2$ will send a retarded wave ξ_2 to the surface Γ. But when ξ_2 reached Γ, ξ_1 has passed Γ long time ago, hence $\xi_1 = 0$ on the surface. ξ_1 and ξ_2 can not be synchronized, hence in general there is above formula.

The mutual energy flow theorem can be summarized as following:

(I) If the surface Γ separated the two volume V_1 and V_2, If J_1 and J_2 are not all retarded wave or all advanced wave there is

$$(\xi_1, \xi_2)_\Gamma = constant \tag{60}$$

Where Γ is arbitrary surface between V_1 and V_2. See Figure 7

(II) If the ξ_1, ξ_2 are all retarded waves or all advanced waves, we have,

$$(\xi_1, \xi_2)_\Gamma = 0 \tag{61}$$

See figure 8.

(III) If ξ_1 and ξ_2 one is retarded wave and one is advanced wave, there current $\boldsymbol{J}_1$ and $\boldsymbol{J}_2$ are inside the volume V. There is

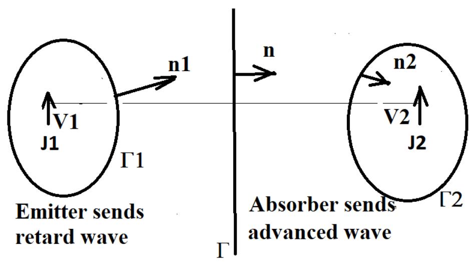

Figure 7: J_1 is inside V_1. J_2 is inside V_2. V_1 and V_2 are inside V. The boundary surface of V_1 is Γ_1. The boundary surface of V_2 is Γ_2 For the surface Γ_1, the surface norm unit vector is $\hat{n}_1$. For the surface Γ_2 the norm unit vector is $\hat{n}_2$, we can see the norm unit vector $\hat{n}_1$, $\hat{n}$ and $\hat{n}_2$ are all at the direction form I to F.

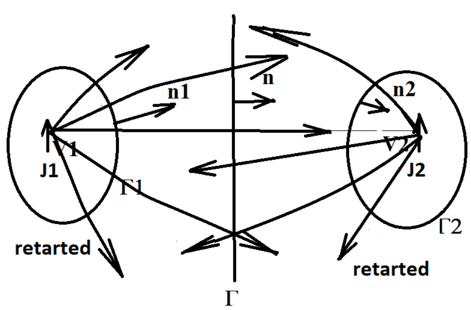

Figure 8: J_1 is inside V_1. J_2 is inside V_2. Γ is a surface between V_1 and V_2. The two wave are all retarded wave. The inner product is 0 on the surface Γ.

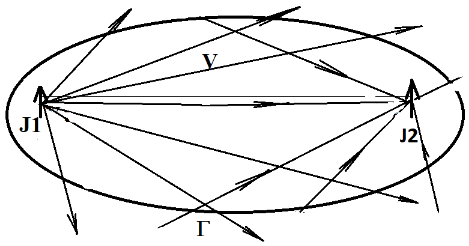

Figure 9: J_1 is inside V_1. J_2 is inside V_2. V_1 and V_2 are inside V. Γ is a boundary surface of V. One of the wave is retarded wave, the other is advanced wave. The inner product is 0 on the surface.

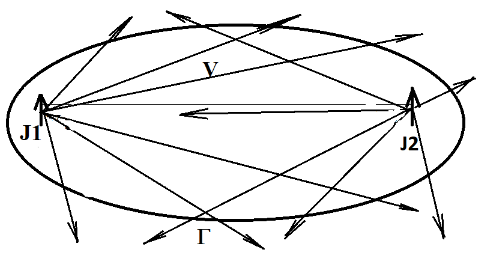

Figure 10: J_1 is inside V_1. J_2 is inside V_2. V_1 and V_2 are inside V. Γ is a boundary surface of V. The two waves are all retarded waves. The inner product is constant.

$$(\xi_1, \xi_2)_{\Gamma_s} = 0 \tag{62}$$

Here Γ_s is infinite big sphere surface. Γ_s is also any surface surrounds the volume of V. See Figure 9

(IV) If ξ_1 and ξ_2 are all retarded wave, we have,

$$(\xi_1, \xi_2)_{\Gamma_s} = constant \tag{63}$$

Here the surface Γ_s is any surface surrounds the volume V, assume the two current sources J_1 and J_2 are all inside the volume V. (IV) is also effective if the two field ξ_1 and ξ_2 are all advanced fields. See Figure 10

(V) We know that a retarded wave with it's source at the center of a infinite big sphere can be seen as an advanced wave if its source at uniformly distributed infinite big sphere. Hence the above (VI) can be also seen as if the field ξ_1 is retarded field, its source is inside V and ξ_2 is an advanced wave and its sink is uniformly distributed on the Γ_s, Here Γ_s is at infinite big sphere. Γ is any surface between V and Γ_s. If ξ_1 is the retarded wave sent from $\boldsymbol{J}_1$. ξ_2 is the advanced wave sent from $\boldsymbol{J}_2$. $\boldsymbol{J}_2$ is uniformly distribute on Γ_S, we have,

$$(\xi_1, \xi_2)_{\Gamma} = constant \tag{64}$$

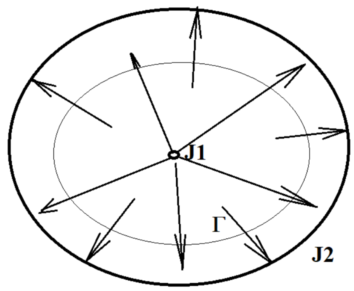

Figure 11: J_1 is inside V_1. J_2 is at outside of infinite big sphere Γ_s. J_2 is produce by uniformly distributed absorber. The field ξ_1 is retarded wave, the field ξ_2 is advanced wave. The inner product is a constant on any surface Γ.

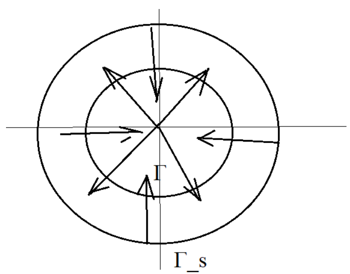

Figure 12: J_1 is inside V_1. J_2 is at outside of infinite big sphere Γ_s. J_2 is uniformly distributed emitters on Γ_s. The field ξ_1 is retarded wave, the field ξ_2 is also retarded wave. The inner product is a 0 on any surface Γ. Γ is inside Γ_s.

See Figure 11.

(VI). Same as (V), but if $\boldsymbol{J}_1$ is emitter and ξ_1 is the retarded wave, J_2 is also emitter and sent the retarded wave we have ξ_2, there will be

$$(\xi_1, \xi_2)_\Gamma = 0 \tag{65}$$

See Figure 12.

3.3 Example

Assume J_1 is the current of a transmitting antenna which is inside the volume V_1 and J_2 is the current of a receiving antenna which is inside of volume V_2. The field $\xi_1 = [\boldsymbol{E}_1, \boldsymbol{H}_1]$ is produced by J_1 and it is a retarded wave. The field $\xi_2 = [\boldsymbol{E}_2, \boldsymbol{H}_2]$ is produced by J_2 and it is an advanced wave wave. (I) tell us there is energy current flow from V_1 to V_2, the energy flow to any surface Γ is all the same. Γ is the arbitrary surface between V_1 and V_2. (III) tell us there is no any energy go outside of our universe. This give a double guarantee for condition (I). The mutual energy theorem Eq(49), Eq(50) or Eq(51) offers the

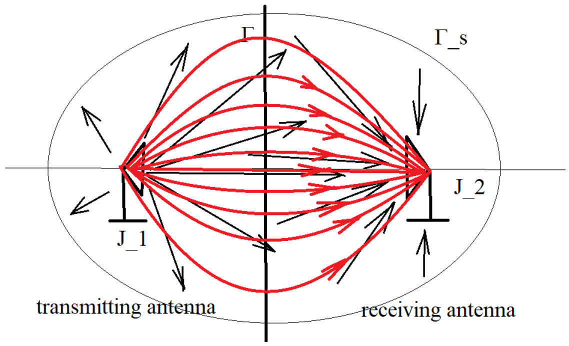

Figure 13: J_1 is inside V_1 which is a transmitting antenna and sends the retarded wave out. J_2 is inside V_2, which is a receiving antenna and sends advanced wave out. The mutual energy current go to outside of the surface Γ_s is 0. The mutual energy current go through Γ is a constant, Γ can be in any place between V_1 and V_2. The red arrowhead is the mutual energy flow which is sent out from V_1 and has been received in V_2

energy send by the transmitting antenna just equal the energy received by the receiving antenna. See Figure 13. The red arrowhead is the mutual energy flow. The mutual energy flow cannot go outside of Γ_s. The mutual energy flow are constant at any surface of Γ. The surface Γ is at any place between the transmitting antenna to the receiving antenna. Through the mutual energy flow, the energy send by the transmitting antenna is received by the receiving antenna.

It should be noticed that for the energy radiate from the transmitting antenna, it is possible to be received by other receiving antenna or background environment. It is important its energy is radiate only by the form of the mutual energy. We will prove that the self-energy do not transfer energy, this prove cannot be done inside the Maxwell's theory. It need to introduce the self-energy principle which will be done in the following sections.

4 The path integral based on the mutual energy flow

Feynman has mentioned Huygens principle in his article about path integral [8]. But he did not offer any formula based on Huygens principle. And the Huygens principle is also not combined with mutual energy flow and mutual energy flow theory[20]. Hence, Feynman did not offer a very clear picture that the path integral related to Huygens principle. In this section I will discusses this in details.

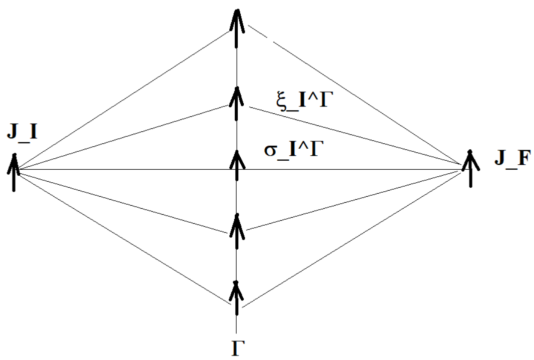

Figure 14: Huygens virtual source, which can replace the field, i.e., $\xi_I^\Gamma \Longleftrightarrow \sigma_I^\Gamma$

4.1 Huygens virtual sources

Assume $\tau_I = [\boldsymbol{J}_I, \boldsymbol{K}_I]$, $\boldsymbol{J}_I$ is a source current which sends the retarded wave to radiate out the energy. $\boldsymbol{K}_I$ is the magnetic current source. Normally we have $\boldsymbol{K}_I = 0$. Assume $\tau_F = [\boldsymbol{J}_F, \boldsymbol{K}_F]$, is a sink which sends the advanced wave to receive the energy. $\boldsymbol{J}_F$ is the electric current of the sink. $\boldsymbol{K}_F$ is the magnetic current of the sink. Normally we have $\boldsymbol{K}_F = 0$. We define the following inner product,

$$(\xi_F^I, \tau_I)_I = \int_{t=-\infty}^{\infty} \int_{V_I} (\boldsymbol{E}_F^I(t) \cdot \boldsymbol{J}_I(t) + \boldsymbol{H}_F^I(t) \cdot \boldsymbol{K}_I(t)) dV dt \tag{66}$$

$$(\tau_F, \xi_I^F)_F = \int_{t=-\infty}^{\infty} \int_{V_F} (\boldsymbol{J}_F(t) \cdot \boldsymbol{E}_I^F(t) + \boldsymbol{K}_F(t) \cdot \boldsymbol{H}_I^F(t)) dV dt \tag{67}$$

Γ can be any surface between the source and sink. I will choice Γ as a infinite big plane. The normal vector $\hat{n}$ is at the direction from the source point to the sink. In the above formula, for the field, for example ξ_I^F, the subscript I is for the source or sink position, the superscript F is the field position. And hence, ξ_I^F is the field at the position F and sends from the source I. Similarly $\boldsymbol{H}_I^\Gamma(t)$ is the magnetic field at the position Γ send by the source I. According the mutual energy theorem we have,

$$(\tau_F, \boldsymbol{\xi}_I^F)_F = -(\xi_F^I, \tau_I)_I \tag{68}$$

Where $\boldsymbol{\xi}_I^F$ is the field at the position of F and produced by the emitter τ_I . ξ_F^I is the advanced field at the position I and produced by the absorber τ_F. The mutual energy flow theorem can be written as,

$$(\tau_F, \boldsymbol{\xi}_I^F)_I = (\xi_F^\Gamma, \xi_I^\Gamma)_\Gamma = -(\xi_F^I, \tau_I)_I \tag{69}$$

where

$$\tau_I = [\boldsymbol{J}_I(t), \boldsymbol{K}_I(t)] \tag{70}$$

$$\tau_F = [\boldsymbol{J}_F(t), \boldsymbol{K}_F(t)] \tag{71}$$

Here $\boldsymbol{K}_F(t) = 0$ and $\boldsymbol{K}_I(t) = 0$. Where ξ_F^Γ is the field at Γ and produced by the absorber F. ξ_I^Γ is the field at Γ and produced by the emitter at I. $(\xi_F^\Gamma, \xi_I^\Gamma)_\Gamma$ is the mutual energy flow, Which is defined as,

$$(\xi_F^\Gamma, \xi_I^\Gamma)_\Gamma = \int_{t=-\infty}^{\infty} \iint_\Gamma (\boldsymbol{E}_I^\Gamma(t) \times \boldsymbol{H}_F^\Gamma(t) + \boldsymbol{E}_F^\Gamma(t) \times \boldsymbol{H}_I^\Gamma(t)) \cdot \hat{n} d\Gamma dt \tag{72}$$

Where Γ is a surface between I and F. We can assume Γ is a infinite big plane. $\hat{n}$ is the surface unit normal vector which is at the direction from I to F.

$$\begin{aligned}(\boldsymbol{E}_F^\Gamma(t) \times \boldsymbol{H}_I^\Gamma(t)) \cdot \hat{n} &= \boldsymbol{E}_F^\Gamma(t) \cdot (\boldsymbol{H}_I^\Gamma(t) \times \hat{n}) \\ &= \boldsymbol{E}_F^\Gamma(t) \cdot (-\hat{n} \times \boldsymbol{H}_I^\Gamma(t))\end{aligned} \tag{73}$$

and

$$\begin{aligned}(\boldsymbol{E}_I^\Gamma(t) \times \boldsymbol{H}_F^\Gamma(t)) \cdot \hat{n} &= \hat{n} \cdot (\boldsymbol{E}_I^\Gamma(t) \times \boldsymbol{H}_F^\Gamma(t)) \\ &= (\hat{n} \times \boldsymbol{E}_I^\Gamma(t)) \cdot \boldsymbol{H}_F^\Gamma(t)\end{aligned} \tag{74}$$

We can define Huygens virtual current source as,

$$\boldsymbol{J}_I^\Gamma(t) = -\hat{n} \times \boldsymbol{H}_I^\Gamma(t) \tag{75}$$

$$\boldsymbol{K}_I^\Gamma(t) = \hat{n} \times \boldsymbol{E}_I^\Gamma(t) \tag{76}$$

Please notice, in the electromagnetic field theory text book, the Huygens virtual current source usually is defined as,

$$\boldsymbol{J}_I^\Gamma(t) = \hat{n}_{outside} \times \boldsymbol{H}_I^\Gamma(t) \tag{77}$$

$$\boldsymbol{K}_I^\Gamma(t) = -\hat{n}_{outside} \times \boldsymbol{E}_I^\Gamma(t) \tag{78}$$

In that situation, the $\hat{n}_{outside}$ is a surface normal unit vector in outside direction. $\hat{n}_{outside}$ is at the direction from Γ to I. In our situation, our $\hat{n}$ is at the direction from Γ to F. Hence, we need a negative sign, so that the formula Eq.(75,76) is correct. We can write Huygens virtual current source as σ_I^Γ,

$$\sigma_I^\Gamma = [\boldsymbol{J}_I^\Gamma(t), \boldsymbol{K}_I^\Gamma(t)] \tag{79}$$

Eq.(72) can be written as,

$$(\xi_F^\Gamma, \xi_I^\Gamma)_\Gamma = \int\limits_{t=-\infty}^{\infty} \iint\limits_{\Gamma} (\boldsymbol{E}_F^\Gamma(t) \cdot \boldsymbol{J}_I^\Gamma(t) + \boldsymbol{H}_F^\Gamma(t) \cdot \boldsymbol{K}_I^\Gamma(t)) d\Gamma = (\xi_F^\Gamma, \sigma_I^\Gamma)_\Gamma \tag{80}$$

$$(\xi_F^\Gamma, \xi_I^\Gamma)_\Gamma = \int\limits_{t=-\infty}^{\infty} \iint\limits_{\Gamma} (\boldsymbol{J}_F^\Gamma(t) \cdot \boldsymbol{E}_I^\Gamma(t) + \boldsymbol{K}_F^\Gamma(t) \cdot \boldsymbol{H}_I^\Gamma(t)) d\Gamma = (\sigma_F^\Gamma, \xi_I^\Gamma)_\Gamma \tag{81}$$

From this formula, we know that the field, ξ_I^Γ is equivalent to the Huygens source, see Figure 14.

$$\xi_I^\Gamma \Longleftrightarrow \sigma_I^\Gamma \tag{82}$$

Certainly, this equivalent is made on different form of the inner product,

$$(\xi_F^\Gamma, \xi_I^\Gamma)_\Gamma = \int\limits_{t=-\infty}^{\infty} \iint\limits_{\Gamma} (\boldsymbol{E}_F^\Gamma(t) \times \boldsymbol{H}_I^\Gamma(t) + \boldsymbol{E}_I^\Gamma(t) \times \boldsymbol{H}_F^\Gamma(t)) \cdot \hat{n} d\Gamma \tag{83}$$

$$(\xi_F^\Gamma, \sigma_I^\Gamma)_\Gamma = \int\limits_{t=-\infty}^{\infty} \iint\limits_{\Gamma} (\boldsymbol{E}_F^\Gamma(t) \cdot \boldsymbol{J}_I^\Gamma(t) + \boldsymbol{H}_F^\Gamma(t) \cdot \boldsymbol{K}_I^\Gamma(t)) d\Gamma \tag{84}$$

but the above inner products are exactly equal. The above can be written as,

$$(\xi_F^\Gamma, \sigma_I^\Gamma)_\Gamma = \int\limits_{t=-\infty}^{\infty} dt \iint\limits_{\Gamma} d\Gamma \sum_{k=1}^{6} \xi_F^{\Gamma k}(t) \sigma_I^{\Gamma k}(t) \tag{85}$$

σ_I^Γ is corresponding to ξ_I^Γ which is the field at Γ produced by the source τ_I. If we only consider one component of the source τ_I for example the j component we have,

$$(\xi_{Fi}^\Gamma, \sigma_{Ij}^\Gamma)_\Gamma = \int\limits_{t=-\infty}^{\infty} dt \iint\limits_{\Gamma} d\Gamma \sum_{k=1}^{6} \xi_{Fi}^{\Gamma k}(t) \sigma_{Ij}^{\Gamma k}(t)) \tag{86}$$

Where ξ_{Fi}^Γ is the i component of the field produced by sink τ_F at F. We can define,

$$\sum = \sum_{\Gamma k} = \int\limits_{t=-\infty}^{\infty} dt \iint\limits_{\Gamma} d\Gamma \sum_{k=1}^{6} \tag{87}$$

We have,

$$(\xi_{Fi}^\Gamma, \tau_{Ij}^\Gamma)_\Gamma = \sum_{\Gamma k} \xi_{Fi}^{\Gamma k}(t) \sigma_{Ii}^{\Gamma k}(t) \tag{88}$$

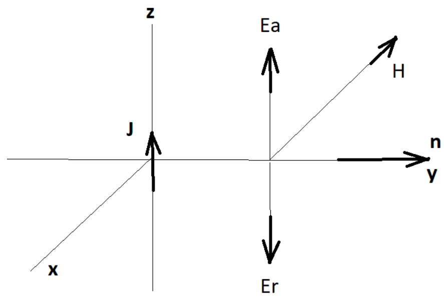

Figure 15: The fields produced by a current source. The magnetic field produced by the current can be decided by the right hand law, and hence only in one direction showed as $\boldsymbol{H}$. There two possibility, for the electric field. It can direct up or direct to down. The the up direction electric field is the retarded field $\boldsymbol{E}_r$ and the down direction electric field is the advanced field $\boldsymbol{E}_a$.

