Elementary Particles, Dark Matter, Dark Energy, Cosmology, and Galaxy Evolution

Thomas J. Buckholtz
*T. J. Buckholtz & Associates**
(Dated: DRAFT' - September 4, 2018)

We suggest united models and specific predictions regarding elementary particles, dark matter, dark energy, aspects of the cosmology timeline, and aspects of galaxy evolution. Results include specific predictions for new elementary particles and specific descriptions of dark matter and dark energy. Some modeling matches known elementary particles and extrapolates to predict other elementary particles, including bases for dark matter. Some models complement traditional quantum field theory. Some modeling features Hamiltonian mathematics and originally de-emphasizes motion. We incorporate results from traditional motion-centric and action-based Lagrangian math into our Hamiltonian-centric framework. Our modeling framework features mathematics for isotropic quantum harmonic oscillators.

Notes about this manuscript:

Keywords - beyond the Standard Model, dark matter, dark energy, cosmology, galaxy evolution, quantum gravity, quantum field theory, unified physics theory

Running title (suggested, assuming a limit of no more than five words) - Dark Matter and Dark Energy

The main text starts on the next PDF page.

* Thomas.J.Buckholtz@gmail.com

I. INTRODUCTION

Physics includes issues that have been unresolved for decades. For example, what elementary particles remain to be found? What is dark matter? Traditional physics theory has bases in adding quantization to classical modeling of the motion of objects. We think that an approach that features, from its beginning, quantized concepts and that does not necessarily originally address motion may prove useful.

II. METHODS

A. Opportunities based on observations

Some data point to quantized phenomena for which models do not necessarily need to have bases in motion, even though observations of motion led to making needed inferences from the data. Examples include quantized phenomena with integer bases, including spin, charge, baryon number, and weak hypercharge; the 24 known elementary particles and some aspects of their properties; and some approximate ratios, including ratios of approximate squares of masses of elementary bosons and ratios of approximate logarithms of known masses of known non-zero-mass elementary fermions. Other data also might be significant. One example features near-integer ratios of dark matter effects to ordinary matter effects. Another example features a numeric relationship between the ratio of the mass of a tauon to the mass of an electron and the ratio, for two electrons, of electromagnetic repulsion to gravitational attraction.

We develop new physics theory that correlates with such observations. We select modeling bases that produce quantized results. Based on quantum modeling techniques that do not necessarily consider motion or theories of motion, we develop models that match known elementary particles and extrapolate to suggest other elementary particles. We see how many observations we can match. This work suggests, for example, descriptions of some components of dark matter. Then, we consider so-called instance-related symmetries. The work then suggests more components for dark matter and suggests theory that explains observed ratios of effects of dark matter to effects of ordinary matter.

B. Mathematics for harmonic oscillators

First, we consider correlations between harmonic oscillator math and theory regarding electromagnetism.

Traditional physics uses two harmonic oscillators to represent excitations of a photon. The non-negative integer n_{SA1} can denote the number of excitations for the left-circularly polarized mode and the non-negative integer n_{SA2} can denote the number of excitations for the right-circularly polarized mode. People consider the modes to be perpendicular to the direction of motion of the photon. People associate with each mode the phrase transverse polarization. For each mode, people use modeling based on a harmonic-oscillator raising operator and a harmonic-oscillator lowering operator. Much physics theory correlates with four dimensions, three spatial and one temporal. We add two oscillators. The integer n_{SA0} correlates with longitudinal polarization. The integer n_{TA0} correlates with temporal excitation. Here, the acronym TA abbreviates the two-word phrase temporal aspects and contrasts with the acronym SA, which abbreviates the phrase spatial aspects. Equations (1) and (2) represent, respectively, the left-circularly and right-circularly polarized modes. The symbol $@_0$ denotes a number that is always zero. Equation (3) algebraically extends the domain of raising operators to include the state $n = -1$. The symbol a_b^+ denotes the raising operator. The subscript b denotes the word boson and anticipates uses of similar techniques regarding fermions. Regarding $n_{SA0} = -1$, $a_b^+|-1>= 0|0>$. Longitudinal polarization does not pertain. Equations (4) and (5) introduce, via the example of photons, a notion of double-entry bookkeeping that pervades ALG modeling. Here ALG correlates with the word algebraic and contrasts with PDE, which is an acronym for the phrase partial differential equation. Equation (6) correlates with double-entry bookkeeping. We use the terms TA-side and SA-side to refer, respectively, to TA-related aspects and SA-related aspects. Per equation (4), the ground state for each photon mode correlates with $A_{TA}^{ALG} = A_{SA}^{ALG} = 1/2$. Absent states that correlate with equations (1) and (2), traditional physics modeling for a three-dimensional isotropic SA-side harmonic oscillator points to a ground state for which each $n_{SA..}$ is zero and $A_{TA}^{ALG} = A_{SA}^{ALG} = 3/2$.

$$n_{TA0} = n_{SA1},\ n_{SA0} = -1,\ n_{SA2} = @_0 \tag{1}$$

$$n_{TA0} = n_{SA2},\, n_{SA0} = -1,\, n_{SA1} = @_0 \tag{2}$$

$$a_b^+ |n >= (1+n)^{1/2} |n+1> ,\ -\infty < n < \infty \tag{3}$$

$$n_{TA0} + 1/2 = A_{TA}^{ALG} = A_{SA}^{ALG} = (n_{SA0} + 1/2) + (n_{SA1} + 1/2) + (n_{SA2} + 1/2) \tag{4}$$

$$n_{TA0} + 1/2 = A_{TA}^{ALG} = A_{SA}^{ALG} = n_{SA1} + n_{SA2} + 1/2 \tag{5}$$

$$A^{ALG} \equiv A_{TA}^{ALG} - A_{SA}^{ALG} = 0 \tag{6}$$

The above re-characterization of aspects of electromagnetism shows promise. The sum of A^{ALG}, over all photons, is zero. This result contrasts with the traditional physics result that, absent cutoffs regarding maximal energy and/or spatial volume, the sum over all photon modes of A_{TA}^{ALG} for the ground state of each mode is infinite. Perhaps, the oscillator SA0 correlates with whether an elementary particle has non-zero mass. Perhaps, the oscillator pair SA1-and-SA2 correlates with aspects related to charge. Regarding elementary bosons and their spins, perhaps the oscillator SA0 correlates with spin-0 and the oscillator pair SA1-and-SA2 correlates with spin-1. Perhaps, the oscillator pair SA3-and-SA4 correlates with spin-2 and/or aspects related to gravitation.

