A Short Disproof of the Riemann Hypothesis

By

Armando M. Evangelista Jr.

armando781973@gmail.com

On

August 28, 2018

ABSTRACT

The **Riemann Hypothesis** is one of the most important unsolved problem in Mathematics: its validity will have have a great consequence on the precise calculation of the number of primes. Riemann developed an explicit formula relating the number of primes with the hypothesized *non-trivial zeros* of the Riemann zeta function. Riemann hypothesis states that all the non-trivial zeros of the zeta function have real part equal to one-half on the *critical strip*.

Despite many attempts to solve it for about 150 years, no one have, so far succeeded. Riemann hypothesis is based on the **existence of the zeros o**f the zeta function. If it can be shown, that, such zeros do not exist, then, the Riemann Hypothesis is false.

The Riemann zeta function (or zeta function) shown below is central to the Riemann Hypothesis,

(1) $$\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s} \qquad s = \sigma + \omega i,$$

where s is a complex variable with real part σ and imaginary part ω. The modulus of $\zeta(s)$, denoted by $|\zeta(s)|$, is a positive number associated with it. The quantity σ has a dampening effect on $\zeta(s)$ while ω acts as a filter that can remove some of its components. Thus, σ and ω can have a great effect on the convergence of the infinite series in (1).

The infinite series in (1) converges absolutely for $\sigma > 1$,

$$|\zeta(\sigma+\omega i)| = \left|\sum_{n=1}^{\infty} n^{-\sigma+\omega i}\right| \leq 1+2^{-\sigma}+3^{-\sigma}+4^{-\sigma}+5^{-\sigma}+\cdots+n^{-\sigma} = \sum_{n=1}^{\infty} n^{-\sigma}.$$

The infinite series in (1) can also be expressed as

(2) $$\zeta(s) = \prod_{p}^{\infty} \frac{1}{1-p^{-s}}.$$

The infinite product in (2) runs through all the prime numbers p and is widely known as the Euler product. Its modulus is given by

(3) $$|\zeta(\sigma+\omega i)| = \prod_{p}^{\infty} \frac{1}{\sqrt{1 - 2p^{-\sigma}\cos(\omega \log p) + p^{-2\sigma}}}.$$

In order for (2) to be valid, every individual term of the infinite product in (3) must satisfy the condition given below,

(4) $$1 - 2p^{-\sigma}\cos(\omega \log p) + p^{-2\sigma} > 0.$$

If only a single term of (3) violates (4) (≤ 0), then, $\zeta(s)$ will be undefined. Now, the least value of (4) is attained if $\cos(\omega \log p) = 1$, and by substituting this to (4), one finds that $\sigma > 0$. Thus, the zeta function $\zeta(s)$ is valid for $\sigma > 0$, and undefined or invalid if $\sigma \leq 0$.

Each individual term in (3) converges absolutely for $\sigma > 0$ while their product converges conditionally if $0 < \sigma \leq 1$, and their product converges absolutely for $\sigma > 1$. In fact, the divergent nature of $\zeta(s)$ at $0 < \sigma \leq 1$, proves the existence of the infinity of primes, but at $\sigma \leq 0$, it is completely invalid.

As a consequence of (4) being satisfied, the zeta function has no zeros in its domain and its modulus is always greater than zero, that is,

(5) $$\zeta(s) \neq 0 \quad \text{and} \quad |\zeta(s)| > 0, \quad \sigma > 0.$$

Each term in (3) can be either greater than one if cos(ω log p) > 0, or less than one if cos(ω log p) < 0; as p becomes larger and larger, each succeeding term approaches unity which prevents $|\zeta(s)|$ from being zero.

The following conclusions can be made as a consequence of (5):

(a) Riemann's functional equation $\zeta(s) = 2^s \pi^{s-1} \sin\left(\frac{\pi s}{2}\right) \Gamma(1-s) \zeta(1-s)$ is not valid.

(b) The equality $\pi^{-\frac{s}{2}} \Gamma\left(\frac{s}{2}\right) \zeta(s) = \pi^{-\left(\frac{1-s}{2}\right)} \Gamma\left(\frac{1-s}{2}\right) \zeta(1-s)$ is invalid.

(c) An approximation on $\zeta(s)$, say $Z(s)$, tends to have a lower value of σ at which it converges than $\zeta(s)$, but $\zeta(s)$ stay the same, since $Z(s) \approx \zeta(s)$ is not the same as $Z(s) = \zeta(s)$. All the pseudo-zeta functions, therefore, are *only* an approximations on $\zeta(s)$ and *not* its analytic continuations, the lowering of the value of σ is due to the approximation method used.

(d) The Riemann zeta function $\zeta(s)$ is defined only on the the right half-plane of the s-domain.

(e) $\zeta(s)$ has no zeros in its domain and its modulus is always greater than zero.

(f) The Riemann hypothesis is not true, since it is based on a false presupposition: the existence of the zeros of the zeta function.

Conclusions (a), (b), and (c) dispense of the notion that the zeta function $\zeta(s)$ can still be extended beyond the right half-plane.

REFERENCE

Riemann, Bernhard (1859). *On the Number of Prime Numbers less than a Given Quantity.*