4.2 The difference between the normal current and Huygens virtual sources

Consider a current $\boldsymbol{J}$ it has the ability to produce two kind of fields, the retarded field and the advanced field. The current $\boldsymbol{J}$ can produce a magnetic field which can be decided by the right hand law, the magnetic field is shown in Figure 15. However if we known the wave propagate along the direction $\hat{\boldsymbol{n}}$, for the electric field there are two possibilities. The electric field can be direct to up or down. These two possibilities are corresponding to the two kind of fields, the retarded wave and the advanced wave. The electric field $\boldsymbol{E}$ at down direction, it is corresponding to the retarded electric field $\boldsymbol{E}_r$. This is because the electric field $\boldsymbol{E}_r$ is at opposite direction of the current $\boldsymbol{J}$. The current $\boldsymbol{J}$ will offer some energy and hence, its field will be retarded field.

If the electric field $\boldsymbol{E}_a$ at up direction, $\boldsymbol{E}_a$ is corresponding to the advanced field. This is because the electric field has the same direction with the current $\boldsymbol{J}$. That means the field consume the energy, hence its field should be advanced wave.

Hence, the fields of the current element radiate in all directions. there are two different electric fields, one is the retarded wave, another is the advanced wave.

The field of Huygens virtual source is different.

The mutual energy flow theorem can be written as,

$$(\tau_F, \boldsymbol{\xi}_I^F)_F = (\xi_F^\Gamma, \xi_I^\Gamma)_\Gamma = -(\xi_F^I, \tau_I)_I \tag{89}$$

where $\tau_I = [\boldsymbol{J}_I, 0]$ is the real source. $\tau_F = [\boldsymbol{J}_F, 0]$ is the real sink. Assume σ_I,σ_F are Huygens virtual sources at the place I and F, we have,

$$(\sigma_F, \boldsymbol{\xi}_I^F)_F = (\xi_F^\Gamma, \xi_I^\Gamma)_\Gamma = (\xi_F^I, \sigma_I)_I \tag{90}$$

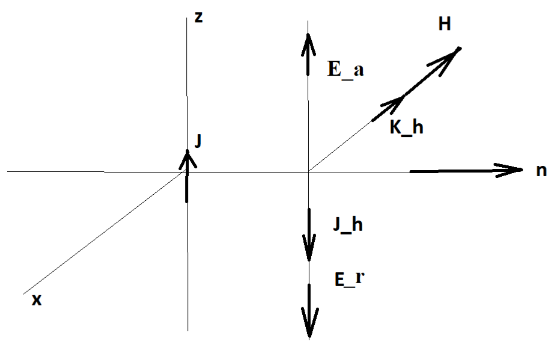

Figure 16: The fields produced by a current source. In the place close to the current source, there are a infinite plane Γ. The field produced by the source $\boldsymbol{J}_I$ can be described by ξ_I^Γ. This field can also be described as Huygens virtual sources $\sigma_I^\Gamma = [\boldsymbol{J}_I^\Gamma, \boldsymbol{K}_I^\Gamma]$.

If we only consider one component, then we have,

$$(\sigma_{Fi}, \boldsymbol{\xi}_I^F)_F = (\xi_{Fi}^\Gamma, \xi_{Ij}^\Gamma)_\Gamma = (\xi_{Fi}^\Gamma, \sigma_{Ij})_I \tag{91}$$

If we use Huygens source and sink to replace the real source and sink, the minus sign in the mutual energy flow theorem disappear, that is the advantage. The formula looks more symmetrical. Huygens source cannot produce two kind of fields like the real current does. If original field is retarded field, the corresponding Huygens source can only produce retarded field. If the original field is advanced field, the corresponding Huygens source can only produce advanced field. This is because Huygens source always have two kind of currents, electric current and magnetic current and the both are not zero.

The above formula looks more simple. In the above the mutual energy flow can be written as,

$$(\xi_{Fi}^\Gamma, \xi_{Ij}^\Gamma)_\Gamma = (\xi_{Fi}^\Gamma, \sigma_{Ij}^\Gamma)_\Gamma = (\sigma_{Fi}^\Gamma, \xi_{Ij}^\Gamma)_\Gamma \tag{92}$$

The above formula, considering

$$(\xi_{Fi}^\Gamma, \sigma_{Ij}^\Gamma)_\Gamma = \sum_{\Gamma k} \xi_{Fi}^{\Gamma k}(t) \sigma_{Ii}^{\Gamma k}(t) \tag{93}$$

we often written as,

$$(\xi_{Fi}^\Gamma, \xi_{Ij}^\Gamma)_\Gamma = (\xi_{Fi}^{\Gamma k}, \xi_{Ij}^{\Gamma k})_\Gamma = (\xi_{Fi}^{\Gamma k}, \sigma_{Ij}^{\Gamma k})_\Gamma = (\sigma_{Fi}^{\Gamma k}, \xi_{Ij}^{\Gamma k})_\Gamma \tag{94}$$

For Huygens source please see Figure 16.

4.3 δ function expansion

We know that, $f(x)$ can be expanded as δ function,

$$f(x) = \int\limits_{-\infty}^{\infty} c(x')\delta(x'-x)dx' \tag{95}$$

where $c(x')$ is unknown coefficient, which can be obtained by

$$(f,\delta) \equiv \int\limits_{-\infty}^{\infty} f(x")\delta(x"-x')dx" \tag{96}$$

$$= \int\limits_{x"=-\infty}^{\infty} \int\limits_{x=-\infty}^{\infty} c(x)\delta(x-x")dx\delta(x"-x')dx"$$

$$= \int\limits_{x=-\infty}^{\infty} c(x) \int\limits_{x"=-\infty}^{\infty} \delta(x-x")\delta(x"-x')dx"dx$$

$$= \int\limits_{x=-\infty}^{\infty} c(x)\delta(x'-x)dx$$

$$= c(x') \tag{97}$$

or

$$c(x') = (f,\delta) \tag{98}$$

Hence we have,hence we always have,

$$f(x) = \int\limits_{-\infty}^{\infty} (f,\delta)\delta(x'-x)dx \tag{99}$$

According to this, we need unit Huygens virtual source in Γ,

$$\delta_{\Gamma'k'}^{\Gamma k} = \delta(t-t')\delta(\Gamma-\Gamma')\delta_{k'}^{k} \tag{100}$$

Here $i = 1, 2 \cdots 6$, corresponding to the 6 component of field. It is similar to j and k,

$$\xi_{F1}^{\Gamma} = [E_{F1}^{\Gamma x}, E_{F1}^{\Gamma y}, E_{F1}^{\Gamma z}, H_{F1}^{\Gamma x}, H_{F1}^{\Gamma y}, H_{F1}^{\Gamma z}] \tag{101}$$

We can expend $\sigma_{Ij}^{\Gamma k}$ with $\delta_{\Gamma' k'}^{\Gamma k}$, and hence we have,

$$\sigma_{Ij}^{\Gamma k} = \sum_{\Gamma' k'} (\delta_{\Gamma" k"}^{\Gamma' k'}, \sigma_{Ij}^{\Gamma" k"})\delta_{\Gamma' k'}^{\Gamma k} \tag{102}$$

where$\sum_{\Gamma' k'} = \int_{t=-\infty}^{\infty} dt \iint_{\Gamma} d\Gamma \sum_{k=1}^{6}$.

4.4 Path integral derivation

Now, let us see the following the field $\boldsymbol{\xi}_{Ij}^{Fi}$ is the field at the position F send by the position I. Assume in the place I there is only the source with j component. In the place F we only care the component of i.

$$\boldsymbol{\xi}_{Ij}^{Fi}=(\delta_F,\boldsymbol{\xi}_{Ij}^{Fi})_F \tag{103}$$

In the above, the inner product definition at the position F and the definition of δ_F has been applied.

$$(\delta_F,\boldsymbol{\xi}_{Ij}^{Fi})_F=(\xi_{Fi}^{\Gamma k},\xi_{Ij}^{\Gamma k})_\Gamma \tag{104}$$

In the above formula, the mutual energy flow theorem Eq.(91) has been applied. Further we have,

$$(\xi_{Fi}^{\Gamma k},\xi_{Ij}^{\Gamma k})_\Gamma=(\xi_{Fi}^{\Gamma k},\sigma_{Ij}^{\Gamma k})_\Gamma \tag{105}$$

We have done $\xi_{Ij}^{\Gamma k}\Longleftrightarrow\sigma_{Ij}^{\Gamma k}$, that means the Huygens principle has been applied. The field can be replaced with its Huygens virtual source.

$$(\xi_{Fi}^{\Gamma k},\sigma_{Ij}^{\Gamma k})_\Gamma=(\xi_{Fi}^{\Gamma k},\sum_{\Gamma'k'}(\delta_{\Gamma"k"}^{\Gamma'k'},\sigma_{Ij}^{\Gamma"k"})\delta_{\Gamma'k'}^{\Gamma k})_\Gamma \tag{106}$$

We have substituted the Eq.(102). Further more we have,

$$(\xi_{Fi}^{\Gamma k},\sum_{\Gamma'k'}(\delta_{\Gamma"k"}^{\Gamma'k'},\sigma_{Ij}^{\Gamma"k"})\delta_{\Gamma'k'}^{\Gamma k})_\Gamma=\sum_{\Gamma'k'}(\xi_{Fi}^{\Gamma k},\delta_{\Gamma'k'}^{\Gamma k})_\Gamma(\delta_{\Gamma"k"}^{\Gamma'k'},\sigma_{Ij}^{\Gamma"k"})_\Gamma \tag{107}$$

Assume the think δ_{Fi} produced the advanced field which is $\xi_{Fi}^{\Gamma k}$, The Huygens source$\delta_{\Gamma'k'}^{\Gamma k}$ produce the retarded field is $\xi_{\Gamma'k'}^{Fi}$. Considering the mutual energy theorem, that is,

$$(\xi_{Fi}^{\Gamma k},\delta_{\Gamma'k'}^{\Gamma k})_\Gamma=(\delta_{Fi},\xi_{\Gamma'k'}^{Fi})_F \tag{108}$$

Considering,

$$\xi_{\Gamma'k'}^{Fi}=G_{\Gamma'k'}^{Fi}\delta_{\Gamma'k'} \tag{109}$$

where $G_{\Gamma'k'}^{Fi}$ is the coefficient of field at Fi produced by the source $\Gamma'k'$, hence we have,

$$(\delta_{Fi},\xi_{\Gamma'k'}^{Fi})_F=(\delta_{Fi},G_{\Gamma k}^{Fi}\delta_{\Gamma'k'}^{\Gamma k})_F \tag{110}$$

Eq(107) can be written as,

$$\sum_{\Gamma'k'}(\xi_{Fi}^{\Gamma k},\delta_{\Gamma'k'}^{\Gamma k})_\Gamma(\delta_{\Gamma"k"}^{\Gamma'k'},\sigma_{Ij}^{\Gamma"k"})_\Gamma=\sum_{\Gamma'k'}(\delta_{Fi},G_{\Gamma k}^{Fi}\delta_{\Gamma'k'}^{\Gamma k})_F(\delta_{\Gamma"k"}^{\Gamma'k'},\sigma_{Ij}^{\Gamma"k"})_\Gamma \tag{111}$$

Considering,

$$\sigma_{Ij}^{\Gamma"k"}=G_{Ij}^{\Gamma"k"}\delta_{Ij} \tag{112}$$

Hence we have,

$$\sum_{\Gamma' k'} (\delta_{Fi}, G^{Fi}_{\Gamma k} \delta^{\Gamma k}_{\Gamma' k'})_F (\delta^{\Gamma' k'}_{\Gamma" k"}, \sigma^{\Gamma" k"}_{Ij})_\Gamma = \sum_{\Gamma' k'} (\delta_{Fi}, G^{Fi}_{\Gamma k} \delta^{\Gamma k}_{\Gamma' k'})_F (\delta^{\Gamma' k'}_{\Gamma" k"}, G^{\Gamma" k"}_{Ij} \delta_{Ij})_\Gamma \tag{113}$$

Considered Eq(103, 104, 105, 106, 107,111, 113) we have,

$$\boldsymbol{\xi}^{Fi}_{Ij} = (\delta_{Fi}, \boldsymbol{\xi}^{Fi}_{Ij}) = \sum_{\Gamma' k'} (\delta_{Fi}, G^{Fi}_{\Gamma k} \delta^{\Gamma k}_{\Gamma' k'})_F (\delta^{\Gamma' k'}_{\Gamma" k"}, G^{\Gamma" k"}_{Ij} \delta_{Ij})_\Gamma \tag{114}$$

This can be written as,

$$\left\langle \delta_{Fi} \middle| \boldsymbol{\xi}^{Fi}_{Ij} \right\rangle = \left\langle \delta_{Fi} \middle| G^{Fi}_{Ij} \middle| \delta_{Ij} \right\rangle = \int\limits_{t=-\infty}^{\infty} dt \iint\limits_{\Gamma} d\Gamma \sum_{k=1}^{6} \left\langle \delta_{Fi} \middle| G^{Fi}_{\Gamma k} \middle| \delta_{\Gamma k} \right\rangle_F \left\langle \delta_{\Gamma k} \middle| G^{\Gamma k}_{Ij} \middle| \delta_{Ij} \right\rangle_\Gamma \tag{115}$$

or

$$\left\langle q_{Fi} \middle| G^{Fi}_{Ij} \middle| q_{Ij} \right\rangle = \int\limits_{t=-\infty}^{\infty} dt \iint\limits_{\Gamma} d\Gamma \sum_{k=1}^{6} \left\langle q_{Fi} \middle| G^{Fi}_{\Gamma k} \middle| q_{\Gamma k} \right\rangle_F \left\langle q_{\Gamma k} \middle| G^{\Gamma k}_{Ij} \middle| q_{Ij} \right\rangle_\Gamma \tag{116}$$

if we do not care the index j and i

$$\left\langle q_F \middle| G^F_I \middle| q_I \right\rangle = \int\limits_{t=-\infty}^{\infty} dt \iint\limits_{\Gamma} d\Gamma \left\langle q_F \middle| G^F_\Gamma \middle| q_\Gamma \right\rangle_F \left\langle q_\Gamma \middle| G^\Gamma_I \middle| q_I \right\rangle_\Gamma \tag{117}$$

or

$$\left\langle q_F \middle| G^F_I \middle| q_I \right\rangle = \sum \left\langle q_F \middle| G^F_\Gamma \middle| q_\Gamma \right\rangle_F \left\langle q_\Gamma \middle| G^\Gamma_I \middle| q_I \right\rangle_\Gamma \tag{118}$$

It should be notice that int the formula $\left\langle q_F \middle| G^F_\Gamma \middle| q_\Gamma \right\rangle_F$, the subscript F means the inner product is take at the place F. The above formula can be seen as insert

$$\sum \left| q_\Gamma \right\rangle_F \left\langle q_\Gamma \right|_\Gamma \equiv 1 \tag{119}$$

to $\left\langle q_F \middle| G^F_I \middle| q_I \right\rangle$ hence we have,

$$\begin{aligned} \left\langle q_F \middle| G^F_I \middle| q_I \right\rangle_F &= \left\langle q_F \middle| G^F_\Gamma G^\Gamma_I \middle| q_I \right\rangle_F \\ &= \left\langle q_F \middle| G^F_\Gamma \sum \middle| q_\Gamma \right\rangle_F \left\langle q_\Gamma \middle| G^\Gamma_I \middle| q_I \right\rangle_\Gamma \\ &= \sum \left\langle q_F \middle| G^F_\Gamma \middle| q_\Gamma \right\rangle_F \left\langle q_\Gamma \middle| G^\Gamma_I \middle| q_I \right\rangle_\Gamma \end{aligned} \tag{120}$$

Consider in the above formula,

$$q_\Gamma \equiv \delta^{\Gamma' k'}_{\Gamma k} \equiv \delta(t' - t)\delta(\Gamma - \Gamma')\delta^k_{k'} \tag{121}$$

$$\left\langle q_\Gamma \middle| G_I^\Gamma \middle| q_I \right\rangle \equiv \left\langle \delta_{\Gamma k}^{\Gamma' k'} \middle| G_{Ij}^{\Gamma' k'} \middle| \delta_{Ij} \right\rangle \tag{122}$$

It should be notice that Eq.(119) is not same as

$$\sum \left| q \right\rangle \left\langle q \right| \equiv 1 \tag{123}$$

For Eq.(123) the bra $\left\langle q \right|$ and kit $\left| q \right\rangle$ are in the same region which is the 3D space. For Eq.(119) the bra and kit do not at the same region. One is at F and the other is at Γ. The reader perhaps noticed we have spend much more inc just to make this small difference from Eq.(123) to Eq.(119) work.

We can see in the proof of Dirac the "proof" is so simple, but in my proof, it is so complicated. However, it should be point out, in my proof,

1. I have proved that the electric fields can be written as a inner product (ξ_1, ξ_2),

2. I have proved the mutual energy theorems that is used in the proof of path integral.

3. I have proved the mutual energy flow theorems that is used in the proof of path integral.

4. I have used Huygens source and sink.

5.The advanced waves are involved, since the mutual energy flow is consist of the retarded wave and the advanced wave.

6. I have defined the integral for the variable q is at the surface instead of 3D volume. Dirac use 3D volume, I think that is a wrong concept.

7. Only mutual energy flow is involved, there is nothing related to the probability flow.

I have only proved one step in the path integral the rest should be a same as Dirac's proof.

It should should be notice that the Dirac's "proof" actually need also the above concepts, He didn't offer the corrected of the proof of his path integral, but he has guessed the correct results. It should be mention, without any of the above 7 things, the path integral cannot work. I have only showed the path integral in the case of electromagnetic fields, however for other particles, for example electron, it can also satisfies the extended Maxwell equations[................]. Normally people said that the electron satisfy the Dirac equation, but there are references that electron also satisfy some kind of the extended Maxwell equations. Hence, if we have correct path integral for electromagnetic field or photon, it can also be extended to electron and other particles. In a following section I will prove the mutual energy flow theorem corresponding the Schrödinger equation and Dirac equation.