Equations (7), (8), (9), (10), (11) and (12) generalize the notion of double-entry bookkeeping. Here, $N_{TA|..}$ denotes the number of TA-side oscillators and $N_{SA|..}$ denotes the number of SA-side oscillators. Regarding a set of $n_{..}$ that satisfy equation (11), we use the term solution.

$$A_{TA}^{ALG} = \Sigma_{j=0}^{N_{TA|..}-1} (n_{TAj} + 1/2), \text{ for } N_{TA|..} \geq 1 \tag{7}$$

$$A_{TA}^{ALG} = 0, \text{ for } N_{TA|..} = 0 \tag{8}$$

$$A_{SA}^{ALG} = \Sigma_{j=0}^{N_{SA|..}-1} (n_{SAj} + 1/2), \text{ for } N_{SA|..} \geq 1 \tag{9}$$

$$A_{SA}^{ALG} = 0, \text{ for } N_{SA|..} = 0 \tag{10}$$

$$0 = A^{ALG} = A_{TA}^{ALG} - A_{SA}^{ALG} \tag{11}$$

$$|N_{TA|..} - N_{SA|..}| \text{ is an even integer} \tag{12}$$

Second, we consider correlations between harmonic oscillator math and symmetries pertaining to Standard Model elementary particle theory.

Regarding equations (1) and (2), SA-side aspects feature two degrees of freedom, with one degree of freedom correlating with the ground state $n_{SA1} = 0$, $n_{SA2} = @_0$ and the other degree of freedom correlating with the ground state $n_{SA1} = @_0$, $n_{SA2} = 0$. We correlate these two degrees of freedom with a symmetry that features, from a group theoretic standpoint, two generators. Traditional Standard Model physics theory correlates $U(1)$ symmetry with electromagnetism. The number of generators for $U(1)$ is two. We correlate $U(1)$ symmetry with pairs of oscillators for which the two ground states $n_{XA(odd)} = 0$, $n_{XA(odd+1)} = @_j$ and $n_{XA(odd)} = @_j$, $n_{XA(odd+1)} = 0$ pertain. Here, XA can be either TA or SA and $@_j$ can be either $@_0$ or $@_{-1}$.

Traditional Standard Model physics theory also features $SU(2)$ symmetry and $SU(3)$ symmetry. $SU(2)$ correlates with three generators. We correlate $SU(2)$ symmetry with pairs of oscillators for which a ground state with one of $n_{XA(odd)} = 0$, $n_{XA(odd+1)} = 0$ and $n_{XA(odd)} = -1$, $n_{XA(odd+1)} = -1$ pertains. Each generator correlates with a Pauli matrix. For integer $l' \geq 2$, we correlate $SU(l')$ symmetry with sets l' XA-side oscillators for which the ground state features $n_{XA..} = j$ for each of the l' oscillators. Here, XA can be either TA or SA and j can be either 0 or -1.

We also correlate some aspects of ground states with no symmetries. For example, for $@_j = @_0$ or $@_j = @_{-1}$, the statement $n_{QA(odd)} = @_j = n_{QA(odd+1)}$ correlates with no relevant symmetry.

Equations (13), (14), (15), and (16) define the concepts of closed pair of harmonic oscillators and open pair of harmonic oscillators. Here, j' is a positive odd integer and $j'' = j' + 1$. One of two cases pertains. For one case, $o2 = TAj''$ and $o1 = TAj'$ pertain. For the other case, $o2 = SAj'$ and $o1 = SAj''$ pertain. Each of a and b is a complex number. We correlate with equations (13), (14), and (15) the phrase closed pair of harmonic oscillators. For cases in which equations (13), (14), and (16) pertain, we use the phrase open pair of harmonic oscillators.

$$a|n_{o2} = -1,\, n_{o1} = 0 > \;\; + \;\, b|n_{o2} = 0,\, n_{o1} = -1 > \text{, in which ...} \tag{13}$$

$$|a|^2 + |b|^2 = 1 \tag{14}$$

$$|a|^2 > 0,\; |b|^2 > 0 \tag{15}$$

$$\text{either ... } |a|^2 = 1 \text{ and } |b|^2 = 0 \text{ ... or ... } |a|^2 = 0 \text{ and } |b|^2 = 1 \tag{16}$$

The numbers a and b correlate with traditional notions of amplitude. We introduce notation to correlate with whether a number of choices of magnitudes of amplitudes is finite or uncountably infinite. For the case of open pair, equation (16) points to two choices. We use the notation that equation (17) shows. We correlate the symbol $\pi_{..}$ with the concept of permutations of the set $\{..\}$ of elements the symbol shows. Notation of the form π_{list} denotes the notion that any permutation of the items in the list can be physics-relevant. For the case of closed pair, equation (15), with no further restrictions, allows for an uncountably infinite number of possible values for each of a and b. We use the notation that equation (18) shows. We correlate the symbol $\kappa_{..}$ with the concept of a such a continuous set of choices regarding the set $\{..\}$.

$$\pi_{0,-1} \tag{17}$$

$$\kappa_{\pi_{0,-1}} \tag{18}$$

Table I correlates, for each of various symmetries, one of more combinations of a set of oscillators and values of $n_{..}$ for the oscillators in the set. For each row in table Ia, either all the oscillators are TA-side or all the oscillators are SA-side. The symbol XA denotes one of TA and SA. The range of indices pertains to oscillator numbers. The symbol $S1G$ denotes a one-generator symmetry. For our work, $S1G$ symmetry pertains regarding, at least, n_{TA0}. We allow use of the symbol $\pi_{\{..\}}$ for lists $\{..\}$ with one element. The right-most column shows the contribution to the relevant $A^{ALG}_{XA..}$. Table Ib shows a symmetry pertaining to fermion solutions and the TA0-and-SA0 oscillator pair.

Third, we discuss mathematics and applications correlating with SA-side PDE isotropic quantum harmonic oscillators.