4.5 Replace the path integral with Energy pipe streamline integral

In the above we have worked with very important step of path integral. After this step, The Eq.(120) can be extended as,

$$\left\langle q_F \middle| G_I^F \middle| q_I \right\rangle_F = \left\langle q_F \middle| G_{\Gamma_{N-1}}^F G_{\Gamma_{N-2}}^{\Gamma_{N-1}} G_{\Gamma_{N-3}}^{\Gamma_{N-2}} \cdots G_{\Gamma_1}^{\Gamma_2} G_I^{\Gamma_1} \middle| q_I \right\rangle_F \tag{124}$$

Hence, we have,

$$\left\langle q_F \middle| G_I^F \middle| q_I \right\rangle_F = \sum_{\Gamma_{N-1}} \sum_{\Gamma_{N-2}} \cdots \sum_{\Gamma_1} \left\langle q_F \middle| G_{\Gamma_{N-1}}^F \middle| q_{\Gamma_{N-1}} \right\rangle_F \left\langle q_{\Gamma_{N-1}} \middle| G_{\Gamma_{N-2}}^{\Gamma_{N-1}} \middle| q_{\Gamma_{N-2}} \right\rangle_{\Gamma_{N-2}}$$

$$\left\langle q_{\Gamma_{N-2}} \middle| G_{\Gamma_{N-3}}^{\Gamma_{N-2}} \cdots G_{\Gamma_1}^{\Gamma_2} q_{\Gamma_1} \right\rangle_{\Gamma_1} \left\langle q_{\Gamma_1} \middle| G_I^{\Gamma_1} \middle| q_I \right\rangle_F \tag{125}$$

Since we have energy flow theorem, the energy flow can be seen as many pipes in the space, each pipe can be seen as streamline. Energy is go through the streamline. The summation, $\sum_{\Gamma_{N-1}} \sum_{\Gamma_{N-2}} \cdots \sum_{\Gamma_1}$ can be simplified to only one summation, since the energy flow goes through only the energy pipe streamline. Hence the path integral can be calculated on the streamline.

$$\sum_{\Gamma_{N-1}} \sum_{\Gamma_{N-2}} \cdots \sum_{\Gamma_1} \Longleftrightarrow \sum_{\Gamma} \tag{126}$$

$$\left\langle q_F \middle| G_I^F \middle| q_I \right\rangle_F$$

$$= \sum_{\Gamma} \left\langle q_F \middle| G_{\Gamma_{N-1}}^F \middle| q_{\Gamma_{N-1}} \right\rangle_F \left\langle q_{\Gamma_{N-1}} \middle| G_{\Gamma_{N-2}}^{\Gamma_{N-1}} \middle| q_{\Gamma_{N-2}} \right\rangle_{\Gamma_{N-2}} \left\langle q_{\Gamma_{N-2}} \middle| G_{\Gamma_{N-3}\cdots}^{\Gamma_{N-2}} \right.$$

$$\left. \cdots G_{\Gamma_1}^{\Gamma_2} q_{\Gamma_1} \right\rangle_{\Gamma_1} \left\langle q_{\Gamma_1} \middle| G_I^{\Gamma_1} \middle| q_I \right\rangle_F \tag{127}$$

The above integral is referred as energy pipe streamline integral. The energy is transferred on the streamline.

$$\left\langle q_F \middle| G_I^F \middle| q_I \right\rangle_F$$

$$= \int_{t=-\infty}^{\infty} dt \iint_{\Gamma} d\Gamma \sum_{k=1}^{6} \left\langle q_F \middle| G_{\Gamma_{N-1}}^F \middle| q_{\Gamma_{N-1}} \right\rangle_F \left\langle q_{\Gamma_{N-1}} \middle| G_{\Gamma_{N-2}}^{\Gamma_{N-1}} \middle| q_{\Gamma_{N-2}} \right\rangle_{\Gamma_{N-2}} \left\langle q_{\Gamma_{N-2}} \middle| G_{\Gamma_{N-3}\cdots}^{\Gamma_{N-2}} \right.$$

$$\left. \cdots G_{\Gamma_1}^{\Gamma_2} q_{\Gamma_1} \right\rangle_{\Gamma_1} \left\langle q_{\Gamma_1} \middle| G_I^{\Gamma_1} \middle| q_I \right\rangle_{\Gamma_1} \tag{128}$$

This means there doesn't need a infinite integral in the path integral. The energy stream line can guarantee the energy flow goes through inside a stream line.

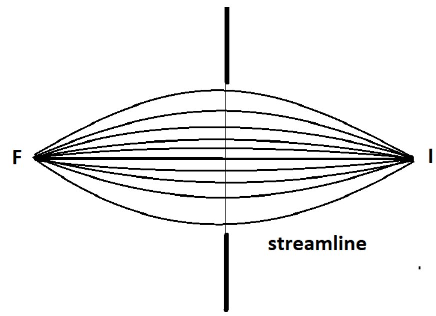

Figure 17: This figure shows there is streamline between the emitter I and the absorber F.

5 From mutual energy pipe streamline to energy pipe streamline

In the above, we have proved that the path integral can be simplified as energy pipe streamline integral. The deference of path integral and the energy pipe streamline integral is that the path integral includes a infinite 3D integral but the streamline integral is a normal 2D surface integral. That vastly simplified the definition that make this technology easy to be implemented as numerical calculations.

But there is one thing still not solved when I speak about the energy pipe streamline that actually that means the mutual energy pipe streamline. This is because all the derivation is based on the mutual energy theorem and mutual energy flow theorem. This two theorems speak all about the mutual energy. The mutual energy is only part of energy, there is also the self-energy. What is the role the self-energy play in the energy transfer?

In this section I will prove the self-energy items has no any contribution to the energy transfer. The energy is transferred only by the mutual energy flow. Hence mutual energy and mutual energy flow can be rectified the name as energy and energy flow. Last section we have introduced the streamline integral, which is based on the mutual energy flow theorem. Only when the mutual energy flow theorem is the energy flow theorem, the definition of energy pipe streamline is meaningful.

In this section we begin to prove the mutual energy flow theorem is energy flow theorem. This proof cannot put inside the Maxwell's theory. It need a totally new theory. I will introduce the mutual energy principle and the self-energy principle.

5.1 The conflict of the classical electromagnetic field theory

We all know that the electromagnetic field theory can be started from a system with N charges. For this system, we have,

I. Maxwell equations. Maxwell equations can be seen as axioms. From Maxwell equation we can obtained the Poynting theorem which can be written as,

$$-\oiint_{\Gamma}(\boldsymbol{E}\times\boldsymbol{H})\cdot\hat{n}d\Gamma=\int_{V}(\boldsymbol{J}\cdot\boldsymbol{E})dV+\int_{V}(\boldsymbol{E}\cdot\frac{\partial\boldsymbol{D}}{\partial t}+\boldsymbol{H}\cdot\frac{\partial\boldsymbol{B}}{\partial t})dV \tag{129}$$

We also know that electromagnetic field also satisfy,

II. The superposition principle. Assume there are N charges, the i-th charge has the fields $\xi_i=[\boldsymbol{E}_i,\boldsymbol{H}_i]$. The superposition principle tell us the total field can be written as,

$$\xi=\sum_{i=1}^{N}\xi_i \tag{130}$$

Normally people often said that Maxwell equations are linear and hence that means, the superposition principle is included inside the Maxwell equations. However, here the Maxwell equations is restrict to only one charge. We use superposition principle to describe the field of many charges. Hence here we can separate the superposition principle to the Maxwell equations. Hence, superposition principle principle can be used as a independent principle.

III. energy conservation. We also know the i-th charge received energy from j-the charge is,

$$\int_{t==\infty}^{\infty}\int_{V}(\boldsymbol{E}_j(t)\cdot\boldsymbol{J}_i(t))dVdt \tag{131}$$

where $J_i(t)$ are current of i-th charge. Assume in the empty space there are only the N charges. It is clear that if i-th charge offer j-th charge some energy, i-th charge will loss some energy hence the energy of the whole system is,

$$\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}\int_{t==\infty}^{\infty}\int_{V}(\boldsymbol{E}_j(t)\cdot\boldsymbol{J}_i(t))dVdt=0 \tag{132}$$

The above formula is the energy conservation of the system with N charges. The above 3 conditions I., II. and III. are self-explanatory. The condition III is an additional condition. It is added to normal electromagnetic field system. Normally a system only has the above first two conditions which is enough to find a solution.

Please notice that in the above formula the summation $\sum_{j=1,j\neq i}^{N}$ has been applied, which means we have assumed that a charge's field cannot offer a force to its self. This is according to the Newton's law. There are a few researcher believe this law can be broken for a charge with radiation. However this author think newton's law still should be insistent here.

Now let us substitute Eq.(130) to Eq.(129), we have,

$$-\sum_{i=1}^{N}\sum_{j=1}^{N}\oiint_{\Gamma}(\boldsymbol{E}_i\times\boldsymbol{H}_j)\cdot\hat{n}d\Gamma$$

$$=\sum_{i=1}^{N}\sum_{j=1}^{N}\int_{V}(\boldsymbol{J}_i\cdot\boldsymbol{E}_j)dV+\sum_{i=1}^{N}\sum_{j=1}^{N}\int_{V}(\boldsymbol{E}_i\cdot\frac{\partial\boldsymbol{D}_j}{\partial t}+\boldsymbol{H}_i\cdot\frac{\partial\boldsymbol{B}_j}{\partial t}))_{\Gamma_s}dV \quad (133)$$

It is clear if we need to prove Eq.(132) from Eq.(133), we need to prove the following 3 conditions,

$$-\sum_{i=1}^{N}\oiint_{\Gamma}(\boldsymbol{E}_i\times\boldsymbol{H}_i)\cdot\hat{n}d\Gamma=\sum_{i=1}^{N}\int_{V}(\boldsymbol{J}_i\cdot\boldsymbol{E}_i)dV+\sum_{i=1}^{N}\int_{V}(\boldsymbol{E}_i\cdot\frac{\partial\boldsymbol{D}_i}{\partial t}+\boldsymbol{H}_i\cdot\frac{\partial\boldsymbol{B}_i}{\partial t})dV=0 \quad (134)$$

This is referred as self-energy formula. The above equation tell us all self energy items should be 0. Substitute the above self-energy formula to the total energy formula Eq.(133) we have,

$$-\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}\oiint_{\Gamma}(\boldsymbol{E}_i\times\boldsymbol{H}_j)\cdot\hat{n}d\Gamma$$

$$=\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}\int_{V}(\boldsymbol{J}_i\cdot\boldsymbol{E}_j)dV+\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}\int_{V}(\boldsymbol{E}_i\cdot\frac{\partial\boldsymbol{D}_j}{\partial t}+\boldsymbol{H}_i\cdot\frac{\partial\boldsymbol{B}_j}{\partial t})dV \quad (135)$$

This is the mutual energy formula, in the above mutual energy formula, if we can prove that,

$$\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}\oiint_{\Gamma}(\boldsymbol{E}_i\times\boldsymbol{H}_j)\cdot\hat{n}d\Gamma=0 \quad (136)$$

and

$$\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}\int_{t==\infty}^{\infty}\int_{V}(\boldsymbol{E}_i\cdot\frac{\partial\boldsymbol{D}_j}{\partial t}+\boldsymbol{H}_i\cdot\frac{\partial\boldsymbol{B}_j}{\partial t})dVdt=0 \quad (137)$$

we have

$$\int_{t==\infty}^{\infty}\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}\int_{V}(\boldsymbol{J}_i\cdot\boldsymbol{E}_j)dVdt=0 \quad (138)$$

This is Eq.(132), i.e., the energy conservation formula. In the following section we will prove Eq.(136, 137). We cannot prove Eq.(133) in the frame of Maxwell's theory. Since the energy conservation should be reserved any way, hence we assume that

$$\oiint_{\Gamma} (\boldsymbol{E}_i \times \boldsymbol{H}_i) \cdot \hat{n} = 0 \tag{139}$$

$$\int_{V} (\boldsymbol{J}_i \cdot \boldsymbol{E}_i) dV = 0 \tag{140}$$

$$\int_{V} (\boldsymbol{E}_i \cdot \frac{\partial \boldsymbol{D}_j}{\partial t} + \boldsymbol{H}_i \cdot \frac{\partial \boldsymbol{B}_j}{\partial t}) dV = 0 \tag{141}$$

This is referred as self-conditions which means all self-energy items are 0. I do not claim the above is correct, but please just accept is for the time being.

5.2 In case there is only two charges

The 3 formula Eq.(139,140 and 141) are referred as self-energy conditions. The self-conditon tell us all self-energy items are 0. We do not claim the above 3 formulas are all correct. But in the time being, we accept that. This 3 formula will guarantee the mutual energy formula Eq(134) succeeds. This further leads the Eq.(135) succeeds. Eq.(135) can be rewritten as,

$$-\sum_{i=1}^{N}\sum_{j=1}^{j<i} \oiint_{\Gamma} (\boldsymbol{E}_i \times \boldsymbol{H}_j + \boldsymbol{E}_j \times \boldsymbol{H}_i) \cdot \hat{n} d\Gamma$$

$$= \sum_{i=1}^{N}\sum_{j=1}^{j<i} \int_{V} (\boldsymbol{J}_i \cdot \boldsymbol{E}_j + \boldsymbol{J}_j \cdot \boldsymbol{E}_i) dV$$

$$+ \sum_{i=1}^{N}\sum_{j=1}^{j<i} \int_{V} (\boldsymbol{E}_i \cdot \frac{\partial \boldsymbol{D}_j}{\partial t} + \boldsymbol{E}_j \cdot \frac{\partial \boldsymbol{D}_i}{\partial t} + \boldsymbol{H}_i \cdot \frac{\partial \boldsymbol{B}_j}{\partial t} + \boldsymbol{H}_j \cdot \frac{\partial \boldsymbol{B}_i}{\partial t}) dV \tag{142}$$

Assume $N = 2$, the above formula can be rewritten as,

$$-\sum_{i=1}^{2}\sum_{j=1}^{j<i} \oiint_{\Gamma} (\boldsymbol{E}_i \times \boldsymbol{H}_j + \boldsymbol{E}_j \times \boldsymbol{H}_i) \cdot \hat{n} d\Gamma$$

$$= \sum_{i=1}^{2}\sum_{j=1}^{j<i} \int_{V} (\boldsymbol{J}_i \cdot \boldsymbol{E}_j + \boldsymbol{J}_j \cdot \boldsymbol{E}_i) dV$$

$$+ \sum_{i=1}^{2}\sum_{j=1}^{j<i} \int_{V} (\boldsymbol{E}_i \cdot \frac{\partial \boldsymbol{D}_j}{\partial t} + \boldsymbol{E}_j \cdot \frac{\partial \boldsymbol{D}_i}{\partial t} + \boldsymbol{H}_i \cdot \frac{\partial \boldsymbol{B}_j}{\partial t} + \boldsymbol{H}_j \cdot \frac{\partial \boldsymbol{B}_i}{\partial t}) dV \tag{143}$$

or

$$-\oiint_{\Gamma}(\boldsymbol{E}_1\times\boldsymbol{H}_2+\boldsymbol{E}_2\times\boldsymbol{H}_1)\cdot\hat{n}d\Gamma$$
$$=\int_V(\boldsymbol{J}_1\cdot\boldsymbol{E}_2+\boldsymbol{J}_2\cdot\boldsymbol{E}_1)dV$$
$$+\int_V(\boldsymbol{E}_1\cdot\frac{\partial\boldsymbol{D}_2}{\partial t}+\boldsymbol{E}_2\cdot\frac{\partial\boldsymbol{D}_1}{\partial t}+\boldsymbol{H}_1\cdot\frac{\partial\boldsymbol{B}_2}{\partial t}+\boldsymbol{H}_2\cdot\frac{\partial\boldsymbol{B}_1}{\partial t})dV \tag{144}$$

This can be rewritten as differential formula,

$$-\nabla\cdot(\boldsymbol{E}_1\times\boldsymbol{H}_2+\boldsymbol{E}_2\times\boldsymbol{H}_1)$$
$$=\boldsymbol{J}_1\cdot\boldsymbol{E}_2+\boldsymbol{J}_2\cdot\boldsymbol{E}_1$$
$$+\boldsymbol{E}_1\cdot\frac{\partial\boldsymbol{D}_2}{\partial t}+\boldsymbol{E}_2\cdot\frac{\partial\boldsymbol{D}_1}{\partial t}+\boldsymbol{H}_1\cdot\frac{\partial\boldsymbol{B}_2}{\partial t}+\boldsymbol{H}_2\cdot\frac{\partial\boldsymbol{B}_1}{\partial t} \tag{145}$$

Considering the following mathematical formulas,

$$\nabla\cdot(\boldsymbol{E}_1\times\boldsymbol{H}_2)=\nabla\times\boldsymbol{E}_1\cdot\boldsymbol{H}_2-\nabla\times\boldsymbol{H}_2\cdot\boldsymbol{E}_1 \tag{146}$$

$$\nabla\cdot(\boldsymbol{E}_2\times\boldsymbol{H}_1)=\nabla\times\boldsymbol{E}_2\cdot\boldsymbol{H}_1-\nabla\times\boldsymbol{H}_1\cdot\boldsymbol{E}_2 \tag{147}$$

We have,

$$-(\nabla\times\boldsymbol{E}_1\cdot\boldsymbol{H}_2-\nabla\times\boldsymbol{H}_2\cdot\boldsymbol{E}_1+\nabla\times\boldsymbol{E}_2\cdot\boldsymbol{H}_1-\nabla\times\boldsymbol{H}_1\cdot\boldsymbol{E}_2)$$
$$=\boldsymbol{J}_1\cdot\boldsymbol{E}_2+\boldsymbol{J}_2\cdot\boldsymbol{E}_1$$
$$+\boldsymbol{E}_1\cdot\frac{\partial\boldsymbol{D}_2}{\partial t}+\boldsymbol{E}_2\cdot\frac{\partial\boldsymbol{D}_1}{\partial t}+\boldsymbol{H}_1\cdot\frac{\partial\boldsymbol{B}_2}{\partial t}+\boldsymbol{H}_2\cdot\frac{\partial\boldsymbol{B}_1}{\partial t} \tag{148}$$

or

$$-(\nabla\times\boldsymbol{E}_1+\frac{\partial\boldsymbol{B}_1}{\partial t})\cdot\boldsymbol{H}_2+(\nabla\times\boldsymbol{H}_2-\boldsymbol{J}_2-\frac{\partial\boldsymbol{D}_2}{\partial t})\cdot\boldsymbol{E}_1$$
$$-(\nabla\times\boldsymbol{E}_2+\frac{\partial\boldsymbol{B}_2}{\partial t})\cdot\boldsymbol{H}_1+(\nabla\times\boldsymbol{H}_1-\boldsymbol{J}_1-\frac{\partial\boldsymbol{D}_1}{\partial t})\cdot\boldsymbol{E}_2=0 \tag{149}$$

It is clear, if

$$\begin{cases}\nabla\times\boldsymbol{E}_1+\frac{\partial\boldsymbol{B}_1}{\partial t}=0\\ \nabla\times\boldsymbol{H}_1-\boldsymbol{J}_1-\frac{\partial\boldsymbol{D}_1}{\partial t}=0\end{cases} \tag{150}$$

$$\begin{cases}\nabla\times\boldsymbol{E}_2+\frac{\partial\boldsymbol{B}_2}{\partial t}=0\\ \nabla\times\boldsymbol{H}_2-\boldsymbol{J}_2-\frac{\partial\boldsymbol{D}_2}{\partial t}=0\end{cases} \tag{151}$$

This are two group of Maxwell equations. If two group Maxwell equations are satisfied, the Eq.(149) can be satisfied.

In other hand, if we assume,

$$\begin{cases} \boldsymbol{E}_2 \equiv 0 \\ \boldsymbol{H}_2 \equiv 0 \end{cases} \tag{152}$$

We obtain,

$$\begin{cases} \nabla \times \boldsymbol{H}_1 - \boldsymbol{J}_1 - \frac{\partial \boldsymbol{D}_1}{\partial t} < \infty \\ \nabla \times \boldsymbol{E}_1 + \frac{\partial \boldsymbol{B}_1}{\partial t} < \infty \end{cases} \tag{153}$$

We have,

$$\begin{cases} \boldsymbol{E}_1 = anything < \infty \\ \boldsymbol{H}_1 = anyting < \infty \end{cases} \tag{154}$$

This is not an acceptable solution.

Similarly if

$$\begin{cases} \boldsymbol{E}_2 \equiv 0 \\ \boldsymbol{H}_2 \equiv 0 \end{cases} \tag{155}$$

we have,

$$\begin{cases} \boldsymbol{E}_2 = anything < \infty \\ \boldsymbol{H}_2 = anyting < \infty \end{cases} \tag{156}$$

This is also not an acceptable solution. Hence, for the equation Eq.(148), the solution are two Maxwell equations, which mast be satisfied in the same time. That means the two fields $\xi_1 = [\boldsymbol{E}_1, \boldsymbol{H}_1]$ must synchronized with $\xi_2 = [\boldsymbol{E}_2, \boldsymbol{H}_2]$.

Now lets to prove the Eq.(136). When $N = 2$, it become,

$$\oiint\limits_{\Gamma} (\boldsymbol{E}_1 \times \boldsymbol{H}_2 + \boldsymbol{E}_2 \times \boldsymbol{H}_1) \cdot \hat{n} d\Gamma = 0 \tag{157}$$

We can chose Γ as big sphere surface with infinite radius. If the two fields are all retarded fields or the two fields are all advanced field, the above formula, is not zero in general. If the two fields are one is retarded wave, another is advanced wave, since two wave reach the sphere surface one is the future, one is in the past. Hence the two field cannot reach the sphere surface in the same time. This will guarantee the Eq.(157) succeeds. Hence the two field $\xi_1$and ξ_2 must one is retarded field and another is the advanced field. We also know the two field must synchronized. The proof can be widened to the situation where N is not 2. Hence Eq.(136) is established.