Equations (19) and (20) correlate with an isotropic quantum harmonic oscillator. Here, r denotes the radial coordinate and has dimensions of length. The parameter η_{SA} has dimensions of length. The parameter η_{SA} is a non-zero real number. The magnitude $|\eta_{SA}|$ correlates with a scale length. The positive integer D correlates with a number of dimensions. Each of ξ and ξ' is a constant. The symbol $\Psi(r)$ denotes a function of r and, possibly, of angular coordinates. The symbol ${\nabla_r}^2$ denotes a Laplacian operator. In some traditional physics applications, Ω_{SA} is a constant that correlates with aspects correlating with angular coordinates. We associate the term SA-side with this use of symbols and mathematics, in

Table I: Symmetries, oscillators, and values of $n_{..}$

(a) Ground-state values of $n_{..}$ for various symmetries that do not mix TA-side with SA-side

Symmetry	Number of		Range of indices	Ground-state	
	Generators	Oscillators		values	$A^{ALG}_{XA..}$
None	-	2	j'(odd) and $j'+1$	$\kappa_{\pi_{0,-1}}$	0
"	"	1	≥ 0	π_0	1/2
"	"	"	"	π_{-1}	$-1/2$
"	"	2	j'(odd) and $j'+1$	$\pi_{@_0,@_0}$	1
"	"	"	"	$\pi_{@_{-1},@_{-1}}$	-1
$S1G$	1	1	≥ 0	π_0	1/2
"	"	"	"	π_{-1}	$-1/2$
$U(1)$	2	2	j'(odd) and $j'+1$	$\pi_{0,@_0}$	1
"	"	"	j'(odd) and $j'+1$	$\pi_{0,-1}$	0
$SU(l')$	$(l')^2-1$	l'	≥ 0	$\kappa_{0,\ldots,0}$	$l'/2$
"	"	"	≥ 0	$\kappa_{-1,\ldots,-1}$	$-l'/2$

(b) Fermion symmetry pertaining to the TA0-and-SA0 oscillator pair

Symmetry	Number of		Oscillators	Values	$A^{ALG}_{TA..}-A^{ALG}_{SA..}$
	Generators	Oscillators			
$U(1)$	2	2	TA0 and SA0	$-1\leq n_{TA0}$ $n_{TA0}=n_{SA0}$ $n_{SA0}\leq 0$	0

anticipation that the symbols used correlate with spatial aspects of physics modeling and in anticipation that TA-side symbols and mathematics pertain for some modeling.

$$\xi\Psi(r)=(\xi'/2)(-(\eta_{SA})^2{\nabla_r}^2+(\eta_{SA})^{-2}r^2)\Psi(r) \tag{19}$$

$${\nabla_r}^2=r^{-(D-1)}(\partial/\partial r)(r^{D-1})(\partial/\partial r)-\Omega_{SA}r^{-2} \tag{20}$$

Including for $D=1$, each of equation (19), equation (20), and the function Ψ pertains for the domain equation (21) shows.

$$0<r<\infty \tag{21}$$

We consider solutions of the form equation (22) shows.

$$\Psi(r)\propto(r/\eta_{SA})^{\nu_{SA}}\exp(-r^2/(2(\eta_{SA})^2)),\text{ with }(\eta_{SA})^2>0 \tag{22}$$

Some traditional applications of equations (19) and (20) correlate with equation (23). To denote this case, we use the term PDE|HER. Solutions include factors that are Hermite polynomial functions of r/η_{SA}. Our work generally de-emphasizes PDE|HER.

$$D=1,\ \Omega_{SA}=0,\ -\infty<r<\infty \tag{23}$$

Equations (24) and (25) characterize solutions. (See discussion pertaining to table II.) The parameter η_{SA} does not appear in these equations. Equation (26) correlates with the domain of D and ν_{SA} for which normalization pertains for $\Psi(r)$. (See discussion related to equation (29).)

$$\xi=(D+2\nu_{SA})(\xi'/2) \tag{24}$$

$$\Omega_{SA}=\nu_{SA}(\nu_{SA}+D-2) \tag{25}$$

Table II: Terms correlating with an SA-side PDE equation (assuming $(\xi'/2) = 1$ and $\eta_{SA} = 1$)

Term/ $\exp(-r^2/2)$	Symbol for term	Change in power of r	Non-zero unless ...	Notes
$-r^{\nu_{SA}+2}$	K_{+2}	$+2$	-	Cancels V_{+2}
$(D+\nu_{SA})r^{\nu_{SA}}$	K_{0a}	0	$D+\nu_{SA}=0$	-
$\nu_{SA}r^{\nu_{SA}}$	K_{0b}	0	$\nu_{SA}=0$	-
$-\nu_{SA}(\nu_{SA}+D-2)r^{\nu_{SA}-2}$	K_{-2}	-2	$\nu_{SA}=0$ or $(\nu_{SA}+D-2)=0$	Cancels V_{-2}
$\Omega_{SA}r^{\nu_{SA}-2}$	V_{-2}	-2	$\Omega_{SA}=0$	Cancels K_{-2}
$r^{\nu_{SA}+2}$	V_{+2}	$+2$	-	Cancels K_{+2}

$$D + 2\nu_{SA} \geq 0 \tag{26}$$

Table II provides details leading to equations (24) and (25). We consider equations (19), (20), and (22). The table assumes, without loss of generality, that $(\xi'/2) = 1$ and that $\eta_{SA} = 1$. More generally, we assume that each of the four terms $K_{..}$ and each of the two terms $V_{..}$ includes appropriate appearances of $(\xi'/2)$ and η_{SA}. The term V_{+2} correlates with the right-most term in equation (19). The term V_{-2} correlates with the right-most term in equation (20). The four $K_{..}$ terms correlate with the other term in equation (20). The sum of the two $K_{0..}$ terms correlates with the factor $D + 2\nu_{SA}$ in equation (24).

The presence of Ω_{SA} summarizes aspects pertaining to angular coordinates. We limit consideration to solutions that comport with equation (27) and, therefore, with equation (28).

$$2\nu_{SA} \text{ is an integer} \tag{27}$$

$$4\Omega_{SA} \text{ is an integer} \tag{28}$$

The set of solutions to which work above points is too broad for for our work. We de-emphasize solutions that do not normalize.