Now let us to prove Eq.(137), if $N = 2$, it can be written as,

$$\int\limits_{t==\infty}^{\infty} \int\limits_{V} (\boldsymbol{E}_1 \cdot \partial \boldsymbol{D}_2 + \boldsymbol{E}_2 \cdot \partial \boldsymbol{D}_1 + \boldsymbol{H}_1 \cdot \partial \boldsymbol{B}_2 + \boldsymbol{H}_2 \cdot \partial \boldsymbol{B}_1) dV dt$$

$$
\begin{aligned}
&= \int_{t=-\infty}^{\infty} dU \\
&= U|_{t=-\infty}^{\infty} \\
&= U(\infty) - U(-\infty) \\
&= 0
\end{aligned} \tag{158}
$$

Where

$$U = \int_V (\boldsymbol{E}_1 \cdot \boldsymbol{D}_2 + \boldsymbol{H}_1 \cdot \boldsymbol{B}_2) dV \tag{159}$$

is the mutual energy in the space, is can be choose so that $U(\infty)$=$U(-\infty)$. This proof can be easily widened to the general situation where N is not equal 2. Hence Eq.(137) is established. The proof of Eq.(157, 158) is first been done on the Welch's reciprocity theorem [28]. Welch first mentioned that only the retarded wave and the advanced wave can made the integral vanish on the infinite big sphere.

Substitute the Eq.(157, 158) to the mutual energy formula Eq.(144) we obtain,

$$\int_{t=-\infty}^{\infty} \int_V (\boldsymbol{J}_1 \cdot \boldsymbol{E}_2 + \boldsymbol{J}_2 \cdot \boldsymbol{E}_1) dV = 0 \tag{160}$$

Assume $\boldsymbol{J}_1$ is inside V_1. $\boldsymbol{J}_2$ is only inside V_2, and there is $V_1 \subset V$ and $V_2 \subset V$, we have,

$$-\int_{t=-\infty}^{\infty} \int_{V_1} \boldsymbol{J}_1(t) \cdot \boldsymbol{E}_2(t) dV = \int_{t=-\infty}^{\infty} \int_{V_2} \boldsymbol{J}_2(t) \cdot \boldsymbol{E}_1(t) dV \tag{161}$$

This is Welch's reciprocity theorem. The formula can also be generalized to obtain the de Hoop's reciprocity theorem,

$$-\int_{t=-\infty}^{\infty} \int_{V_1} \boldsymbol{J}_1(t) \cdot \boldsymbol{E}_2(t+\tau) dV = \int_{t=-\infty}^{\infty} \int_{V_2} \boldsymbol{J}_2(t+\tau) \cdot \boldsymbol{E}_1(t) dV \tag{162}$$

de Hoop's reciprocity theorem is also referred as cross correlation reciprocity theorem. According the Fourier transform of the correlation function, the above theorem can be written as,

$$-\int_{V_1} \boldsymbol{J}_1(\omega) \cdot \boldsymbol{E}_2(\omega) dV = \int_{V_2} \boldsymbol{J}_2(\omega) \cdot \boldsymbol{E}_1(\omega) dV \tag{163}$$

This is the reciprocity theorem of Rumsey's reciprocity theorem and also Zhao's mutual energy theorem. It should be notice that, in my derivation of the

Welch's reciprocity theorem, I started from Poynting theorem, Welch started from Maxwell equations. Welch would like to prove his reciprocity theorem is correct, so it can be derived from Maxwell equations. I started from Poynting theorem, this way to show this theorem is really a energy theorem. In the time I derived the mutual energy theorem, I plan to prove it from Poynting theorem, but I didn't realized this. After around 30 years past, in the second time I work on this problem I realized it[13]. When there is the concept mutual energy, I begin to ask the energy flow. Usually energy flow is corresponding to the Poynting vector and Poynting theorem. Hence I call this new energy flow mutual energy flow. When I have the concept mutual energy flow, I get the mutual energy flow theorem Eq.(56).

5.3 There is the conflict between the Maxwell theory and energy conservation

Eq.(158, 159, 160) can be generalized to Eq.(136, 137, 138). From the derivation of last sub-section, we know that the 2 waves must be synchronized, the two wave one must be retarded wave and another is advanced wave. This results is also correct to the situation where have more charges instead of only two charges.

We also know the wave should still satisfy the Maxwell equations. There are additional requirement for the electromagnetic fields, which means that for the pair waves, one must be retarded and another must be an advanced field. And the two must be synchronized. The concept of the synchronization for the retarded wave and the advanced wave is similar to the transactional process in the transactional interpretation of quantum mechanics of John Cramer [5, 6]. John Cramer guess that the retarded wave and advanced wave have a handshake or transaction. The above is a proof of that.

The problem we have derived that the wave still satisfy Maxwell equations Eq.(150,151). If the Maxwell equations satisfies, the Poynting theorem must also be satisfied and hence, cannot have the self-energy condition Eq.(140,141,142). Hence cannot have Eq.(134). But without Eq.(134), we cannot obtained the energy conservation formula Eq.(132). However energy conservation should be reserved first. What is wrong? I assume the wrong side is at Maxwell equations. This conflict cannot solved inside the theory of Maxwell.

5.4 Self-energy principle

In order to solve the conflict, I assume the electromagnetic field is not only with the retarded field and the advanced field. There is two time-reverse fields which satisfy the time-reversal Maxwell equations, which can cancel all the energy of the retarded wave and the advanced wave. The time-reversal electromagnetic fields satisfies the time-reversal Maxwell equations. The time-reversal transformation $\mathbb{R}$ can be written as following,

$$\mathbb{R}t = -t = \tau \tag{164}$$

where t is time. $\tau = -t$, is the new time after the time reversal transform. Hence we have,

$$\mathbb{R}(x(t)) = x(-t) = x(\tau) \tag{165}$$

similarly we have,

$$\mathbb{R}[\boldsymbol{E}(t), \boldsymbol{H}(t), \boldsymbol{D}(t), \boldsymbol{B}(t), \mu(t), \epsilon(t)] = [\boldsymbol{E}(-t), \boldsymbol{H}(-t), \boldsymbol{D}(-t), \boldsymbol{B}(-t), \mu(-t), \epsilon(-t)]$$
$$= [\boldsymbol{E}(\tau), \boldsymbol{H}(\tau), \boldsymbol{D}(\tau), \boldsymbol{B}(\tau), \mu(\tau), \epsilon(\tau)] \tag{166}$$

$$\mathbb{R}\boldsymbol{v} = \mathbb{R}(\frac{dx(t)}{dt}) = \frac{dx(-t)}{dt} = -\frac{dx(-t)}{d(-t)} = -\frac{dx(\tau)}{d(\tau)} = -\boldsymbol{v} \tag{167}$$

Hence the speed $\boldsymbol{v}$ will change the sign after the time reversal transform, this also leads the current change the sign:

$$\mathbb{R}\boldsymbol{J}(t) = \mathbb{R}(q\boldsymbol{v}) = -q\boldsymbol{v} = -\boldsymbol{J}(\tau) \tag{168}$$

similarly, we have,

$$\mathbb{R}\boldsymbol{K}(t) = -\boldsymbol{K}(\tau) \tag{169}$$

$$\mathbb{R}\frac{\partial \boldsymbol{E}(t)}{\partial t} = \frac{\partial \boldsymbol{E}(-t)}{\partial t} = -\frac{\partial \boldsymbol{E}(-t)}{\partial(-t)} = -\frac{\partial \boldsymbol{E}(\tau)}{\partial(\tau)} \tag{170}$$

or

$$\mathbb{R}[\frac{\partial}{\partial t}\boldsymbol{E}(t), \frac{\partial}{\partial t}\boldsymbol{H}(t), \frac{\partial}{\partial t}\boldsymbol{D}(t), \frac{\partial}{\partial t}\boldsymbol{B}(t)]$$
$$= [-\frac{\partial}{\partial \tau}\boldsymbol{E}(\tau), -\frac{\partial}{\partial \tau}\boldsymbol{H}(\tau), -\frac{\partial}{\partial \tau}\boldsymbol{D}(\tau), -\frac{\partial}{\partial \tau}\boldsymbol{B}(\tau)] \tag{171}$$

Assume

$$\zeta = [\boldsymbol{E}(t), \boldsymbol{H}(t), \boldsymbol{D}(t), \boldsymbol{B}(t), \epsilon(t), \mu(t),$$
$$t, \boldsymbol{J}(t), \boldsymbol{K}(t), \frac{\partial}{\partial t}\boldsymbol{E}(t), \frac{\partial}{\partial t}\boldsymbol{H}(t), \frac{\partial}{\partial t}\boldsymbol{D}(t), \frac{\partial}{\partial t}\boldsymbol{B}(t)] \tag{172}$$

then we have,

$$\mathbb{R}\zeta = [\boldsymbol{E}(\tau), \boldsymbol{H}(\tau), \boldsymbol{D}(\tau), \boldsymbol{B}(\tau), \epsilon(\tau), \mu(\tau),$$
$$\tau, -\boldsymbol{J}(\tau), -\boldsymbol{K}(\tau), -\frac{\partial}{\partial \tau}\boldsymbol{E}(\tau), -\frac{\partial}{\partial \tau}\boldsymbol{H}(\tau), -\frac{\partial}{\partial \tau}\boldsymbol{D}(\tau), -\frac{\partial}{\partial \tau}\boldsymbol{B}(\tau)] \tag{173}$$

We know the Maxwell equations are the following,

$$\nabla \cdot \boldsymbol{D}(t) = \rho(t) \tag{174}$$

$$\nabla \cdot \boldsymbol{B}(t) = \rho_M(t) \tag{175}$$

$$\nabla \times \boldsymbol{H}(t) = \boldsymbol{J}(t) + \frac{\partial}{\partial t}\boldsymbol{D}(t) \tag{176}$$

$$\nabla \times \boldsymbol{H}(t) = -\boldsymbol{K}(t) - \frac{\partial}{\partial t}\boldsymbol{B}(t) \tag{177}$$

$$\begin{cases} \boldsymbol{D}(t) = \epsilon(t)\boldsymbol{E}(t) \\ \boldsymbol{B}(t) = \mu(t)\boldsymbol{H}(t) \end{cases} \tag{178}$$

After the time reversal transform we have the time reversal Maxwell equations:

$$\nabla \cdot \boldsymbol{D}(\tau) = \rho(\tau) \tag{179}$$

$$\nabla \cdot \boldsymbol{B}(\tau) = \rho_M(\tau) \tag{180}$$

$$\nabla \times \boldsymbol{H}(\tau) = -\boldsymbol{J}(\tau) - \frac{\partial}{\partial \tau}\boldsymbol{D}(\tau) \tag{181}$$

$$\nabla \times \boldsymbol{H}(\tau) = +\boldsymbol{K}(\tau) + \frac{\partial}{\partial \tau}\boldsymbol{B}(\tau) \tag{182}$$

$$\begin{cases} \boldsymbol{D}(\tau) = \epsilon(\tau)\boldsymbol{E}(\tau) \\ \boldsymbol{B}(\tau) = \mu(\tau)\boldsymbol{H}(\tau) \end{cases} \tag{183}$$

It should be notice the time reversal Maxwell equations are not the Maxwell equations, Maxwell equations are not time reversible. Considering after the time reversal transform, the field is not the normal electromagnetic fields which satisfy Maxwell equations, we give another symbol. The time τ is also change back to t. Hence the time-reversal equation can be written as,

$$\nabla \cdot \boldsymbol{d}(t) = \varrho(t) \tag{184}$$

$$\nabla \cdot \boldsymbol{b}(t) = \varrho_M(t) \tag{185}$$

$$\nabla \times \boldsymbol{e}(t) = -\boldsymbol{j}(t) - \frac{\partial}{\partial \tau}\boldsymbol{d}(t) \tag{186}$$

$$\nabla \times \boldsymbol{h}(t) = +\boldsymbol{k}(t) + \frac{\partial}{\partial \tau}\boldsymbol{b}(t) \tag{187}$$

$$\begin{cases} \boldsymbol{d}(t) = \epsilon(t)\boldsymbol{e}(t) \\ \boldsymbol{b}(t) = \epsilon\boldsymbol{h}(t) \end{cases} \tag{188}$$

The Poynting theorem,

$$-\nabla \cdot (\boldsymbol{E} \times \boldsymbol{H}) = \boldsymbol{J} \cdot \boldsymbol{E} + \boldsymbol{K} \cdot \boldsymbol{H} + \boldsymbol{E} \cdot \frac{\partial \boldsymbol{D}}{\partial t} + \boldsymbol{H} \cdot \frac{\partial \boldsymbol{B}}{\partial t} \tag{189}$$

After time-reversal transform we obtain the Poynting theorem for time reversal field, which is,

$$-\nabla \cdot (\boldsymbol{e} \times \boldsymbol{h}) = -\boldsymbol{j} \cdot \boldsymbol{e} - \boldsymbol{k} \cdot \boldsymbol{h} - \boldsymbol{e} \cdot \frac{\partial \boldsymbol{d}}{\partial t} - \boldsymbol{h} \cdot \frac{\partial \boldsymbol{b}}{\partial t} \tag{190}$$

or

$$\nabla \cdot (\boldsymbol{e} \times \boldsymbol{h}) = +\boldsymbol{j} \cdot \boldsymbol{e} + \boldsymbol{k} \cdot \boldsymbol{h} + \boldsymbol{e} \cdot \frac{\partial \boldsymbol{d}}{\partial t} + \boldsymbol{h} \cdot \frac{\partial \boldsymbol{b}}{\partial t} \tag{191}$$

The items,

$$\boldsymbol{j}\cdot\boldsymbol{e}+\boldsymbol{k}\cdot\boldsymbol{h} \tag{192}$$

this corresponding to heat energy loss.

$$\boldsymbol{e}\cdot\frac{\partial\boldsymbol{d}}{\partial t}+\boldsymbol{h}\cdot\frac{\partial\boldsymbol{b}}{\partial t}=\frac{\partial U}{\partial t} \tag{193}$$

where

$$U=\frac{1}{2}(\boldsymbol{e}\cdot\boldsymbol{d}+\boldsymbol{h}\cdot\boldsymbol{b}) \tag{194}$$

is the energy of the time reversal wave, $\frac{\partial U}{\partial t}$ is the energy increase. It is the energy from outside flow in to the inside. It is possible to make,

$$\boldsymbol{J}\cdot\boldsymbol{E}+\boldsymbol{j}\cdot\boldsymbol{e}=0 \tag{195}$$

$$\boldsymbol{K}\cdot\boldsymbol{H}+\boldsymbol{k}\cdot\boldsymbol{h}=0 \tag{196}$$

$$\boldsymbol{E}\cdot\frac{\partial\boldsymbol{D}}{\partial t}+\boldsymbol{e}\cdot\frac{\partial\boldsymbol{d}}{\partial t}=0 \tag{197}$$

$$\boldsymbol{H}\cdot\frac{\partial\boldsymbol{B}}{\partial t}+\boldsymbol{h}\cdot\frac{\partial\boldsymbol{b}}{\partial t}=0 \tag{198}$$

$$-\nabla\cdot(\boldsymbol{E}\times\boldsymbol{H})+\nabla\cdot(\boldsymbol{e}\times\boldsymbol{h})=0 \tag{199}$$

The two Poynting theorem put together we have,

$$\begin{aligned}&-\nabla\cdot(\boldsymbol{E}\times\boldsymbol{H})+\nabla\cdot(\boldsymbol{e}\times\boldsymbol{h})\\ &=\boldsymbol{J}\cdot\boldsymbol{E}+\boldsymbol{K}\cdot\boldsymbol{H}+\boldsymbol{j}\cdot\boldsymbol{e}+\boldsymbol{k}\cdot\boldsymbol{h}+\boldsymbol{E}\cdot\frac{\partial\boldsymbol{D}}{\partial t}+\boldsymbol{H}\cdot\frac{\partial\boldsymbol{B}}{\partial t}+\boldsymbol{e}\cdot\frac{\partial\boldsymbol{d}}{\partial t}+\boldsymbol{h}\cdot\frac{\partial\boldsymbol{b}}{\partial t}\\ &=0\end{aligned} \tag{200}$$

If time reversal field exist, the Poynting theorem doesn't transfer energy, because all self-items are 0. This can be widen to if the charge equal to N. For every charge the self energy are 0. Eq.(134) can be replaced as,

$$\begin{aligned}&-\sum_{i=1}^{N}\oiint_{\Gamma}(\boldsymbol{E}_i\times\boldsymbol{H}_i-\boldsymbol{e}_i\times\boldsymbol{h}_i)\cdot\hat{n}d\Gamma\\ &=\sum_{i=1}^{N}\int_{V}(\boldsymbol{J}_i\cdot\boldsymbol{E}_i+\boldsymbol{K}_i\cdot\boldsymbol{H}_i+\boldsymbol{j}_i\cdot\boldsymbol{e}_i+\boldsymbol{k}_i\cdot\boldsymbol{h}_i)dV\\ &+\sum_{i=1}^{N}\int_{V}(\boldsymbol{E}_i\cdot\frac{\partial\boldsymbol{D}_i}{\partial t}+\boldsymbol{H}_i\cdot\frac{\partial\boldsymbol{B}_i}{\partial t}+\boldsymbol{e}_i\cdot\frac{\partial\boldsymbol{d}_i}{\partial t}+\boldsymbol{h}_i\cdot\frac{\partial\boldsymbol{b}_i}{\partial t})dV\\ &=0\end{aligned} \tag{201}$$

This is updated Eq.(134), it tell us self energy items doesn't carry any energy. This formula do not conflict with Maxwell equations which led Poynting theorem. For Poynting theorem the self-items are not as 0. If the items Poynting theorem is 0, that will leads the Maxwell equation also have only 0 solution. In this way Poynting theorem is not 0, but the total self-energy includes the self-energy items of time reversal wave will vanish.

5.5 Mutual energy principle

The above discussion can be referred as self-energy principle. The self-energy principle tell us in the space not only have the retarded wave and advanced wave, but there are time reversal wave corresponding to the retarded wave and the time reversal wave corresponding to the advanced wave. The total energy of these 4 waves are completely canceled or balanced out. And hence energy is only transferred by the mutual energy items.

After I have introduced the self energy principle I will also call the mutual energy formula Eq.(135) as the mutual energy principle. The reason is that the Maxwell equations cannot correctly describe the electromagnetic phenomenal. For example we cannot derive the time reverse waves from Maxwell equations. Hence I have to found other formula to as axioms to replace the Maxwell equations. The mutual energy formula is a very good candidate to do so. From mutual energy formula we can derive Maxwell equations. Important thing is that the derived Maxwell equations must be paired. Each pair need to have the retarded wave and advanced wave, and the two waves have to be synchronized.

If we started from Maxwell equations adding also the time reversal Maxwell equations, it is still difficult to obtained the concept of advanced wave. Because even Maxwell equations can derive the the advanced wave, but the advanced wave derived from Maxwell equations is not clear which is a physical solution all only a Mathematics formula. In the other hand, started from the mutual energy formula, the advanced wave cannot be avoid. I believe the advance wave, hence, I choose the mutual energy formula as the axioms and call it as mutual energy principle.

We have derived the mutual energy theorem and mutual energy flow theorem from mutual energy principle with only a system only having two charges. This result can be widened to there are N charges. From the mutual energy principle Eq.(135) we can derive the mutual energy theorem with N charges Eq.(138).

After we have the self-energy principle the self-energy items do no contributed to the energy transfer, the mutual energy theorem become the energy conservation theorem. The mutual energy flow theorem can be also referred as energy flow theorem.

The mutual energy pipe streamline integral now can be referred as energy pipe streamline integral. The word “mutual” can be dropped off. This is very important. The path integral and streamline integral all should base on only energy flow!