We consider normalization with respect to D^*_{SA} dimensions. A factor $r^{(D^*-1)}$ correlates with the expression $\int r^{(D^*_{SA}-1)}dr$. A factor $r^{2\nu_{SA}}$correlates with the multiplicative product $(\Psi(r))^* \times \Psi(r)$, regarding which the symbol * denotes complex conjugate. For $r \to 0^+$, the integrand behaves like $r^{(D^*_{SA}-1)+2\nu_{SA}}$. The following three possibilities pertain.

- For $D^*_{SA} + 2\nu_{SA} > 0$, normalization occurs for any $(\eta_{SA})^2 > 0$. We correlate solutions that correlate with this case with the term volume-like. Solutions pertain to the domain that equation (21) specifies.
- For $D^*_{SA} + 2\nu_{SA} = 0$, normalization occurs only in the limit $(\eta_{SA})^2 \to 0^+$. We correlate solutions that correlate with this case with the term point-like. In some sense, solutions pertain in the limit $r \to 0^+$.
  - Relevant math correlates with an expression for a delta function. Note equation (29). (See reference [8].) Given that $-r^2/(2(\eta_{SA})^2) + \{-r^2/(2(\eta_{SA})^2)\}$ equals $-r^2/(\eta_{SA})^2$, we correlate $(\eta_{SA})^2$ with 4ϵ. We correlate r^2 with x^2. People use equation (29) with the domain $-\infty < x < \infty$. We use the domain $0 < x < \infty$. (Note equation (21).) We posit that the answer to the question of whether a function Ψ normalizes does not depend on our choice of domain.

$$\delta(x) = \lim{}_{\epsilon\to 0^+}(1/(2\sqrt{\pi\epsilon}))e^{-x^2/(4\epsilon)} \tag{29}$$

- For $D^*_{SA} + 2\nu_{SA} < 0$, normalization fails. We de-emphasize solutions that do not normalize.

For PDE-based modeling, features and applications include the following.

Possibly, PDE-based modeling correlates with some aspects of unification of the strong, electromagnetic, and weak interactions. We consider modeling for which $2\nu_{SA}$ is a non-negative integer. Based on the r^{-2} spatial factor, the V_{-2} term might correlate with the square of an electrostatic potential. Based on the r^2 spatial factor, the V_{+2} term might correlate (at least, within hadrons) with the square of a potential correlating with the strong interaction. The sum $K_{0a} + K_{0b}$ might correlate with the strength of the weak interaction. (The effective range of the weak interaction is much smaller than the size of a hadron. Perhaps, the spatial characterization r^0 correlates with an approximately even distribution, throughout a hadron, for the possibility of a weak interaction occurring.) When coupled with a TA-side term and possibly with a term that includes a factor of a square of mass, the model conceptually offers bound-state similarities to the plane-wave Klein-Gordon equation. The overall $\Psi(t,r)$ is the product of the TA-side $\Psi(t)$ and SA-side $\Psi(r)$. Based on the V_{-2} term, we expect that ξ' includes a factor $\hbar^2$.

Possibly, PDE-based modeling correlates with a complement to traditional physics QFT (or, quantum field theory) for elementary particles. We consider modeling for which $2\nu_{SA}$ is a negative integer. For elementary fermions, solutions correlating with $\nu_{SA} = -1/2$ are volume-like and correlate with fields. Solutions correlating with $\nu_{SA} = -3/2$ are point-like and correlate with aspects of interaction vertices. For non-zero-mass elementary bosons, $\nu_{SA} = -1$ correlates with volume-like and with fields. After separating harmonic oscillator equations into equations correlating with pairs of oscillators (Examples of pairs include TA2-and-TA1, TA0-and-SA0, SA1-and-SA2, and SA3-and-SA4.), $\nu_{SA} = -1$ correlates with point-like and with interaction vertices. For each pair, we denote the relevant D^* by D''. Here, $D'' = 2$ and $D'' + 2\nu_{SA} = 0$.

PDE modeling has a role in modeling elementary particles. Equations (19) through (28) (except equation (23)) include solutions for which equations (30), (31), and (32) pertain. Here, $2S$ is a non-negative integer.

$$\Omega_{SA} = \sigma S(S + D - 2) \tag{30}$$

$$\sigma = \pm 1 \tag{31}$$

$$\nu_{SA} < 0 \tag{32}$$

Each known elementary particle has a spin $S\hbar$ that comports with equations (33) and (34).

$$S(S+1) = S(S + D^*_{SA} - 2) \tag{33}$$

$$D^*_{SA} = 3 \tag{34}$$

Except for zero-mass bosons, each known elementary particle and each elementary particle that our work suggests comports, for some choice of D and σ, with equations (30) through (34). (See table XIV.) Here, D does not necessarily equal D^*_{SA}. Here, $\sigma = -1$ correlates with the notion of a particle's existing only in hadron-like particles or in seas the feature elementary particles that otherwise exist only in hadron-like particles. Here, $\sigma = +1$ correlates with the notion of free-ranging.

III. RESULTS

A. Elementary particles and elementary long-range forces

We use the two-word phrase elementary particles and the three-element phrase elementary long-range forces.

Table III alludes to all, but does not directly show some of, the ALG solutions that our work suggests have physics-relevance regarding elementary particles and elementary long-range forces. In the symbol $\Sigma\Phi$, the symbol Σ denotes twice the spin S. For example, for 1N (which correlates with neutrinos), $S = 1/2$ and $\Sigma = 1$. Each Φ correlates with a family of solutions. Each row in table III comports with ALG double-entry bookkeeping. Regarding labeling for some columns, SA0 correlates with the SA0 oscillator, for which n_{SA0} pertains, and SA1,2 correlates with the SA1-and-SA2 pair of oscillators, for which n_{SA1} and n_{SA2} pertain. People can consider that, regarding oscillator-centric columns, in each

Table III: Sub-families

$\Sigma\Phi$	σ	$\vert\leftarrow$ ..	..	TA	..	.. $\rightarrow\vert$	$\vert\leftarrow$ ..	..	SA	..	.. $\rightarrow\vert$
		8,7	6,5	4,3	2,1	0	0	1,2	3,4	5,6	7,8
0H	+1					0	0				
1N	+1	$\pi_{0,-1}$		$\kappa_{-1,-1}$		−1	−1	$\pi_{0,-1}$	$\kappa_{-1,-1}$		$\pi^L_{0,-1}$
1C	+1	$\pi_{0,-1}$		$\kappa_{0,0}$		0	0	$\pi_{0,-1}$	$\kappa_{0,0}$		$\pi^L_{0,-1}$
1R	−1	$\pi_{0,-1}$		$\kappa_{-1,-1}$	$\pi_{0,-1}$	−1	−1	$\pi_{0,-1}$	$\kappa_{-1,-1}$		$\pi^L_{0,-1}$
1Q	−1	$\pi_{0,-1}$		$\kappa_{0,0}$	$\pi_{0,-1}$	0	0	$\pi_{0,-1}$	$\kappa_{0,0}$		$\pi^L_{0,-1}$
2U	−1		$^\dagger U_{TA}$			$^\dagger U_{TA}$	$^\dagger U_{SA}$			$^\dagger U_{SA}$	
2W	+1			$^\dagger W_{TA}$		$^\dagger W_{TA}$	$^\dagger W_{SA}$	$^\dagger W_{SA}$			
2T	−1				$^\dagger T_{TA}$	$^\dagger T_{TA}$	$^\dagger T_{TA}$		$^\dagger T_{TA}$		
2G	+1					0	−1	$\pi_{0,@_0}$			
4G	+1					0	−1		$\pi_{0,@_0}$		
6G	+1					0	−1			$\pi_{0,@_0}$	
..G	+1					0	−1				..
Footnotes:											
					$^\dagger U_{TA}$	$\pi_{0,-1,-2}$	$^\dagger U_{SA}$	$\kappa_{-1,-1,-1}$			
					$^\dagger W_{TA}$	$\kappa_{0,0,0}$	$^\dagger W_{SA}$	$\pi_{0,@_0,@_0}$			
					$^\dagger T_{TA}$	$\pi_{0,@_0,@_0}$	$^\dagger T_{TA}$	$\kappa_{0,0,0}$			

blank cell in the table correlates with $\kappa_{\pi_{0,-1}}$. Traditional physics would consider $\sigma = +1$ to correlate with the term free-ranging. Elementary particles for which $\sigma = -1$ exist only in hadron-like particles or in seas that feature elementary particles for which $\sigma = -1$. For $\sigma = +1$, SA-side aspects correlate with numbers of elementary particles and with interactions in which the particles partake. TA-side aspects correlate with notions of instance-related symmetries. For $\sigma = -1$, TA-side aspects correlate with numbers of elementary particles and with interactions in which the particles partake. SA-side aspects correlate with notions of instance-related symmetries. The symmetry $^\dagger U_{TA}$ $\pi_{0,-1,-2}$ correlates with the traditional physics strong interaction $SU(3)$ symmetry. Regarding a traditional representation, oscillator TA0 correlates with the color green. The other two relevant TA-side oscillators correlate, respectively in some order, with red and blue. Of the six permutations of 0, −1, and −2, the three correlate with, say, cyclic order correlate with interactions with matter 1Q particles and 1R matter particles and the other three correlate with interactions with antimatter 1Q particles and antimatter 1R particles. The value −2 correlates with erasing a color. The value 0 correlates with painting a color. Paralleling traditional physics theory, $SU(3)$ symmetry correlates with sums of terms, with each term correlating with an erase-and-paint pair of solutions. The item $^\dagger W_{SA}$ $\pi_{0,@_0,@_0}$ correlates with the traditional physics weak interaction $SU(2) \times U(1)$ symmetry. W bosons intermediate interactions that change charge. Z bosons intermediate interactions that do not change charge. For charged matter leptons and neutrinos, two charge states pertain. A charge of $q_e = -|q_e|$ pertains for matter charged leptons. A charge of $0|q_e|$ pertains for neutrinos. For matter quarks, the relevant charges are $-(1/3)|q_e|$ and $+(2/3)|q_e|$. In each case, based on the two choices (one of a change in charge and one of no change in charge), an SA-side $U(1)$ symmetry pertains. In each case, a notion of three fermion generations pertains. An SA-side $SU(2)$ symmetry pertains. An overall SA-side symmetry of $SU(2) \times U(1)$ pertains. For elementary bosons for which $\sigma = +1$, the table shows ground states. Remarks below provide further insight regarding ΣG, 2W, and 2T. We correlate some aspects of ΣG with the phrase elementary long-range forces.

The following paragraphs discuss individual rows in table III.

The 0H subfamily includes one solution. The solution correlates with the Higgs boson. The SA0 column correlates with abilities to interact with (at least) fermions for which $n_{SA0} = 0$. (In traditional QFT, the Higgs boson can interact with elementary bosons for which $n_{SA0} = 0$. In our complementary QFT, elementary bosons do not interact directly with elementary bosons, but do interact with indirectly with elementary bosons via fermion pair production, fermion-boson interaction vertices, and fermion pair destruction.) For the Higgs boson, the spin ($S = 0$) correlates with the SA-side relevance of just SA0.

Generally, $n_{SA0} = 0$ correlates with two aspects. One is aspect is that (at least, fermion) particles interact directly with the Higgs boson. The other is that particles have non-zero mass. Generally, $n_{SA0} = -1$ correlates with two aspects. One is aspect is that particles do not interact directly with the Higgs boson. The other is that particles have zero mass. (Regarding 1N particles, see the discussion below regarding neutrino oscillations, neutrino masses, and Majorana neutrinos.)

The 1N family includes three solutions correlating with matter elementary particles and three solutions correlating with antimatter elementary particles. The SA1-and-SA2 oscillators correlate with abilities to absorb a charge-related quantity of $\chi = \pm 3$. (Here, $\chi = q/|q_d|$, in which q denotes the charge of a

Table IV: Excitations for the H-family and for G-family sub-families

ΣΦ	σ	|← ..	..	TA	..	.. →|	|← ..	..	SA	..	..	.. →|
		8,7	6,5	4,3	2,1	0	0	1,2	3,4	5,6	7,8	...
0H	+1					n	n					
2G	+1					n	−1	$\pi_{n,@_0}$				
4G	+1					n	−1		$\pi_{n,@_0}$			
6G	+1					n	−1			$\pi_{n,@_0}$		
8G	+1					n	−1				$\pi_{n,@_0}$	
...	+1					n	−1					...

particle and q_d denotes the charge of a down quark.) Here, the SA1-and-SA2 oscillators correlate with the topic of matter and antimatter. (Regarding Majorana fermions, see the discussion below regarding neutrino oscillations, neutrino masses, and Majorana neutrinos.) The SA3-and-SA4 oscillators correlate with $SU(2)$ symmetry and three generations. The SA7-and-SA8 oscillators correlate with topics including at least some of handedness, chirality, and helicity and including weak hypercharge. The symbol $\pi^L_{0,-1}$ correlates with observations of only left-handedness.