5.6 Action-at-a-distance vs Mutual energy principle

The theory of action-at-a-distance are introduced by K. Schwarzschild, H. Tetrode and A.D. Fokker. According to this theory, a electric current will produce two electromagnetic potentials or two electromagnetic waves: one is the retarded wave, another is advanced wave. The emitter can send the retarded wave, but in the same time it also sends an advanced wave. The absorber can send the advanced wave, but in the same time it also sends a retarded wave. According to this theory, the sun cannot send the radiation wave out, if it stayed alone in the empty space. Infinite absorbers are the reason that the sun can radiate its light. The action formula can be written as following,

$$S=-\sum_{i} m_i c\int(\frac{dx_{i\mu}}{d\tau_i}\frac{dx_i^{\mu}}{d\tau_i})^{\frac{1}{2}}d\tau_i-\sum_{i}\sum_{j<i}\frac{e_ie_j}{c}\int\int\delta(s_{ij}^2)\frac{dx_{i\mu}}{d\tau_i}\frac{dx_j^{\mu}}{d\tau_j}d\tau_id\tau_j$$

$$= \text{extremum} \tag{202}$$

where m_i is mass of the i-th charge, c is the speed of light, e_i is the charge amount of the i-th charge, $x_{i\mu}$ is the 4-D space-time coordinates and,

$$s_{ij}^2=(x_{i\mu}-x_{j\mu})(x_i^{\mu}-x_j^{\mu}) \tag{203}$$

$$ds=c^2dt^2-dx_1^2-dx_2^2-dx_3^2 \tag{204}$$

It is possible that the mutual energy principle are equivalent to the above action at distance principle. The mutual energy principle is still a field theory that is different compare to the action at a distance theory. For a field theory, the field value can be obtained form only its neighborhood. For an action at distance formula, at least two points and with a distance will be involved, the two points in space separated with a distance can perhaps in different time-space coordinates, things become very complicate. I think they are equivalent is because in both formula they have a same summation $\sum_{i=1}^{N}\sum_{j<i}$ (This can also be written as $\sum_{i=1}^{N}\sum_{i=1,j\neq i}^{N}$). We have known to compare to the Maxwell theory, the theory of action at a distance has many advantages. If the mutual energy principle is equivalent to the theory action at a distance, the mutual energy principle can absorber all advantages from the theory of action at a distance. The mutual energy principle can derive the Maxwell equations and hence inherit all correct results from Maxwell equations.

I think that from the above action distance principle we can also obtain that the two Maxwell equations must synchronized. This is because of the same summation $\sum_{i=1}^{N}\sum_{j<i}$. But is not so clear like the mutual energy principle. From mutual energy principle is easy to obtained two group Maxwell equations and which must synchronized. From synchronization we can get the conclusion that the two waves obtained from the two Maxwell equations must one is a retarded wave and a advanced wave. If this is still not clear. Welch's condition in the infinite big sphere only a retarded wave and advanced wave can make the surface integral vanished Eq.(47 or 157). This conditions will strongly suggest there must exist the advanced wave. Only a retarded wave and an advanced wave can be synchronized in 3D space, can make the surface integral vanish at infinite big sphere. This will guarantees the mutual energy theorem and mutual energy flow theorem can be established. Mutual energy theorem actually is same to the energy conservation condition that further suggests that the self-energy con not send or carry any energy and the further suggest the self-energy

principle. All this further guarantees the energy pipe streamline integral can be defined properly. All this kind thing is not easy to obtained from starting from the action-at-a-distance principle. Hence, using the mutual energy principle and self-energy principle as axioms is reasonable.

5.7 Mutual energy principle for the time reversal waves

The time reversal wave will have also the mutual energy principle, which can be obtained by applying the time-reversal transform $\mathbb{R}$ to the mutual energy principle Eq.(135), we obtained,

$$\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}\oiint_{\Gamma}(\boldsymbol{e}_i\times\boldsymbol{h}_j)\cdot\hat{n}d\Gamma$$

$$=\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}\int_{V}(\boldsymbol{j}_i\cdot\boldsymbol{e}_j)dV+\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}\int_{V}(\boldsymbol{e}_i\cdot\frac{\partial\boldsymbol{d}_j}{\partial t}+\boldsymbol{h}_i\cdot\frac{\partial\boldsymbol{b}_j}{\partial t})dV \qquad (205)$$

From the above the time reversal Maxwell equations Eq.(184-187) can be derived. There is also the concept of time reversal mutual energy theorem and time reversal mutual energy flow theorem. It should be notice that the time reversal mutual energy flow can cancel the mutual energy flow. In case of race, for example there are two absorbers and one emitter in a system. The two advanced waves synchronized to one retarded wave. Each absorber can only obtain a half or a part of photon. In this case, the current $\boldsymbol{J}_2$ jump to a half way to the higher level and then, there is a time reversal current $\boldsymbol{j}_2$ take place, which produced a time reversal wave and bring the energy back to the emitter. In the emitter, there is a current $\boldsymbol{j}_1$ which cancels $\boldsymbol{J}_1$. Hence, we can say the time reversal mutual energy principle is responsible to bring the half or the part photon back to the emitter. That is the reason we can only find the whole photon and did not find any thing like a half photon.

The two mutual energy principles Eq.(135) and Eq.(205) are axioms. The self-energy principle Eq.(200) is also a axioms. The static field equation Eq.(179, 180) and Eq.(184185) can not be derived from mutual energy principle. It can be taken also as the axioms. The superposition principle is also belong to the axioms. It should be notice that the superposition principle cannot work with Poynting theorem that will leads the conflict. We have spend a lot of inc to discuss that conflict, and that lead to introduce two principles, i.e. the self-energy principle and the mutual energy principle. However the superposition principle can work with the mutual energy principle. All this formula become a axioms system.

This axioms system has much less formulas than other axioms system. There 2 mutual energy principle formula, 1 self-energy principle formula, 2 Gauss formula and 2 Gauss formula for time-reversal field, 1 superposition principle together 9 formulas. If we use Maxwell equations as axioms we have 4 Maxwell equations and 4 time-reversal Maxwell equations and 1 superposition principle that is also 9 formula. But in addition, there still need additional formula to discuss the advanced wave and additional formula to deal the problem of the connection between the time-reversal wave and the normal waves. Hence things become more complicate. According the principle the axioms should be simple we should take the axioms system with the mutual energy principle.

It should be notice that since there is no counterpoint of self-energy principle in the action-at-a-distance axiom system, the axiom system of the mutual energy principle and the self-energy principle is much more complete than action-at-a-distance axiom system.

6 The macroscopic wave

In last section we have spent a lot inc to prove the mutual energy flow theorem is a energy flow theorem, the mutual energy theorem is a energy conservation theorem. That will allow use to drop off the "mutual" form the streamline integral, otherwise we must speak about the "mutual" energy pipe streamline instead just energy pipe streamline.

Lets come back to the topic of path integral or streamline integral. In the derivation of path integral. Assume from point I to point F is a straight line. In the derivation of path integral, Dirac and Feynman applied the following method.

$$\left\langle q_F \middle| G_I^F \middle| q_I \right\rangle = \langle q_F | \, e^{-iHt} \, | q_I \rangle \tag{206}$$

That means

$$G_I^F = e^{-iHt} \tag{207}$$

or

$$||G_I^F|| = ||e^{-iHt}|| = 1 \tag{208}$$

This means from point I to the point F the field has the same aptitude. We know that in 3D space the field $\boldsymbol{E}$ and $\boldsymbol{H}$ decrease with the distance. Then which physical amount go from I to to F does not decrease? Dirac and Feynman did not offer a clear explanation. Feynman try to let us to accept that is the probability, the probability is not decrease from a point in space to another point. However the probability is related to the squire of amplitude of the fields. If the fields is decrease with the distance how can the probability does not decrease with the distance? I would like to study this with details that need some background information.

6.1 Wave in wave cylinder guide

Assume we have a wave guide, see Figure 18. In one side I there is a source which is a current $\boldsymbol{J_1}$, and in another ends F, there is a sink or load which has also a current J_2. In subsection 4.2 we have show a current have two possibility, sends a retarded wave or sends an advanced wave. If it sends the retarded wave, it is a source, if it sends advanced wave it is a sink. Since the wave guide is

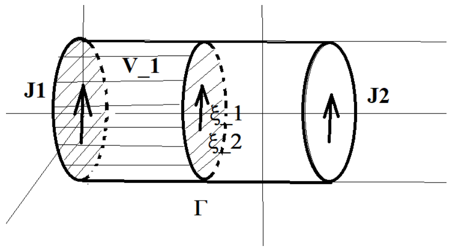

Figure 18: The wave inside the wave guide. Assume there is a source $\boldsymbol{J}_1$ which sends the retarded filed ξ_1 and a load $\boldsymbol{J}_2$ which sends the advanced wave ξ_2. In this wave guide, we can calculate the energy flow with Maxwell equations and Poynting theorem or calculate the energy flow with mutual energy principle.

1-D structure, in this special situation, if the retarded wave sent from $\boldsymbol{J}_1$ and the advanced wave sent from $\boldsymbol{J}_2$ are synchronized, the two waves are exactly same. The synchronization of the retarded wave and the advanced wave is a requirement of the mutual energy principle. This principle should be also work inside the wave guide. The mutual energy principle Eq.(144) in side the wave guide can be written as,

$$-\oiint_{\Gamma}(\boldsymbol{E}_1\times\boldsymbol{H}_2+\boldsymbol{E}_2\times\boldsymbol{H}_1)\cdot\hat{n}d\Gamma$$

$$=\int_V(\boldsymbol{J}_1\cdot\boldsymbol{E}_2+\boldsymbol{J}_2\cdot\boldsymbol{E}_1)dV$$

$$+\int_V(\boldsymbol{E}_1\cdot\frac{\partial\boldsymbol{D}_2}{\partial t}+\boldsymbol{E}_2\cdot\frac{\partial\boldsymbol{D}_1}{\partial t}+\boldsymbol{H}_1\cdot\frac{\partial\boldsymbol{B}_2}{\partial t}+\boldsymbol{H}_2\cdot\frac{\partial\boldsymbol{B}_1}{\partial t})dV \tag{209}$$

Assume the field of the retarded field and the advanced field are exactly same wave $\xi_1=\xi_2$ that is,

$$\begin{cases}\boldsymbol{E}_1=\boldsymbol{E}_2\\ \boldsymbol{H}_1=\boldsymbol{H}_2\end{cases} \tag{210}$$

Hence, we have

$$-\oiint_{\Gamma}(\boldsymbol{E}_1\times\boldsymbol{H}_1+\boldsymbol{E}_1\times\boldsymbol{H}_1)\cdot\hat{n}d\Gamma$$

$$=\int_V(\boldsymbol{J}_1\cdot\boldsymbol{E}_1+\boldsymbol{J}_2\cdot\boldsymbol{E}_1)dV$$

$$+\int_V(\boldsymbol{E}_1\cdot\frac{\partial\boldsymbol{D}_1}{\partial t}+\boldsymbol{E}_1\cdot\frac{\partial\boldsymbol{D}_1}{\partial t}+\boldsymbol{H}_1\cdot\frac{\partial\boldsymbol{B}_1}{\partial t}+\boldsymbol{H}_1\cdot\frac{\partial\boldsymbol{B}_1}{\partial t})dV \tag{211}$$

In the above formula the volume V can be taken in any place. If we take it close to the region of source, i.e.,

$$V = V_1 \tag{212}$$

Inside V_1 we have $\boldsymbol{J}_2 = 0$, hence, we have,

$$\begin{aligned}
&-\oiint_{\Gamma_1} (\boldsymbol{E}_1 \times \boldsymbol{H}_1 + \boldsymbol{E}_1 \times \boldsymbol{H}_1) \cdot \hat{n} d\Gamma \\
&= \int_{V_1} (\boldsymbol{J}_1 \cdot \boldsymbol{E}_1) dV \\
&+ \int_{V_1} (\boldsymbol{E}_1 \cdot \frac{\partial \boldsymbol{D}_1}{\partial t} + \boldsymbol{E}_1 \cdot \frac{\partial \boldsymbol{D}_1}{\partial t} + \boldsymbol{H}_1 \cdot \frac{\partial \boldsymbol{B}_1}{\partial t} + \boldsymbol{H}_1 \cdot \frac{\partial \boldsymbol{B}_1}{\partial t}) dV
\end{aligned} \tag{213}$$

Γ_1 is the boundary surface of volume V_1, or

$$\begin{aligned}
&-2 \oiint_{\Gamma_1} (\boldsymbol{E}_1 \times \boldsymbol{H}_1) \cdot \hat{n} d\Gamma \\
&= \int_{V_1} (\boldsymbol{J}_1 \cdot \boldsymbol{E}_1) dV \\
&+ 2 \int_{V_1} (\boldsymbol{E}_1 \cdot \frac{\partial \boldsymbol{D}_1}{\partial t} + \boldsymbol{H}_1 \cdot \frac{\partial \boldsymbol{B}_1}{\partial t}) dV
\end{aligned} \tag{214}$$

We know that the whole field are the retarded field and the advanced field, hence the total field $\boldsymbol{E}$ are two times of the retarded field, i.e.,

$$\begin{cases} \boldsymbol{E}_1 = \frac{1}{2} \boldsymbol{E} \\ \boldsymbol{H}_1 = \frac{1}{2} \boldsymbol{H} \end{cases} \tag{215}$$

Considering this, we have,

$$\begin{aligned}
&-\oiint_{\Gamma_1} (\boldsymbol{E} \times \boldsymbol{H}) \cdot \hat{n} d\Gamma \\
&= \int_{V_1} (\boldsymbol{J}_1 \cdot \boldsymbol{E}) dV + \int_{V_1} (\boldsymbol{E} \cdot \frac{\partial \boldsymbol{D}}{\partial t} + \boldsymbol{H} \cdot \frac{\partial \boldsymbol{B}}{\partial t}) dV
\end{aligned} \tag{216}$$

This is Poynting theorem with the source $\boldsymbol{J}_1$ and inside the volume V_1.

This means, the field in the wave guide can be calculated with also Poynting theorem which get the same result with the mutual energy theorem. The Poynting theorem can be rewritten as,

$$-\nabla\cdot(\boldsymbol{E}\times\boldsymbol{H})=\boldsymbol{J}_1\cdot\boldsymbol{E}+\boldsymbol{E}\cdot\frac{\partial\boldsymbol{D}}{\partial t}+\boldsymbol{H}\cdot\frac{\partial\boldsymbol{B}}{\partial t} \tag{217}$$

Consider

$$\nabla\cdot(\boldsymbol{E}\times\boldsymbol{H})=\nabla\times\boldsymbol{E}\cdot\boldsymbol{H}-\boldsymbol{E}\cdot\boldsymbol{\nabla}\times\boldsymbol{H} \tag{218}$$

we have,

$$-\nabla\times\boldsymbol{E}\cdot\boldsymbol{H}+\boldsymbol{E}\cdot\boldsymbol{\nabla}\times\boldsymbol{H}=\boldsymbol{J}_1\cdot\boldsymbol{E}+\boldsymbol{E}\cdot\frac{\partial\boldsymbol{D}}{\partial t}+\boldsymbol{H}\cdot\frac{\partial\boldsymbol{B}}{\partial t} \tag{219}$$

or

$$-(\nabla\times\boldsymbol{E}+\frac{\partial\boldsymbol{B}}{\partial t})\cdot\boldsymbol{H}+\boldsymbol{E}\cdot(\nabla\times\boldsymbol{H}-\boldsymbol{J}_1-\frac{\partial\boldsymbol{D}}{\partial t})=0 \tag{220}$$

or

$$\begin{cases}\nabla\times\boldsymbol{E}+\frac{\partial\boldsymbol{B}}{\partial t}=0\\ \nabla\times\boldsymbol{H}-\boldsymbol{J}_1-\frac{\partial\boldsymbol{D}}{\partial t}=0\end{cases} \tag{221}$$

This is the Maxwell equations for current source $\boldsymbol{J}_1$. Hence even here the axioms is self-energy principle and mutual energy principle, according which the retarded wave and Poynting vector do not carry energy, but the mutual energy of the retarded wave and the advanced wave together still make the Poynting theorem succeeds and hence make the Maxwell equation succeeds. It should notice these Maxwell equations is for the macroscopic wave. This means in macroscopic situation, inside a cylinder wave guide, the Poynting theorem and Maxwell equations are still correct!

6.2 Self-energy items in cylinder guide

In the last sub-section I have discussed the contribution of the mutual energy items, all mutual energy items together can have the same effect with the result of Poynting theorem and Maxwell equations. Now let us to study the contribution of the self-energy items. In the cylinder guide, we can have the Poynting theorem for the source and sink,

$$-\oint\!\!\!\oint_{\Gamma_1}(\boldsymbol{E}_1\times\boldsymbol{H}_1)\cdot\hat{n}d\Gamma$$

$$=\int_{V_1}(\boldsymbol{J}_1\cdot\boldsymbol{E}_1)dV$$

$$+\int_{V_1}(\boldsymbol{E}_1\cdot\frac{\partial\boldsymbol{D}_1}{\partial t}+\boldsymbol{H}_1\cdot\frac{\partial\boldsymbol{B}_1}{\partial t})dV \tag{222}$$

Compare the items $\oint\!\!\oint_{\Gamma_1}(\boldsymbol{E}_1\times\boldsymbol{H}_1)\cdot\hat{n}d\Gamma$ in the above equation with Eq.(214) we know that the self-energy of the retarded wave can have the contribution of a half of the contribution from mutual energy items. The self-energy of the

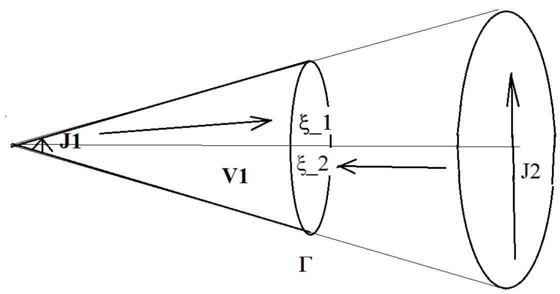

Figure 19: The wave inside cone beam wave guide. Assume there is a source $\boldsymbol{J}_1$ which sends the retarded filed ξ_1 and a load $\boldsymbol{J}_2$ which sends the advanced wave ξ_2. In this wave guide, we can calculate the energy flow with Maxwell equations and Poynting theorem or calculate the energy flow with mutual energy flow.

advanced wave will have the same contribution with the retarded wave. Hence together the 2 self-energy items of the retarded wave and advanced wave will transferred energy same as that from the mutual energy items. Hence if the mutual energy and self-energy items all contribute to the energy transfer, the energy transfer will be doubled.

We have known in the electronic engineering that the cylinder wave guide, the energy transferred can be described by Poynting theorem. If self-energy items also have contribution for energy transfer, the energy transferred will be doubled than that of Poynting theorem. That is clear some thing wrong. Hence the self-energy flow cannot have any contribution to the energy transfer.

The contribution of self-energy items are canceled by the time-reversal waves. There are two time reversal waves in the wave guide, which can transfer the same energy as self-energy items, but the direction of energy flow are negative. Hence, together with the contribution of self-energy items and the contribution of the time-reversal waves, the total contribution for self-energy is 0. Hence we can say that the self-energy items do not have any contribution to the energy transfer. The mutual energy flow actually is the the energy flow in the wave guide. In the above we have proved in the cylinder wave guide, the energy contribution of the mutual energy items is same as the contribution with Poynting theorem.

Hence, for engineering started from the mutual energy principle and self-energy principle will have the same result with that if you started from Maxwell equations and Poynting theorem for the cylinder wave guide.

6.3 Wave in cone-beam wave guide

Assume that we have a cone beam wave guide. In the vertex of the cone there is a source current $\boldsymbol{J}$, In another end of the cone, the absorbers are distributed uniformly. In this situation the advanced wave produced by the absorbers can also be same with the retarded wave. This cannot be seen very clearly, hence we make this as a presumption, i.e, a uniformly distributed absorb can be seen as black body which can absorb all radiation sends from the source, this kind of sink car produce a advanced wave in the cone beam wave guide which is exactly same as the retarded wave sends from the source $\boldsymbol{J}$.

According this assumption, the the same result can be achieve like last subsection. Hence in the cone-beam wave guide, the macroscopic wave satisfy Maxwell equations and Poynting theorem can also be derived from the mutual energy principle and self-energy principle. It should be noticed that this is only correct in macroscopic view. In microscopic wave, the source $\boldsymbol{J}$ send the retard wave, the absorber sends the advanced wave. There are also self-energy items which is canceled by the time reversal waves.

6.4 Wave in free space

For a free 3D space, it can be seen as also a special situation of the cone beam wave guide, where the cone angle is 4π. Hence in the 3D space, if the absorber are uniformly distribute at infinite big sphere, the Poynting theorem and Maxwell equations also succeeds. This means even we have started from self-energy principle, mutual energy principle and we have assumed the advanced wave and time reversal wave, the calculation result is same as the traditional way (only with the retarded wave) to calculate . The only thing we need is the absorbers must equally distribute on the infinite big sphere. This point view is agree with the absorber theory[1, 2]. In the absorber theory Wheeler and Feynman also mentioned the uniformly distributed absorbers are need to the Maxwell equations.