Generally, the SA3-and-SA4 oscillators correlate with three generations and with the topic of mass and/or gravitation. For elementary fermions, results that table Ib shows pertain. For elementary fermions, oscillators in the TA0-and-SA0 pair do not participate in instance symmetries. However, between each of the two pairs of similar sets of elementary fermions, a $U(1)$ symmetry pertains. (See table Ib. One pair of similar sets of solutions features 1N and 1C. The 1C subfamily correlates with charged leptons. The other pair of similar sets of solutions features 1R and 1Q.)

The 1Q subfamily features three generations (correlating with the SA3-and-SA4 $\kappa_{0,0}$) of each of four sets of particles. For each generation, each of the four particles includes one particle that can absorb (via a W boson) a charge-related $\chi = \pm 3$ (This correlates with SA1-and-SA2.) or (via a charged T boson) a charge-related $\chi = \pm 1$ (This correlates with TA2-and-TA1.). Interactions with W bosons preserve matter and/or antimatter. Interactions with charged tweaks (or, charged T bosons) convert matter quarks into antimatter quarks or antimatter quarks into matter quarks.

1R elementary fermions have zero charge and, we think, zero mass.

For each of 1N, 1C, 1R, and 1Q, the TA4-and-TA3 oscillators show a result that balances (in the sense of $A^{ALG} = 0$) the SA3-and-SA4 entry. Three TA4-and-TA3 instance-related generators pertain. The The TA8-and-TA7 oscillators show a result that balances (in the sense of $A^{ALG} = 0$) the SA7-and-SA8 entry. Two TA8-and-TA7 instance-related generators pertain. Six is the multiplicative product of three and two.

For each of 2U, 2W, and 2T, an eight-instance symmetry pertains, based on either $\kappa_{-1,-1,-1}$ (for 2U, or gluons) or $\kappa_{0,0,0}$ (for 2W and 2T). This symmetry should not be conflated with a 48-instance symmetry that correlates with each of the W boson and the $T^{\pm}$ boson. (See remarks related to table VIIa.)

We pursue the topic of instance symmetries. Six-generator symmetries for 1R and 1Q plus eight-generator symmetries for 2U and 2T suggest that, at least mathematically, 48 instances of hadron-like particles pertain. We symbolize hadrons via 1Q⊗2U. Here, we recognize that (based on the notion that complementary QFT need not necessarily include interaction vertices in which a gluon becomes two gluons) that, for some modeling, 1Q⊗2U hadrons may be considered as including virtual 1R particles. Perhaps, an appropriate statement is that hadrons contain 1Q valence fermions and do not contain 1R valence fermions. Possibly, 1R⊗2U hadron-like particles exist and boson 1R⊗2U particles serve some roles that people might correlate with roles of (hypothetical elementary particles called) axions. Our work does not point to an axion-like elementary particle. Possibly, 1Q⊗2T hadron-like particles exist and serve some roles that people might correlate with roles of (hypothetical elementary particles called) WIMPs (or, weakly interacting massive particles). (For PR001INe models, our work does not point to a WIMP-like elementary particle. For PR006INe models, PR0048INe models, and PR288INe models, people might consider some components of dark matter to correlate with the term WIMP. See table Va.)

Table IV shows, for the H-family and for G-family sub-families, traditional physics representations for excitations. Here, n denotes the number of excitations for a state. Here, $n = 0$ correlates with a ground state and n is a non-negative integer. In traditional physics, 2G correlates with electromagnetism and $S = 1$. 4G might correlate with $S = 2$ and gravitation. Other ΣG might correlate with long-range interactions other that electromagnetism and gravitation.

Table V: Models and abbreviations regarding dark matter and dark energy

(a) Models regarding dark matter and dark energy density

Model	Complementary physics theory		Traditional physics theory	
	Dark matter	Dark energy density	Dark matter	Dark energy density
PR001INe	Dark matter may be hadron-like particles (other than hadrons).	Dark energy density correlates with notions such as vacuum energy, vacuum fluctuations, or quintessence.	Dark matter may be axions and/or WIMPs.	Dark energy density correlates with notions such as vacuum energy, vacuum fluctuations, or quintessence.
PR006INe	Dark matter is mostly five somewhat copies of ordinary matter, plus some hadron-like particles (other than hadrons).	Ditto.	(Not applicable)	
PR048INe	Ditto.	Dark energy density correlates with 42 other somewhat copies of ordinary matter.	(Not applicable)	
PR288INe	Ditto.	Ditto.	(Not applicable)	

(b) Some abbreviations regarding ordinary matter, dark matter, and dark energy stuff.

Abbreviation and phrase
• OM denotes ordinary matter.
○ OM|DI denotes ordinary matter density or impact.
○ OM|DI|ST denotes stuff that people correlate with the term ordinary matter.
○ OM|ENS denotes the ordinary matter ensemble.
○ OM|ENS|ST denotes stuff correlating with the OM|ENS.
• DM denotes dark matter.
○ DM|DI denotes dark matter density or impact.
○ DM|DI|ST denotes stuff that people correlate with the term dark matter.
○ DM|ENS denotes one or more dark matter ensembles.
○ DM|ENS|ST denotes stuff correlating with one or more DM|ENS.
• OM|ENS|ST-DM|DI denotes stuff, correlating with the OM|ENS, for which people interpret effects as being DM|DI.
• OMDM denotes ordinary matter plus dark matter.
○ The symbols ..|DI, ..|DI|ST, ..|ENS, and ..|ENS|ST pertain.
• DE denotes dark energy stuff. (DE does not denote dark energy forces.)
○ The symbols ..|DI, ..|DI|ST, ..|ENS, and ..|ENS|ST pertain.

B. Instance symmetries and PRnnnINe models

Work, regarding instance-related symmetries, above suggests possibilities for six instances of some elementary fermions, eight instances of some elementary bosons, and 48 instances of hadron-like particles.

Table V describes four modeling cases and defines acronyms. To some extent, it can be useful to think of a PR006INe universe as including six PR001INe (somewhat) sub-universes that gravity unites. To some extent, it can be useful to think of a PR048INe universe as consisting of eight PR006INe (somewhat) sub-universes.

For each case that table Va shows, the characters PRnnnINe denote that notion that the number of physics-relevant (or, PR) instances (or, IN) of the electron (or, e) is nnn (or, 1, 6, 48, or 288). The PR048INe case includes the notion the dark energy densities correlate with stuff and not necessarily with fluctuations. Above, we pointed to possibilities for 48 copies of electrons (and other charged elementary particles) and for 48 copies of hadron-like particles. PR288INe correlates with possible interactions (that, with respect to our universe, would not conserve energy at the instant of the big bang for our

Table VI: Instance symmetries and interactions for elementary long-range forces

$\Sigma\Phi\Gamma$	σ	TA-side	$\vert\leftarrow$ ..	..	TA	.. $\rightarrow\vert$	$\vert\leftarrow$ ..	..	SA	..	.. $\rightarrow\vert$
		symmetry	6,5	4,3	2,1	0	0	1,2	3,4	5,6	7,8
2G2	$+1$	ToBeDet				0	-1	$\pi_{0,@_0}$			
4G4	$+1$	$SU(3)$			0,0	0	-1	A0	$\pi_{0,@_0}$		
ΣG24	$+1$	ToBeDet				0	-2	$\pi_{0,@_0}$	$\pi_{0,@_0}$		
6G6	$+1$	$SU(5)$		0,0	0,0	0	-1	A0	A0	$\pi_{0,@_0}$	
ΣG26	$+1$	$SU(3)$			0,0	0	-2	$\pi_{0,@_0}$	A0	$\pi_{0,@_0}$	
ΣG46	$+1$	$SU(3)$			0,0	0	-2	A0	$\pi_{0,@_0}$	$\pi_{0,@_0}$	
ΣG246	$+1$	ToBeDet				0	-3	$\pi_{0,@_0}$	$\pi_{0,@_0}$	$\pi_{0,@_0}$	
8G8	$+1$	$SU(7)$	0,0	0,0	0,0	0	-1	A0	A0	A0	$\pi_{0,@_0}$
ΣG28	$+1$	$SU(5)$		0,0	0,0	0	-2	$\pi_{0,@_0}$	A0	A0	$\pi_{0,@_0}$
ΣG48	$+1$	$SU(5)$		0,0	0,0	0	-2	A0	$\pi_{0,@_0}$	A0	$\pi_{0,@_0}$
ΣG68	$+1$	$SU(5)$		0,0	0,0	0	-2	A0	A0	$\pi_{0,@_0}$	$\pi_{0,@_0}$
ΣG248	$+1$	$SU(3)$			0,0	0	-3	$\pi_{0,@_0}$	$\pi_{0,@_0}$	A0	$\pi_{0,@_0}$
ΣG268	$+1$	$SU(3)$			0,0	0	-3	$\pi_{0,@_0}$	A0	$\pi_{0,@_0}$	$\pi_{0,@_0}$
ΣG468	$+1$	$SU(3)$			0,0	0	-3	A0	$\pi_{0,@_0}$	$\pi_{0,@_0}$	$\pi_{0,@_0}$
ΣG2468	$+1$	ToBeDet				0	-4	$\pi_{0,@_0}$	$\pi_{0,@_0}$	$\pi_{0,@_0}$	$\pi_{0,@_0}$

universe) between our universe and up to five other universes. (Reference [2] provides details about such non-conservation of energy.)

The word ensemble denotes all span-1 $\sigma = +1$ elementary particles and all hadron-like particles. The set of span-1 $\sigma = +1$ elementary particles varies with the choice of PRnnnINe model. (See tables VIIa and IX.)

Regarding the numbers one, six, and 48, we note that $1 = 48/48$, $6 = 48/8$, and $48 = 48/1$ and that (regarding denominators) 48 is the number of generators of $SU(7)$ and eight is the number of generators of $SU(3)$. Also, 288 is the number of generators of $SU(17)$ and $288/48 = 6$.

Table Vb shows some abbreviations that we use regarding ordinary matter, dark matter, and dark energy. The notion that, for PR006INe, PR048INe, and PR288INe models, some stuff that measures as dark matter correlates with the five DM ensembles and some stuff that measures as dark matter correlates with the OM ensemble leads to needs to define concepts carefully.

C. Elementary long-range forces, their instances, and their spans

Some aspects of physics regarding the elementary particles and elementary long-range forces to which table III alludes are context sensitive. For example, within hadrons, W-boson interaction vertices do not necessary conserve fermion generation; however, isolated interactions involving the W boson conserve fermion generation. Also, the photon modes of a cavity resonator are not the same as the photon modes of free-ranging photons.

We explore long-range aspects of G-family physics. We emphasize SDF (or, spatial dependence of force) and instances of forces.

Table VI alludes to all the ALG solutions that we suggest have physics-relevance regarding elementary long-range forces. This table summarizes information about instances and interactions. Each row in the table pertains to ground states and comports with ALG double-entry bookkeeping. Each A0 denotes $@_0@_0$ and correlates with an oscillator pair that does not excite. For each SA-side $\pi_{0,@_0}$, a first conceptual excitation can be either to $n_{SA\text{odd}} = 1$ and $n_{SA\text{even}} = 0$, which correlates with left-circular polarization, or to $n_{SA\text{odd}} = 0$ and $n_{SA\text{even}} = 1$, which correlates with right-circular polarization. (We use the two-word phrase conceptual excitation because we are discussing symmetries that correlate with, at least, ground states and because interactive excitation correlates with table IV.) For each ΣGΓ, the number of SA-side oscillator pairs that correlate with conceptual excitation is $-n_{SA0}$. Regarding the Σ in ΣGΓ, Σ denotes both $2S$ and the absolute value of the arithmetic combination across excitable SA-side oscillators of $+2S_{\text{oscillator}}$ for each left-circular excitation and $-2S_{\text{oscillator}}$ for each right-circular excitation. For example, for ΣG24, Σ can be two, as in $|-2+4|$, or six, as in $|+2+4|$. For each relevant TA-side oscillator, $n_{TA..} = 0$. In the column labeled TA-side symmetry, we show (when applicable) an instance symmetry based on relevant TA-side oscillators. The characters ToBeDet abbreviate the phrase to be determined.