6.5 Wave in the path or streamline

In the energy pipe of a streamline, let us look the mutual energy flow

$$(\xi_1, \xi_2)_\Gamma = \oiint_\Gamma (\boldsymbol{E}_1 \times \boldsymbol{H}_2 + \boldsymbol{E}_2 \times \boldsymbol{H}_1) \tag{223}$$

$$\begin{cases} \xi_1 = [\boldsymbol{E}_1, \boldsymbol{H}_1] \\ \xi_2 = [\boldsymbol{E}_2, \boldsymbol{H}_2] \end{cases} \tag{224}$$

ξ_1 is decrease from the source place I to the sink place F. But the ξ_2 is decrease from F to I. Hence the advanced wave is increase from I to F. When we put ξ_1, ξ_2 the together the mutual energy flow $(\xi_1, \xi_2)_\Gamma$ is not changed in any surface Γ.

In the streamline the field amputate changes, however we can define a effective field which is,

$$\begin{cases} ||\xi_1^e|| = \sqrt{(\xi_1, \xi_2)_\Gamma} \\ ||\xi_2^e|| = \sqrt{(\xi_1, \xi_2)_\Gamma} \end{cases} \tag{225}$$

For this effective field ξ_1^e, ξ_2^e they are looks like the field of cylinder wave guide, and hence the amplitude is not changed. Hence the amplitude do not change is come from the energy flow which does not change. In case energy flow in the streamline doesn't change, we can define the the effective field, the amplitude of which will not change. In case the amplitude doesn't change we can have,

$$(\xi_2^e, \xi_1^e)_F = (\sigma_2^e, G\sigma_1^e)_F \tag{226}$$

σ_1^e is Huygens source. We have proved that the field ξ_1^e can be replaced as Huygens source σ_1^e. This replace is taken place at the point I. The field at the F will be $\xi_1^e = G\sigma_1^e$. G is propagation coefficient. Since for the effective fields, the amplitude do not change, we know inside this wave guide can only have plane wave, hence we have,

$$G = \exp(something) \tag{227}$$

For the quantum mechanics we know the plane wave is Schrödinger equation, the plane wave can be written as

$$G = \exp(-iHT) \tag{228}$$

$$(\xi_2^e, G\sigma_1^e)_F = (\sigma_2^e, \exp(-iHT)\sigma_1^e)_F \tag{229}$$

or

$$\left\langle \sigma_2^e \middle| \exp(-iHT) \middle| \sigma_1^e \right\rangle \tag{230}$$

It is notice in all my derivation, the fields are all electromagnetic field, however we know there has some theory that the field of electron also satisfied a extended Maxwell equations instead of Dirac equation or Schrödinger equation. We can see the Dirac equation and Schrödinger equation are simplified version of that extended Maxwell equations. In this way, the discussion of this article for electromagnetic field theory should be also correct for the case of the field of electron or other particles. In section 8 we will discuss in case of Schrödinger equation replace the Maxwell equation.

In this sub-section, we can see that the field amplitude doesn't change with the time and distance is because the effective field ξ_1^e, ξ_2^e and the fact there is the mutual energy flow theorem which guarantees the energy flow inside the streamline is not changed. It should be notice that the amplitude of the actual field ξ_1 and ξ_2 are always change in the path or streamline!

7 Important notices

7.1 It is not possible to have the other path than the streamline

It is often heard that the path integral includes all path for example in the figure we have showed 3 paths The first one is a strait line. The second line is a streamline. The third is a arbitrary path. We have know that the first

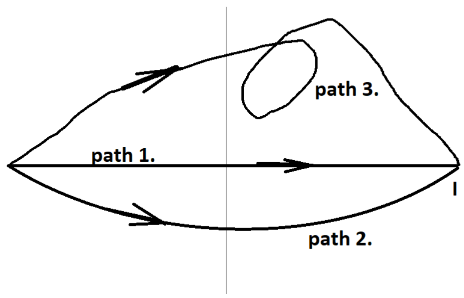

Figure 20: In the figure there are 3 paths. Path (1) is straight line. The path (2) is a streamline, this is a line the mutual energy flow will go. The path (3) path is arbitrary path.

two paths are allowed in by the mutual energy flow theorem and belong to the energy pipe stream line. The third line doesn't belong the streamline, many quantum mechanics text book showed this kind of paths. The question is that is this kind of path really allowed in the path integral?

First let us see if we can create a path like this. The energy pipe stream line is decided by the retarded wave and the advanced wave, in that line we can define the energy flow stream line. in the other arbitrary line it is not possible to define the energy flow, there is no any physics variable not changed along that line. Hence it is not possible to get the result,

$$\left\langle \sigma_2^e \middle| \exp(-iHT) \middle| \sigma_1^e \right\rangle_F = \left\langle \sigma_2^e \middle| \exp(-iH_2T) \middle| \xi_\Gamma \right\rangle_F \left\langle \xi_\Gamma \middle| \exp(-iH_1T) \middle| \sigma_1^e \right\rangle_\Gamma \quad (231)$$

where H is Hamilton,

$$H = H_1 + H_2 \quad (232)$$

ξ_Γ is the field at the a meddle point Γ between the two ends I and F. This means there is no any reason that the path integral can includes that kind of paths. That kind of paths make the concept path integral become very confuse!

In general it is not possible to build a arbitrary path in which can define a physical amount with amplitude is constant. If that physical amount is energy, we have to adjust the section area of the pipe to keep the energy inside keep as a constant. That become very strange.

7.2 Probability

According to the mutual energy principle, in the photon situation, the probability comes from the following reason. The emitter randomly sends the retarded wave. The absorber randomly sends the advanced wave. Since there are many absorbers, which absorber can send the advance wave and synchronized with the retarded wave is also random event. If an advanced wave sends from an absorber charge just win the synchronization, it absorber a photon. This events is clear a random event and hence the probability comes.

Even which absorber is randomly decided, but once it is decided, the energy flow is real physical energy flow. Hence, the streamline or path integral is based on energy and not the probability!

About why the probability of a place received photons is proportional to the square of the amplitude of the field is because that normally an absorber in the beginning can only received a part of photon instead of a whole photon, this part of photon is returned to the emitter and re-sends from the emitter. After the energy is resented, some absorber can win the energy from its neighbor. Hence the energy received on a region can be received finally by only one absorber. this energy will equal to the area of the region multiplies the square of the amplitude of the field. This made the probability of receiving a photon for an absorber is proportional to the square of the amplitude of the field.

The energy can return from the absorber to the emitter is because of the time reversal mutual energy flow which is responsible to return all part photon or the half photon. This is also the reason we cannot receive the half photon. It should be notice, the energy return to the emitter from absorber that uses a negative time, this is because of the time-reverse wave. The total time the energy send from the emitter to the absorber and then return to the emitter is 0. The streamline and path integral is really because of the energy not the probability!

7.3 Streamline integral is a well better formalism than Schrödinger equation

It is often found in the quantum mechanic text book that path integral formalism is equal to other formalism for example the formalism with Schrödinger equation. This is also not correct. The stream line integral is based on mutual energy theorem, mutual energy flow theorem. After we have accept the self-energy principle, the mutual energy theorem and mutual energy flow theorem become the energy conservation theorem and energy flow theorem. the word "mutual" can be taken away. We also know that the mutual energy theorem and the mutual energy flow theorem is based on the mutual energy principle and self-energy principle which is not equal to the Maxwell equations. The system with mutual energy principle and self-energy principle have 4 waves which is more than that a system of Maxwell equations which at most to have 2 waves. The system with mutual energy flow principle have successfully interpreted all phenomenon of wave particle duality. It is not possible to achieve this by Maxwell equation or Schrödinger equation.

Hence, the streamline integral is a well better formalism than the formalism of Schrödinger equation, Dirac equation or Maxwell equations. Here the streamline integral is updated version of the path integral.

8 The mutual energy flow for the Schrödinger equation

For photon we have obtained the results that the waves of photon obey the mutual energy principle and self-energy principle. In this section we will extended the results from photon to other quanta. The mutual energy principle and self energy principle corresponding to the Schrödinger equation are introduced. The results are that a electron, for example, travel in the empty space from point A to point B, there are 4 different waves: the retarded wave started from point A to infinite big sphere; the advanced wave started from point B to infinite big sphere; the return wave corresponding to the above retarded wave which is a time reversal wave of the retarded wave; the return wave corresponding to the above advanced wave which is a time reversal wave of the advanced wave. There are 6 different energy flows corresponding to these waves: the self-energy flow corresponding to the retarded wave; the self-energy flow corresponding to the advanced wave; the return flows corresponding to the above two time reversal wave; the mutual energy flow of the retarded wave and the advanced wave. The time-reversal mutual energy flow. It is found that the mutual energy flow is the energy flow, or the charge intensity flow, or electric current of the electron. Hence, the electron travel in the empty space is a complicated process and do not only obey one Schrödinger equation. This result can also extend to Dirac equations. This 4 waves and 6 flows together can offer a correct interpretation for the duality of the quantum.

We assume the quantum for example electron runs in the empty space from point $\boldsymbol{a}$ to $\boldsymbol{b}$. This electron must satisfy in the Schrödinger equation

$$i\hbar\frac{\partial}{\partial t}\Psi(\boldsymbol{r},t)=\left[\frac{-\hbar^2}{2\mu}\nabla^2+V(\boldsymbol{r},t)\right]\Psi(\boldsymbol{r},t) \tag{233}$$

where $i=\sqrt{-1}$. $\Psi(\boldsymbol{r},t)$ is the wave function.

8.1 The retarded equation for point $\boldsymbol{a}$

In empty space there is,

$$V(\boldsymbol{r},t)=0 \tag{234}$$

We have know that the wave $\Psi_a(\boldsymbol{r},t)$ is retarded wave started from point $\boldsymbol{a}$ and spread to the infinite big sphere. This wave satisfies,

$$i\hbar\frac{\partial}{\partial t}\Psi_a(\boldsymbol{r},t)=\left[\frac{-\hbar^2}{2\mu}\nabla^2\right]\Psi_a(\boldsymbol{r},t) \tag{235}$$

We do not know the exact wave should be, but we know that this wave should be a retarded wave, from the experience of photon we know that a retarded wave should look like the following,

$$\Psi_a(\boldsymbol{r},t)=\frac{1}{|\boldsymbol{r}-\boldsymbol{a}|}\exp_T(i\omega(t-\frac{|\boldsymbol{r}-\boldsymbol{a}|}{\frac{\omega}{k}}+t_a)) \tag{236}$$

where $\frac{\omega}{k} = v$ is the speed of the particle.

$$\exp_T(i\tau) = \begin{cases} \exp(i\tau) & 0 < \tau < 2\pi \\ 0 & otherwise \end{cases} \tag{237}$$

Where t_a is a initial constant, since we do not assume the wave is started at time $t = 0$. We have assume the wave has been truncated with only one wave length. This may be not true, perhaps the wave has a life time more than one wave length. Since the frequency of electron is very high, for example if the electron have speed of $v = \frac{c}{10}$, where c is light speed. Then moment of the electron is around,

$$\begin{aligned} p = mv &= 9 * 10^{-31} kilogram * (3 * 10^8 meter * \frac{1}{10}) \\ &= 2.7 * 10^{-23} [kg][m]/[s] \end{aligned} \tag{238}$$

The wave length of the electron is,

$$\lambda = \frac{h}{p} = \frac{6.62607004 * 10^{-34} [kg][m]^2/s}{2.7 * 10^{-23} [kg][m]/s} = 2.4541 * 10^{-11} [m] \tag{239}$$

The frequency of the wave is,

$$\lambda f = v \tag{240}$$

$$f = \frac{v}{\lambda} = \frac{3 * 10^8 [m]/[s] * 0.1}{2.4541 * 10^{-11}} = 1.22244407 * 10^{18} \tag{241}$$

Assume the period of the wave is

$$fT = 1 \tag{242}$$

$$T = \frac{1}{f} = 8.1803 * 10^{-19} [s] \tag{243}$$

if we assume the wave is only have a length of wave length, then the wave will appear in space with the $\lambda = 2.4541 * 10^{-11} [m]$. The wave can also have a life time $t = 8.1803 * 10^{-19} [s]$. This is very short wave.

We assume that the distance from point $\boldsymbol{a}$ to the origin point of the coordinates $\boldsymbol{r} = \boldsymbol{o}$ point is $|\boldsymbol{o} - \boldsymbol{a}| = l$, we assume when this retarded wave reach the point $\boldsymbol{o}$ the time is $t = 0$, hence we have,

$$(0 - \frac{|\boldsymbol{o} - \boldsymbol{a}|}{\frac{\omega}{k}} + t_a) = 0 \tag{244}$$

hence,

$$t_a = \frac{l}{\frac{\omega}{k}} \tag{245}$$

$$\Psi_a(\boldsymbol{r},t) = \frac{1}{|\boldsymbol{r}-\boldsymbol{a}|}\exp_T(j\omega(t-\frac{|\boldsymbol{r}-\boldsymbol{a}|-l}{\frac{\omega}{k}})) \tag{246}$$

This wave when $\boldsymbol{r}=\boldsymbol{a}$, $|\boldsymbol{r}-\boldsymbol{a}|=0$

$$(t-\frac{0-l}{\frac{\omega}{k}})=0 \tag{247}$$

$$t+\frac{l}{v}=0 \tag{248}$$

$$t=-\frac{l}{v} \tag{249}$$

This means $t=-\frac{l}{v}$, the wave is at the $\boldsymbol{r}=\boldsymbol{a}$. The wave is started at $t=-\frac{l}{v}$.

This wave when $\boldsymbol{r}=\boldsymbol{b}$,

$$|\boldsymbol{r}-\boldsymbol{a}|=|\boldsymbol{b}-\boldsymbol{a}|=2l \tag{250}$$

$$(t-\frac{2l-l}{\frac{\omega}{k}})=0 \tag{251}$$

$$t=\frac{l}{v} \tag{252}$$

This means that, when $t=\frac{l}{v}$, the wave come to the point $\boldsymbol{b}$. We also obtained, that if $t=0$ there is $\boldsymbol{r}=\boldsymbol{o}$.

8.2 The advanced wave started from point $\boldsymbol{b}$

According to the experience with photon, the retarded wave and the advanced wave satisfy the same Maxwell equations. This should be also true for other particles, hence here for the advanced wave it should also satisfy same Schrödinger equation (if Schrödinger equation cannot offer a correct format of advanced wave, we believe at least the Dirac equation should be, which will be discussed in section 10, here we assume Schrödinger equation is possible to described the advanced wave),

$$i\hbar\frac{\partial}{\partial t}\Psi_b(\boldsymbol{r},t) = \left[\frac{-\hbar^2}{2\mu}\nabla^2\right]\Psi_b(\boldsymbol{r},t) \tag{253}$$

We have write τ as t. $\Psi_b(\boldsymbol{r},t)$ is the advanced wave starting from point $\boldsymbol{b}$.

$$\Psi_b(\boldsymbol{r},t) = \frac{1}{|\boldsymbol{r}-\boldsymbol{b}|}\exp_T(\omega j(t+\frac{|\boldsymbol{r}-\boldsymbol{b}|}{\frac{k}{\omega}}+t_b)) \tag{254}$$

We assume when $t=0$ the advanced wave just pass the origin point $\boldsymbol{r}=\boldsymbol{o}$ and

$$|\boldsymbol{o}-\boldsymbol{b}|=l \tag{255}$$

$$(0 + \frac{l}{\frac{k}{\omega}} + t_b) = 0 \tag{256}$$

hence we have

$$t_b = -\frac{l}{\frac{k}{\omega}} \tag{257}$$

$$\Psi_b(\boldsymbol{r}, t) = \frac{1}{|\boldsymbol{r} - \boldsymbol{b}|} \exp_T(j\omega(t + \frac{|\boldsymbol{r} - \boldsymbol{b}| - l}{\frac{k}{\omega}}) \tag{258}$$

8.3 The advanced wave is synchronized with the retarded wave

Advanced wave and the retarded wave can be synchronized, this section we will show this. For the above advanced wave, when $\boldsymbol{r} = \boldsymbol{a}$,

$$|\boldsymbol{r} - \boldsymbol{b}| = 2l \tag{259}$$

$$(t + \frac{2l - l}{\frac{k}{\omega}}) = 0 \tag{260}$$

$$t = -\frac{l}{v} \tag{261}$$

For this wave, when $\boldsymbol{r} = \boldsymbol{b}$

$$|\boldsymbol{r} - \boldsymbol{b}| = |\boldsymbol{b} - \boldsymbol{b}| = 0 \tag{262}$$

$$(t + \frac{0 - l}{\frac{k}{\omega}}) = 0 \tag{263}$$

$$t = \frac{l}{v} \tag{264}$$

We have evaluated that the wave retarded $\Psi_a(\boldsymbol{r}, t)$ and the advanced wave $\Psi_b(\boldsymbol{r}, t)$ are reach the points $\boldsymbol{a}$, $\boldsymbol{o}$, $\boldsymbol{b}$ at time $t = -\frac{l}{v}$, $t = 0$, and $t = \frac{l}{v}$. Hence these two waves are synchronized at this 3 points. Actually the wave are synchronized at the who line from point $\boldsymbol{a}$ to $\boldsymbol{b}$.

This way the wave $\Psi_b(\boldsymbol{r}, t)$ is said synchronized with $\Psi_a(\boldsymbol{r}, t)$. We look the wave on the connect line between $\boldsymbol{a}$ and $\boldsymbol{b}$. This means that on this line when the retarded wave just started from point $\boldsymbol{a}$ the advanced wave has from infinite big sphere runs to reached the point $\boldsymbol{a}$, when the retarded wave reach the point $\boldsymbol{o}$ the advanced wave also reached the point $\boldsymbol{o}$. When the retarded wave reach the point $\boldsymbol{b}$ the advanced wave also reach the point $\boldsymbol{b}$. We can see the Figure 21 about the synchronization of the two waves. It is clear the most energy flow are go through the region close to the line between $\boldsymbol{a}$ to $\boldsymbol{b}$.

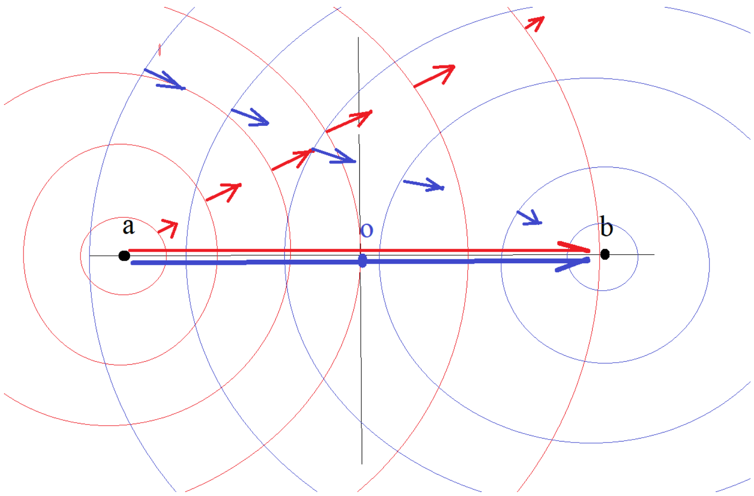

Figure 21: Retarded wave and the advanced wave of the particle, the particle move from point $\boldsymbol{a}$ to point $\boldsymbol{b}$. In the time $t = 0$ the both wave reach the point $\boldsymbol{r} = \boldsymbol{o}$. The red wave is the retarded wave. And the blue wave is the advanced wave. The retarded wave is a divergent wave. The advanced wave is convergent wave. The two waves is synchronized along the line from a to b.

8.4 The mutual energy flow from $\boldsymbol{a}$ to $\boldsymbol{b}$

Using Ψ_b^* multiply Eq(235) from right we have

$$(i\hbar\frac{\partial}{\partial t}\Psi_a)\Psi_b^* = \left[\frac{-\hbar^2}{2\mu}\nabla^2\right]\Psi_a\Psi_b^* \tag{265}$$

Using Ψ_a multiply the complex conjugate of the Eq(253) from the left, we have

$$-i\hbar\Psi_a\frac{\partial}{\partial t}\Psi_b^* = \Psi_a\left[\frac{-\hbar^2}{2\mu}\nabla^2\right]\Psi_b^* \tag{266}$$

Subtract the Eq.(266) from Eq.(265) we obtain

$$\begin{aligned}&(i\hbar\frac{\partial}{\partial t}\Psi_a)\Psi_b^* + i\hbar\Psi_a\frac{\partial}{\partial t}\Psi_b^* \\ &= \frac{-\hbar^2}{2\mu}(\nabla^2\Psi_a\Psi_b^* - \Psi_a\nabla^2\Psi_b^*)\end{aligned} \tag{267}$$

or

$$\frac{\partial}{\partial t}(\Psi_a\Psi_b^*) = -\frac{\hbar}{2\mu i}\nabla\cdot(\nabla\Psi_a\Psi_b^* - \Psi_a\nabla\Psi_b^*) \tag{268}$$

or

$$\frac{\partial}{\partial t}(\rho_{ab}) = -\nabla\cdot\boldsymbol{J}_{ab} \tag{269}$$

where

$$\rho_{ab} = \Psi_a \Psi_b^* \tag{270}$$

$$\boldsymbol{J}_{ab} = \frac{\hbar}{2\mu i}(\nabla\Psi_a\Psi_b^* - \Psi_a\nabla\Psi_b^*) \tag{271}$$

The above formula are mutual energy flow principle. $\boldsymbol{J}_{ab}$ are mutual energy flow.

$$\frac{d}{dt}\iiint\limits_V \rho_{ab} dV = -\oiint\limits_\Gamma \cdot\boldsymbol{J}_{ab}\hat{n}d\Gamma \tag{272}$$

This flow is not a divergence flow $\boldsymbol{J}_{ab}$. It is a point to point converged flow. This can be proved similar to the photon as following, assume Γ is big sphere, the radius of the big sphere is infinity. Assume the wave $\Psi_a(\boldsymbol{r},t)$ is a short time wave. In the time $t_{a0}=0$ the wave is at the place of point $\boldsymbol{a}$. afterwards the wave begin to spread out. When the wave reached the big sphere surface Γ, it happened at a future time

$$t_a = \frac{R}{v} \tag{273}$$

, where R is the radius of the sphere.

The advanced wave started at the time when the retarded wave reached the point $\boldsymbol{b}$, which is the time $t_{b0}=\frac{2l}{v}$, where $2l$ is the distance from point $\boldsymbol{a}$ to point $\boldsymbol{b}$.

v is the speed of the wave. The advanced wave $\Psi_b(\boldsymbol{r},t)$ reach the big sphere is at the past time

$$t_b = \frac{2l}{v} - \frac{R}{v} \tag{274}$$

. We have assume

$$2l \ll R \tag{275}$$

Since the retarded wave come to the big sphere in the future, the advanced wave come to the big sphere in the past. The retarded wave and the advanced wave are not nonzero in the same time at the big sphere Γ, hence

$$\nabla\Psi_a\Psi_b^* - \Psi_a\nabla\Psi_b^* = 0 \tag{276}$$

at the sphere Γ. The $\boldsymbol{J}_{ab}$ has no any flux go out the big sphere Γ.

$$\int\limits_{-\infty}^{\infty}\oiint\limits_\Gamma \boldsymbol{J}_{ab}\cdot\hat{n}d\Gamma dt = 0 \tag{277}$$

This means that mutual energy flow $\boldsymbol{J}_{ab}$ do not go outside our universe. Inside the volume V their is only the two sources for the charges at $\boldsymbol{a}$ and $\boldsymbol{b}$ hence the flow can only started from $\boldsymbol{a}$ to $\boldsymbol{b}$. The flow $\boldsymbol{J}_{ab}$ is very thin in the two ends point $\boldsymbol{a}$ and $\boldsymbol{b}$. The flow $\boldsymbol{J}_{ab}$ are very thick in the middle between the two points $\boldsymbol{a}$ and $\boldsymbol{b}$. The flow will has the same flux integral with time in any surface between the two point $\boldsymbol{a}$ and $\boldsymbol{b}$. If the particle is a electron, this flow $\boldsymbol{J}_{ab}$ is the

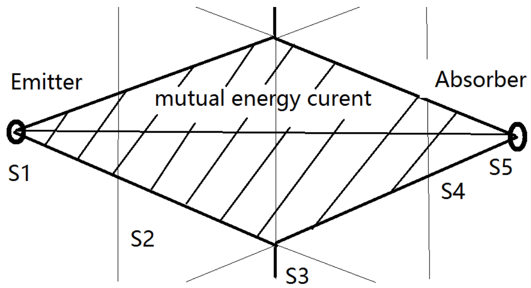

Figure 22: In this picture we assume between the two points points $\boldsymbol{a}$ and $\boldsymbol{b}$, we put a partition board with big hole on it. The mutual energy flow theorem tell us the time integral of the mutual energy flow $\boldsymbol{J}_{ab}$ will be same at any surface S_i where $i = 1, 2, 3, 4, 5$, between the two points $\boldsymbol{a}$ and $\boldsymbol{b}$. The shape of the mutual energy flow is shown in this picture.

current. This flow is the electron itself, it is the electric current between $\boldsymbol{a}$ and $\boldsymbol{b}$.

The above formula also means that

$$\int_{-\infty}^{\infty} \oiint_{S_i} \boldsymbol{J}_{ab} \cdot \hat{n}_{abi} dS = const, \qquad i = 1, 2, ..n \tag{278}$$

See Figure 22, where $\hat{n}_{abi}$ is unit vector of the surface S_i, the direction of $\hat{n}_{abi}$ is from $\boldsymbol{a}$ to $\boldsymbol{b}$. This can be referred as the mutual energy flow theorem, The time integral of the total flux of the flows in any different surface S_i are same. This is same as the photon situation.

Assume there is a partition board. The mutual energy flow between point $\boldsymbol{a}$ and $\boldsymbol{b}$, see Figure 23. If there are double slits on the partition board, it is no any problem for this kind of mutual energy flow to go through the two slits.

Since the mutual energy flow go through the double slits in the same time, and the flow at two end points $\boldsymbol{a}$ and $\boldsymbol{b}$ looks like a particle, and at the middle between two end points $\boldsymbol{a}$ and $\boldsymbol{b}$ looks like wave. This explains the particle and wave duality for all particle includes electron, see Figure 24.

8.5 Inner product for the wave satisfies Schrödinger equation

Inner product can be defined as,

$$(\Psi_b, \Psi_a) = \int_{-\infty}^{\infty} \oiint_{\Gamma} \cdot \boldsymbol{J}_{ab} \hat{n} d\Gamma dt$$

$$= \int_{-\infty}^{\infty} \oiint_{\Gamma} \frac{\hbar}{2\mu i} (\Psi_b^* \nabla \Psi_a - \nabla \Psi_b^* \Psi_a) \cdot \hat{n} d\Gamma dt \tag{279}$$

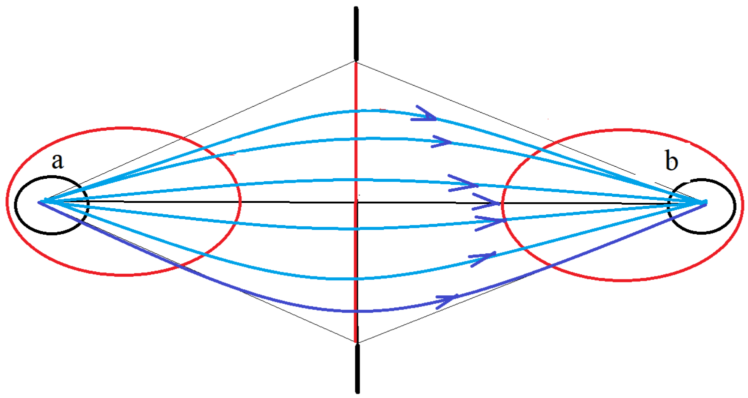

Figure 23: The mutual energy flow between the two point $\boldsymbol{a}$ and $\boldsymbol{b}$. Assume there is a partition board. This wave is quasi-plane wave.

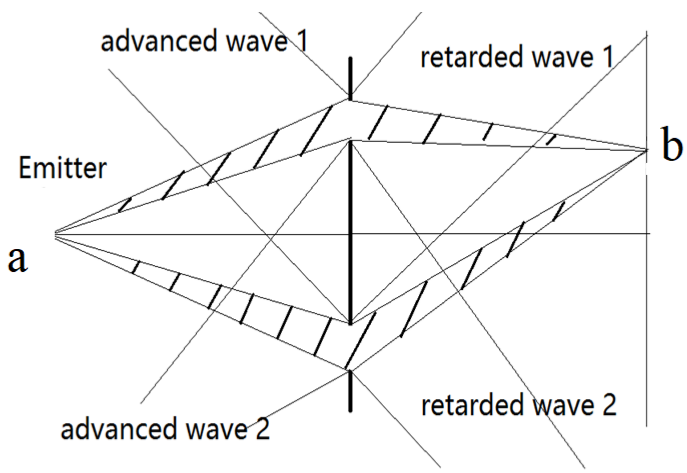

Figure 24: Assume there is a partition board which is put between the point $\boldsymbol{a}$ and $\boldsymbol{b}$. Double slits are opened on the partition board which allow the particle to go through. The shape of the mutual energy flow for the double slits are shown.

Hence we have,

$$(\Psi_b,\Psi_a)^* = \int\limits_{-\infty}^{\infty} \oiint\limits_{\Gamma} \frac{\hbar}{2\mu(-i)}(\nabla\Psi_a^*\Psi_b - \Psi_a^*\nabla\Psi_b)\cdot\hat{n}d\Gamma dt$$

$$= \int\limits_{-\infty}^{\infty} \oiint\limits_{\Gamma} \frac{\hbar}{2\mu i}(\Psi_a^*\nabla\Psi_b - \nabla\Psi_a^*\Psi_b)\cdot\hat{n}d\Gamma dt$$

$$= (\Psi_a,\Psi_b) \tag{280}$$

We also have,

$$(\Psi_b,\Psi_{a_1}+\Psi_{a_2})$$

$$= \int\limits_{-\infty}^{\infty} \oiint\limits_{\Gamma} \frac{\hbar}{2\mu i}(\Psi_b^*\nabla(\Psi_{a_1}+\Psi_{a_2}) - \nabla\Psi_b^*(\Psi_{a_1}+\Psi_{a_2})\cdot\hat{n}d\Gamma dt$$

$$= \int\limits_{-\infty}^{\infty} \oiint\limits_{\Gamma} \frac{\hbar}{2\mu i}(\Psi_b^*\nabla\Psi_{a_1} - \nabla\Psi_b^*\Psi_{a_1})\cdot\hat{n}d\Gamma dt$$

$$+ \int\limits_{-\infty}^{\infty} \oiint\limits_{\Gamma} \frac{\hbar}{2\mu i}(\Psi_b^*\nabla\Psi_{a_2} - \nabla\Psi_b^*\Psi_{a_2})\cdot\hat{n}d\Gamma dt$$

$$= (\Psi_b,\Psi_{a_1}) + (\Psi_b,\Psi_{a_2}) \tag{281}$$

We can easy to see that we have,

$$(\Psi_b,k\Psi_{a_1}) = k(\Psi_b,\Psi_{a_1}) \tag{282}$$

And hence, (Ψ_b,Ψ_a) indeed is a inner product. Hence that can be applied in streamline integral. Apply this kind of inner product that has the mutual energy flow theorem to guarantees we can correctly define the streamline integral. It is better than to use the following definition.

$$(\Psi_b,\Psi_a) = \int\limits_{-\infty}^{\infty} \iiint\limits_{V} (\Psi_b^*\Psi_a)dVdt \tag{283}$$

If we use the above definition we can only define the path integral, and can not define the streamline integral. However the problem of path integral in section 1.2 cannot be solved. Please see Figure 1.

8.6 Self energy flow

We also know that for the retarded wave started from point $\boldsymbol{a}$ there is,

$$\frac{\partial}{\partial t}(\rho_a) = -\nabla \cdot \boldsymbol{J}_a \tag{284}$$

For the advanced wave started from point B there is

$$\frac{\partial}{\partial t}(\rho_b) = -\nabla \cdot \boldsymbol{J}_b \tag{285}$$

where

$$\boldsymbol{J}_a = \frac{\hbar}{2\mu i}(\nabla\Psi_a\Psi_a^* - \Psi_a\nabla\Psi_a^*) \tag{286}$$

$$\boldsymbol{J}_b = \frac{\hbar}{2\mu i}(\nabla\Psi_b\Psi_b^* - \Psi_b\nabla\Psi_b^*) \tag{287}$$

$\boldsymbol{J}_a$ is the so called probability current of retarded wave Ψ_a which is a current sends energy from point $\boldsymbol{a}$ to infinite big sphere.

$\boldsymbol{J}_b$ is the so called probability current of advanced wave Ψ_b which is a current send energy from point **b** to infinite big sphere. Since this is advanced wave the energy current is at reversal direction. The energy flux is go from infinite big sphere Γ to the point $\boldsymbol{b}$.

It should notice here, in this article we do not call $\boldsymbol{J}_a$ and $\boldsymbol{J}_b$ probability current instead we call them self-energy flows. The reason will be cleared at section 10.6.

We know that

$$\int_{t=\infty}^{\infty} \oiint_{\Gamma} \boldsymbol{J}_a \cdot \hat{n} d\Gamma dt = const \tag{288}$$

The wave started from point $\boldsymbol{a}$ is retarded wave and hence this part of energy is at a future time to reach the big sphere Γ.

$$\int_{t=\infty}^{\infty} \oiint_{\Gamma} \cdot \boldsymbol{J}_b \hat{n} d\Gamma dt = -const \tag{289}$$

The negative symbol on the left of the above formula "$-$" is because this is a advanced wave, hence the result is a negative constant. The wave started from point $\boldsymbol{b}$ is advanced wave, this is part of energy will at past time to reach the big sphere. Unless our universe at the infinite big sphere is connected from future to the past, the energy send form point $\boldsymbol{a}$ can be received by the point $\boldsymbol{b}$. Otherwise the retarded flow $\boldsymbol{J}_a$ from $\boldsymbol{a}$ will lose some energy in a future time at infinite big sphere Γ. The advanced flow $\boldsymbol{J}_b$ started from $\boldsymbol{b}$ will receive some energy in the past time at the infinite big sphere Γ. All these are not possible. This violate the energy conservation law. Our solution for this is described in the following section.

9 The return waves (time reversal waves)

9.1 The equation of the return wave

Advanced wave is obtained by a time reversal transform $\mathbb{R}$ which is defined by

$$\mathbb{R}\Psi(\boldsymbol{r},t) = \Psi_r(\boldsymbol{r},-t) \tag{290}$$

Assume the Schrödinger equation is,

$$i\hbar\frac{\partial}{\partial t}\Psi(\boldsymbol{r},t) = \left[\frac{-\hbar^2}{2\mu}\nabla^2 + V(\boldsymbol{r},t)\right]\Psi(\boldsymbol{r},t) \tag{291}$$

In empty space there is,

$$V(\boldsymbol{r},t) = 0 \tag{292}$$

$$i\hbar\frac{\partial}{\partial t}\Psi(\boldsymbol{r},t) = \left[\frac{-\hbar^2}{2\mu}\nabla^2\right]\Psi(\boldsymbol{r},t) \tag{293}$$

The returned wave corresponding retarded wave are,

$$i\hbar\frac{\partial}{\partial t}\Psi_r(\boldsymbol{r},-t) = \left[\frac{-\hbar^2}{2\mu}\nabla^2\right]\Psi_r(\boldsymbol{r},-t) \tag{294}$$

or

$$-i\hbar\frac{\partial}{\partial(-t)}\Psi_r(\boldsymbol{r},-t) = \left[\frac{-\hbar^2}{2\mu}\nabla^2\right]\Psi_r(\boldsymbol{r},-t) \tag{295}$$

Let $-t = \tau$

$$-i\hbar\frac{\partial}{\partial(\tau)}\Psi_r(\boldsymbol{r},\tau) = \left[\frac{-\hbar^2}{2\mu}\nabla^2\right]\Psi_r(\boldsymbol{r},\tau) \tag{296}$$

We also know that $\Psi^*(\boldsymbol{r},\tau)$ also satisfy

$$-i\hbar\frac{\partial}{\partial(\tau)}\Psi^*(\boldsymbol{r},\tau) = \left[\frac{-\hbar^2}{2\mu}\nabla^2\right]\Psi^*(\boldsymbol{r},\tau) \tag{297}$$

Compare the above formula we have the flowing results,

$$\Psi_r(\boldsymbol{r},\tau) = \Psi^*(\boldsymbol{r},\tau) \tag{298}$$

The return wave is just the conjugate wave. The return wave can be obtained from the original wave by change the sign before the items of $\frac{\partial}{\partial t}$.

9.2 The flow of the return waves

According discussion in the end of last section, we assume there are return waves for $\boldsymbol{J}_a$ and $\boldsymbol{J}_b$. The return wave for $\boldsymbol{J}_a$ is a wave from infinite big sphere at future time to the point $\boldsymbol{a}$. The return wave for $\boldsymbol{J}_b$ is a wave start from infinite big sphere at a past time to the point $\boldsymbol{b}$.

Hence for a quantum travel from $\boldsymbol{a}$ to $\boldsymbol{b}$ there 4 different waves, and 5 flows:
(1) retarded wave started from point $\boldsymbol{a}$, which is referred as $\boldsymbol{J}_a$
(2) advanced wave started from point $\boldsymbol{b}$, which is referred as $\boldsymbol{J}_b$
(3) return wave for (1), which is referred as $\boldsymbol{J}_{ar}$
(4) return wave for (2), which is referred as $\boldsymbol{J}_{br}$
The return wave for (1) satisfy

$$-i\hbar\frac{\partial}{\partial t}\Psi_{ar}(\boldsymbol{r},t)=\left[\frac{-\hat{\hbar^2}}{2\mu}\nabla^2\right]\Psi_{ar}(\boldsymbol{r},t) \tag{299}$$

It has the same equation with conjugate wave. The advanced wave is send from point $\boldsymbol{a}$, in the $t = now$ to the time $t = past$. The returned wave Ψ_{ar} is from start from big sphere at time $t = future$ to the point $\boldsymbol{a}$ at time $t = now$.

The return wave for (2) satisfy

$$-i\hbar\frac{\partial}{\partial t}\Psi_{br}(\boldsymbol{r},t)=\left[\frac{-\hat{\hbar^2}}{2\mu}\nabla^2\right]\Psi_{br}(\boldsymbol{r},t) \tag{300}$$

It has the same equation with the complex conjugate wave. The retarded wave from now to the future. $\Psi_{br}(\boldsymbol{r},t)$ is from big sphere at time $t = past$ to the point $\boldsymbol{b}$ at time $t = now$. The two return flow can be defined as following,

$$\begin{aligned}\boldsymbol{J}_{ar} &= \frac{\hbar}{2\mu i}(\nabla\Psi_{ar}\Psi_{ar}^* - \Psi_{ar}\nabla\Psi_{ar}^*)\\ &= \frac{\hbar}{2\mu i}(\nabla\Psi_a^*\Psi_a - \Psi_a^*\nabla\Psi_a)\\ &= -\frac{\hbar}{2\mu i}(\Psi_a^*\nabla\Psi_a - \nabla\Psi_a^*\Psi_a)\\ &= -\boldsymbol{J}_a\end{aligned} \tag{301}$$

Hence we have,

$$\boldsymbol{J}_a + \boldsymbol{J}_{ar} = 0 \tag{302}$$

Similarly we also have,

$$\boldsymbol{J}_b + \boldsymbol{J}_{br} = 0 \tag{303}$$

We assume that the wave Ψ_{br} and Ψ_{ar} can interfere. If it can interfere the mutual energy flow $\boldsymbol{J}_{ab}$ will be canceled by $\boldsymbol{J}_{abr}$ which is the time-reversal mutual energy flow. The time-reversal mutual energy flow is responsible for to return the half photon from the absorber to the emitter (or from the sink to the source). The above two formula tells us the $\boldsymbol{J}_a$ is offset by $\boldsymbol{J}_{ar}$ and $\boldsymbol{J}_b$ is

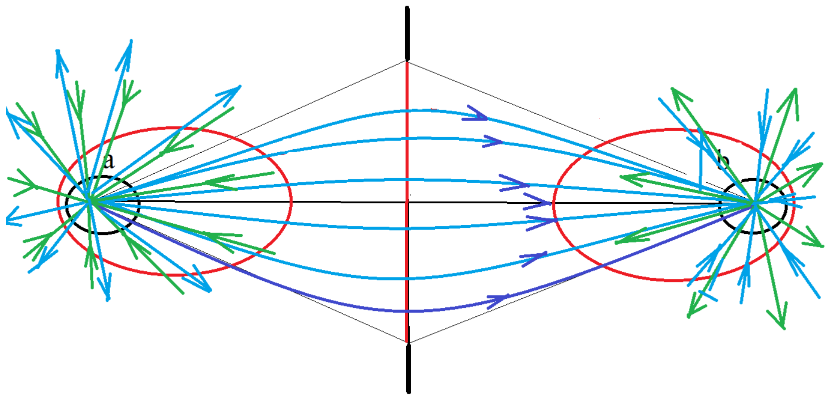

Figure 25: The mutual energy flow between the two point $\boldsymbol{a}$ and $\boldsymbol{b}$. Assume there is a partition board. This wave looks like a quasi-plane wave. In the point of $\boldsymbol{a}$, there is a return wave, the direction of energy flow of this return wave is point to the point $\boldsymbol{a}$. In the point $\boldsymbol{b}$ there is a return wave, the direction of energy flow is starts from the point $\boldsymbol{b}$. The return waves are show with green.

canceled by $\boldsymbol{J}_{br}$ hence the self-energy flows have no contribution to the energy flow from point $\boldsymbol{a}$ to the point $\boldsymbol{b}$.

The energy flow with the mutual energy flow and the return wave is show in the Figure 25. In the figure we have only shown only 3 flows which are $\boldsymbol{J}_a$,$\boldsymbol{J}_{ar}$, $\boldsymbol{J}_b$,$\boldsymbol{J}_{br}$, $\boldsymbol{J}_{ab}$. Actually there are 6 flows: $\boldsymbol{J}_a$,$\boldsymbol{J}_{ar}$, $\boldsymbol{J}_b$,$\boldsymbol{J}_{br}$, $\boldsymbol{J}_{ab}$, $\boldsymbol{J}_{abr}$.

We have to assume that Ψ_{ar} do not interfere with Ψ_a and Ψ_b and Ψ_{br}, Ψ_{br} do not interfere with Ψ_a and Ψ_b and Ψ_{ar}. The return wave (Ψ_{ar}, Ψ_{br}) are different fields with (Ψ_a, Ψ_b), they satisfy different equations.

Schrödinger equation is not a good example to show the author's theory with 4 waves and 6 flows, because in Schrödinger equation, actually cannot put the retarded wave and advanced wave to a same equation. In the above derivation we know the problem, this problem can solved in the following section where the Dirac equation or Maxwell equations is applied.

The retarded wave and the advanced wave should satisfy same equation. The two time-reversal waves should satisfy same equation. Maxwell equations are like this. Schrödinger equation is not. Anyway Schrödinger equation is simplified model. Hence I guess that perhaps the electron satisfy some kind of extension of the Maxwell equation. I know there are a few authors have that kind of theory. I will research it in the future.

10 In case of Dirac equation

10.1 Dirac equation

We have know that the Dirac equation can be written as

$$\frac{1}{c}\frac{\partial \psi_\mu}{\partial t} + \boldsymbol{\alpha}_{\mu\nu} \cdot \frac{\partial \psi_v}{\partial \boldsymbol{x}} + \frac{imc}{\hbar}\beta_{\mu\nu}\psi_v = 0 \tag{304}$$

$$\mu, \nu = 1, 2, 3, 4... \tag{305}$$

Where $\boldsymbol{\alpha} = [\alpha_x, \alpha_y, \alpha_z]$, The components of α_x is $\alpha_{x\mu\nu}$.β is a no dimension unit constant. $i = \sqrt{-1}$. m is the mass of the quantum. And

$$\boldsymbol{\alpha}^\dagger \equiv [\boldsymbol{\alpha}^*]^T = \alpha \tag{306}$$

$$\beta^\dagger \equiv [\beta^*]^T = \beta \tag{307}$$

$$\alpha_x^2 = \alpha_y^2 = \alpha_z^2 = \beta^2 = I \tag{308}$$

$$\frac{\beta_{\sigma\mu}}{c}\frac{\partial \psi_\mu}{\partial t} + \beta_{\sigma\mu}\boldsymbol{\alpha}_{\mu\nu} \cdot \frac{\partial \psi_v}{\partial \boldsymbol{x}} + \frac{imc}{\hbar}\psi_\sigma = 0 \tag{309}$$

or

$$\frac{\boldsymbol{\gamma^0_{\sigma v}}}{c}\frac{\partial \psi_v}{\partial t} + \boldsymbol{\gamma^1_{\sigma v}} \cdot \frac{\partial \psi_v}{\partial x} + \boldsymbol{\gamma^2_{\sigma v}} \cdot \frac{\partial \psi_v}{\partial y} + \boldsymbol{\gamma^3_{\sigma v}} \cdot \frac{\partial \psi_v}{\partial z} + \frac{imc}{\hbar}\psi_\sigma = 0 \tag{310}$$

Hence the Dirac equation also can be written as,

$$\gamma^\mu_{\sigma v}\frac{\partial}{\partial x^\mu}\psi_v + \frac{imc}{\hbar}\psi_\sigma = 0 \tag{311}$$

$$\mu, \nu = 1, 2, 3, 4... \tag{312}$$

10.2 Mutual energy flow corresponding to Dirac equation

Take complex conjugate to the Eq.(304) , we have,

$$\frac{1}{c}\frac{\partial \psi_\mu^*}{\partial t} + [\boldsymbol{\alpha}_{\mu\nu} \cdot \frac{\partial \psi_v}{\partial \boldsymbol{x}}]^* - \frac{imc}{\hbar}[\beta_{\mu\nu}\psi_v]^* = 0 \tag{313}$$

or

$$\frac{1}{c}\frac{\partial \psi_\mu^*}{\partial t} + [\frac{\partial \psi_v}{\partial \boldsymbol{x}}]^*\boldsymbol{\alpha}^{*T}_{\nu\mu} - \frac{imc}{\hbar}\psi_v{}^*[\beta_{\mu\nu}]^{*T} = 0 \tag{314}$$

Considering Eq.(307) we have,

$$\frac{1}{c}\frac{\partial \psi_\mu^*}{\partial t} + [\frac{\partial \psi_v}{\partial \boldsymbol{x}}]^*\boldsymbol{\alpha}_{\mu\nu} - \frac{imc}{\hbar}[\psi_v]^\dagger\beta_{\nu\mu} = 0 \tag{315}$$

or

$$\frac{1}{c}\frac{\partial\psi^\dagger}{\partial t}+[\frac{\partial\psi}{\partial \boldsymbol{x}}]^\dagger\boldsymbol{\alpha}-\frac{imc}{\hbar}[\psi]^\dagger\boldsymbol{\beta}=0 \tag{316}$$

Assume ϕ is also a wave function similar to ψ. We useϕ left multiply to the above formula we get:

$$\frac{1}{c}\frac{\partial\psi^\dagger}{\partial t}\phi+[\frac{\partial\psi}{\partial \boldsymbol{x}}]^\dagger\boldsymbol{\alpha}\phi-\frac{imc}{\hbar}[\psi]^\dagger\beta\phi=0 \tag{317}$$

In the similar way we can obtains,

$$\frac{1}{c}\phi^\dagger\frac{\partial\psi}{\partial t}+\phi^\dagger\boldsymbol{\alpha}[\frac{\partial\psi}{\partial \boldsymbol{x}}]+\frac{imc}{\hbar}\phi^\dagger\beta[\psi]=0 \tag{318}$$

Add the two formula together we have,

$$\frac{1}{c}(\frac{\partial\psi^\dagger}{\partial t}\phi+\phi^\dagger\frac{\partial\psi}{\partial t})+[\frac{\partial\psi}{\partial \boldsymbol{x}}]^\dagger\boldsymbol{\alpha}\phi+\phi^\dagger\boldsymbol{\alpha}[\frac{\partial\psi}{\partial \boldsymbol{x}}]=0 \tag{319}$$

or

$$\frac{1}{c}\frac{\partial}{\partial t}(\psi^\dagger\phi)+\frac{\partial}{\partial \boldsymbol{x}}(\psi^\dagger\boldsymbol{\alpha}\phi)=0 \tag{320}$$

Write

$$\rho=\psi^\dagger\phi \tag{321}$$

$$\boldsymbol{J}=c\psi^\dagger\boldsymbol{\alpha}\phi \tag{322}$$

we have,

$$\frac{\partial\rho}{\partial t}+\frac{\partial}{\partial \boldsymbol{x}}\cdot\boldsymbol{J}=0 \tag{323}$$

or

$$\frac{\partial\rho}{\partial t}+\nabla\cdot\boldsymbol{J}=0 \tag{324}$$

We have know that the retarded wave and advanced wave corresponding to Dirac equation all satisfy the same equation. Assume ψ is the retarded wave send from point $\boldsymbol{a}$, and ϕ is the advanced wave send from point $\boldsymbol{b}$. $\boldsymbol{J}$ will be the mutual energy flow of the wave function ψ and ϕ. similar to the last section the mutual energy flow cannot go to outside of the infinite big sphere. Hence the mutual energy flow theorem should be also established for $\boldsymbol{J}$ which is corresponding to the wave function of Dirac.

10.3 Inner product

The inner product for the Dirac equation can be defined as

$$(\psi,\phi)=\oiint_\Gamma c\psi^\dagger\boldsymbol{\alpha}\phi\cdot\hat{n}d\Gamma \tag{325}$$

10.4 The self energy flow of the Dirac equation

Similarly we can have the self energy flow for the retarded wave send from $\boldsymbol{a}$,

$$\rho_\psi = \psi^\dagger \psi \tag{326}$$

$$\boldsymbol{J_\psi} = c\psi^\dagger \boldsymbol{\alpha} \psi \tag{327}$$

and for the advanced wave send from $\boldsymbol{b}$,

$$\rho_\phi = \phi^\dagger \phi \tag{328}$$

$$\boldsymbol{J_\phi} = c\phi^\dagger \boldsymbol{\alpha} \phi \tag{329}$$

10.5 For the return wave of Dirac waves

Considering the return operator Eq.(290) we have the return wave equation,

$$-\frac{1}{c}\frac{\partial \psi_\mu^r}{\partial t} + \boldsymbol{\alpha}_{\mu\nu} \cdot \frac{\partial \psi_v^r}{\partial \boldsymbol{x}} + jmc\beta_{\mu\nu}\psi_v^r = 0 \tag{330}$$

$$\mu, \nu = 1, 2, 3, 4... \tag{331}$$

We obtained the return wave by change the sign before the items of $\frac{\partial}{\partial t}$. superscript r in ψ^r means the return wave. We also can assume this return wave do not interfere with the original retarded and the advanced Dirac waves. This two return waves corresponding to the retarded wave and the advanced wave also do not interfere. We have

$$\rho_\psi^r = \psi^{r\dagger} \psi^r \tag{332}$$

$$\boldsymbol{J}_\psi^r = c\psi^{r\dagger} \boldsymbol{\alpha} \psi^r \tag{333}$$

similarly we can have the return wave for the advanced wave, and hence,

$$\rho_\phi^r = \phi^{r\dagger} \phi^r \tag{334}$$

$$\boldsymbol{J}_\psi^r = c\phi^{r\dagger} \boldsymbol{\alpha} \phi^r \tag{335}$$

And the return flow of the self energy flow should offset the original self energy flows,

$$\boldsymbol{J}_\psi + \boldsymbol{J}_\psi^r = 0 \tag{336}$$

$$\boldsymbol{J}_\phi + \boldsymbol{J}_\phi^r = 0 \tag{337}$$

Hence in the empty space, the quantum from point $\boldsymbol{a}$ move to point $\boldsymbol{b}$ is down by the mutual energy flow $\boldsymbol{J} = c\psi^\dagger \boldsymbol{\alpha} \phi$.

10.6 Summary

The author found that for a quantum for example an electron, it travel from point $\boldsymbol{a}$ to point $\boldsymbol{b}$ in the empty space, there are 4 different waves instead one Schrödinger/Dirac wave. The 4 waves are retarded wave sends from $\boldsymbol{a}$ go to the big sphere surface Γ. The advanced wave sends from $\boldsymbol{b}$ and go to the big sphere surface Γ, the return waves for the retarded wave and the return wave for the advanced wave. Between point $\boldsymbol{a}$ and point $\boldsymbol{b}$ there is mutual energy flow $\boldsymbol{J}_{ab}$ which is transfer the energy or amount of charge from point $\boldsymbol{a}$ to point $\boldsymbol{b}$. This flow is from point to point and do not diffused. This flow is very thin in the two ends and hence, it looks like a particle. This flow is very thick in the middle between the points $\boldsymbol{a}$ and $\boldsymbol{b}$, and hence it looks a wave. In the middle if there are double slits, the mutual energy flow will go through the two slits in the same time. This explained the duality of the quantum or particle.

The self-energy flow for $\boldsymbol{J}_a$ and $\boldsymbol{J}_b$ do not transfer and energy or amount of charge. We can think they are offset by the return flow $\boldsymbol{J}_{ar}$ and $\boldsymbol{J}_{br}$. It is important to say that, the above flows $\boldsymbol{J}_{ab}$, $\boldsymbol{J}_a$, $\boldsymbol{J}_b$, $\boldsymbol{J}_{ar}$,$\boldsymbol{J}_{br}$ are all physics flow with energy or amount of the charge and they are not the probability flows.

We know the electromagnetic field has sources which is electric current. We assume there are also some sources we do not know for the wave $\Psi_a(\boldsymbol{r},t)$ and $\Psi_b(\boldsymbol{r},t)$ which is stayed at the point $\boldsymbol{a}$ and point $\boldsymbol{b}$. The source at point $\boldsymbol{a}$ can randomly sends the retarded wave. The source at $\boldsymbol{b}$ randomly send advanced wave. Point $\boldsymbol{b}$ is the target, actually on the place close to $\boldsymbol{b}$ there are thousands points similar to point $\boldsymbol{b}$ for example: $\boldsymbol{b}_1$, $\boldsymbol{b}_2$, ... $\boldsymbol{b}_n$... they all randomly send the advanced waves.

The probability come from the sources of the retarded wave starts at point $\boldsymbol{a}$ and the sources of the advanced wave started at point $\boldsymbol{b}$, they are synchronized concurrently, the mutual energy flow $\boldsymbol{J}_{ab}$ is produced. The retarded wave $\Psi_a(\boldsymbol{r},t)$ is a random events, the advanced wave $\Psi_b(\boldsymbol{r},t)$ is also a random events, the two random events just meet together is also a random events. This leads to the position of the particle has been received with a probability. We do not know exactly which advanced wave started at points $\boldsymbol{b}_1$, $\boldsymbol{b}_2$, ... $\boldsymbol{b}_n$... will finally synchronized with the retarded wave $\Psi_a(\boldsymbol{r},t)$.

This can be referred as the interpretation using the mutual energy and self-energy principle for the quantum mechanics. This interpretation is enhancive the transactional interpretation of John Cramer[5, 6].

If the retarded wave flow $\Psi_a(\boldsymbol{r},t)$ cannot meet a advanced wave which is synchronized to the retarded wave $\Psi_a(\boldsymbol{r},t)$. This retarded wave flow $\boldsymbol{J}_a$ just returned through the corresponding return wave $\boldsymbol{J}_{ar}$. If it meet the advanced wave $\Psi_b(\boldsymbol{r},t)$ which is synchronized with the retarded wave $\Psi_a(\boldsymbol{r},t)$, the mutual energy flow $\boldsymbol{J}_{ab}$ is produced. After the $\boldsymbol{J}_{ab}$, there is the return flow $\boldsymbol{J}_{ar}$. Hence no matter the mutual energy flow is produced or not the self-energy flow $\boldsymbol{J}_a$ always returned through $\boldsymbol{J}_{ar}$. For the advanced wave, the similar things also happens. No mater $\boldsymbol{J}_{ab}$ is produced or not, there is $\boldsymbol{J}_{br}$ to offset $\boldsymbol{J}_b$. Hence the self-energy flows do not transfer the energy and also do not lose the energy at infinite big sphere Γ. The energy is transferred by the mutual energy flow which is from point to point do not divergent.

There is also time-reversal mutual energy flow $\boldsymbol{J}_{ab}$ which is responsible to return the half quantum or part quantum back to the source from the sink. Hence there will no any half particle appear. This also leads the probability of the particle appearing is proportional to the square of the amplitude of the wave. Since after the energy is resent, some sink in a region will win all the energy originally send to its neighbor.

11 Conclusion

This article achieve the following conclusion:

1. The stream line integral is defined on 2D surface instead of a 3D volume The path integral is defined on a infinite 3D volume integral, In the stream line integral we have replace the 3D volume as a surface. The surface is saved 1D which is simpler than the volume integral. The reason Dirac defined the path integral on 3D volume is he has only the formula,

$$\sum \Big| q \Big\rangle \Big\langle q \Big| \equiv 1 \tag{338}$$

But we have proved a new formula from the mutual energy flow theorem,

$$\sum \Big| q_\Gamma \Big\rangle_F \Big\langle q_\Gamma \Big|_\Gamma \equiv 1 \tag{339}$$

Hence, I can define the streamline integral on the surface. In the new formula the bra and kit do not in the same surface. It should be notice that the formula $\sum \Big| q \Big\rangle \Big\langle q \Big| \equiv 1$ actually doesn't work. Even the integral region are all 3D volume, but the center of the region for the different q is different and hence the definition and the derivation of path integral is wrong.

2. The the path integral can be simplified on the streamline integral The streamline integral do not need a infinite more integrals. No one can prove the infinite more integrals can converge to something! For the streamline integral there is only one surface integral. But this simplification is because the mutual energy flow theorem. Only if the energy have the form of energy flow, we can define the energy pipe and streamline.

3. The reason amplitude of field doesn't change The reason the amplitude of field doesn't change and hence we have,

$$\Big\langle \sigma_1^e \Big| \exp(-iHT) \Big| \sigma_1^e \Big\rangle \tag{340}$$

is because

(1) in the wave guide, even we started from the mutual energy principle, the Poynting theorem still work for the cylinder wave guide.

(2) In the energy pipe or streamline we can define energy flow or the inner product which doesn't change, hence we can define the effective wave field ξ^e. Even in the 3D space the amplitude of wave is changed but the amplitude of the effective wave ξ^e doesn't change!

This guarantees we can add all effective amplitude of the effective field to get the same energy contribution on the sink point.

4. The streamline integral is not only simplified the definition of path integral We know that the definition of the energy pipe streamline integral is much simpler than the path integral. Hence from this definition the numerical calculation can be much easy. But we should notice that the streamline integral is not only simplified the concept of path integral. The concept of path integral itself is wrong. In general it is not possible to build a arbitrary path in which we can define a physical amount with its amplitude is a constant. If that physical amount is energy, we have to adjust the section area of the pipe to keep the energy inside keep as a constant. That become very strange.

5. The mutual energy principle and self-energy principle The fundamental base of the mutual energy flow theorem is the self-energy principle and the mutual energy principle. According to these principle, there are 4 waves for any particles: The retarded wave, the advanced wave and the 2 time reversal waves. Each wave has a self-energy flow. Hence there are 4 self-energy flow. There are two mutual energy flows. Hence for a particle there is 6 energy flows. All self-energy flow canceled. The mutual energy flow is responsible to send the energy. The time-reversal mutual energy is responsible to bring the half particle back to the source from the sink. This should be true also for any particles, for example electron.

6. It is energy and energy flow and not the probability In this article, we have change the streamline or path integral based on energy and energy flow instead of the probability. Hence, the streamline integral is corresponding to the real energy transferred from I to F. It is not the probability transferred from I to F.

7. The streamline integral is a well better formalism than Schrödinger equation Streamline integral is based on the mutual energy principle and self-energy principle and energy flow theorem which is well better than the formalism with Schrödinger equation, Dirac equation or Maxwell equations.

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