We do not extend table VI to include more items. We think that the notion that, for $\Sigma = 10$ and $\Gamma = \Sigma$, ΣGΓ would correlate with $SU(9)$ correlates with such a limit. The number of generators for $SU(9)$, $SU(7)$, $SU(5)$, and $SU(3)$ is, respectively, 80, 48, 24, and eight. Eight divides each of 24 and 48 evenly and that 24 divides 48 evenly. Neither 24 nor 48 divides 80 evenly. Work below regarding spans might run into difficulties if $SU(9)$ symmetry pertains. Other concepts correlate with the limit regarding table VI. One such concept correlates with a relationship between the ratio of the tauon mass to the electron mass and the ratio, for two electrons, of electromagnetic repulsion to gravitational attraction. (See discussion regarding equation (37).)

Table VII shows all G-family solutions that table VI lists. For each row in table VIIa, the symbol Γ correlates with the corresponding list of one, two, three, or four even positive integers that the first column in the table shows. The column labeled count shows a number of solutions. For each GΓ, the number of solutions is $2^{|n_{SA0}|}$. Paralleling uses in traditional physics theory of terms such as monopole and dipole, the column labeled interaction seems to provide a useful characterization. SDF abbreviates the term spatial dependence of force. The SDF column shows a characteristic of the force that correlates with the solution. The characteristic correlates with uses, in Galilean-Newtonian models, of terms such monopole and dipole. Here, r denotes the distance between the appropriate centers of two interacting entities. We assume that appropriate treatments for, for example, special relativity models and general relativity models, can deal with relevant concepts such as concepts correlating with the not infinite speed of light. People also can arrive at each result for SDF as follows. An oscillator pair for which, in table VI, $\pi_{0,@_0}$ pertains correlates with a square of potential that correlates with r^{-2}. For a list Γ with n elements, the square of the overall potential correlates with r^{-2n}, the potential correlates with r^{-n}, and SDF correlates with r^{-n-1}. For SDF of r^{-3}, the interaction that, in effect, the solution intermediates, has dipole-like characteristics. An SDF of r^{-3} dovetails with traditional notions of dipole. Information in the span column and the TA symmetry column reflects PR048INe modeling. The multiplicative product of the span and the number of TA-symmetry generators is 48. (The number of generators equals a number of instances.) This work reflects the PR006INe notion that most dark matter is five copies of (approximately) ordinary matter, that 4G4 correlates with gravity, and the PR006INe notion that each instance of 4G4 interacts with six instances of (for example) electrons. Regarding the column labeled TA symmetry, we address the notion of ToBeDet in table VI. An instance of traditional physics (long-range) photons interacts with only one instance of each charged elementary fermion. We assume that, for 2G2, a span of one and a TA-side symmetry of $SU(7)$ pertains. Later, we note that 2G24 correlates with interactions with elementary fermion nominal magnetic dipole moments. (See table VIII.) Based on such, we assume that, for each ToBeDet in table VI, a span of one and a TA-side symmetry of $SU(7)$ pertains. Table VIIb reorganizes, based on spin, items in table VIIa for which $2S \geq 2$.

For each GΓ for which the solution count is three or seven, table VII reflects a notion that a mathematically possible solution for which Σ equals zero is not G-family physics-relevant because the solution would correlate with $S = 0$. Such a solution would correlate, in physics, with possible non-zero longitudinal polarization. Regarding G-family forces, we de-emphasize arithmetic results for ΣG for which $\Sigma = 0$. We suggest that 0G246 correlates with the Z boson and the T^0 boson (or, zero-charge tweak), 0G268 correlates with the W boson and the $T^{\pm}$ boson (or, non-zero-charge tweak), and 0G2468 correlates with the Higgs boson. If such is true, then table VIIa provides support for the notion that charged elementary bosons have spans of one and that the spin-1 zero-charge non-zero-mass elementary bosons have spans of six. Possibly, some aspects of theory are invariant with respect to a choice between the Higgs boson having a span of 48 (as might be suggested by table IV) or having a span of one (as might be suggested by table VIIa.) Possibly, a construct that we can label as 0G$\emptyset$ correlates with 2U solutions. (See reference [2].) Possibly, modeling that we have not developed could provide further insight regarding, in effect, theoretical unification for all elementary bosons and all elemental long-range forces.

Table VIII discusses modeling related to electromagnetism and gravity. The table could make essentially similar points about bar magnets as the table makes about the earth. (In general, 2G2 intermediates interactions based on the charges of interacting objects and on motions of those charged objects. But, we have yet to introduce motion into our discussion.)

Tables IX and X summarize information based on mathematics solutions that correlate with the G family. The tables use parentheses (that is, (..)) to call attention to solutions that seem to correlate with physics-relevant forces other than G-family forces. The forces other than G-family forces are the strong interaction; the weak interaction; and, to the extent people categorize interactions mediated by the Higgs boson separately from the weak interaction, interactions mediated by the Higgs boson (or, H^0). Interactions mediated by T-family bosons correlate with 0G246 and 0G268. The acronym CHAR denotes the net charge of an object. The symbol q denotes net charge. The symbol m denotes rest mass of an object. (Technically, regarding elementary fermions, 4G4 interacts with generation.) BNUM denotes baryon number. The symbol B denotes baryon number. (G-family interactions correlating with span-2 pertain only to objects that include more than one elementary particle. Baryons are not elementary particles. The concept of baryon number pertains for quarks, as well as for baryons, which include quarks.) WHCH denotes weak hypercharge. The symbol Y_{W} denotes weak hypercharge. More generally, the acronym WHCHCH correlates with aspects of the traditional physics topics of WHCH, handedness,

Table VII: G-family solutions

(a) Solutions, organized by SDF

ΣΦΓ	Σ=2S	S	Count	Interaction	SDF	n_{SA0}	TA-side symmetry	Span
ΣG2	2	1	1	monopole	r^{-2}	-1	$SU(7)$	1
ΣG4	4	2	1	monopole	r^{-2}	-1	$SU(3)$	6
ΣG6	6	3	1	monopole	r^{-2}	-1	$SU(5)$	2
ΣG8	8	4	1	monopole	r^{-2}	-1	$SU(7)$	1
ΣG24	2, 6	1, 3	2	dipole	r^{-3}	-2	$SU(7)$	1
ΣG46	2, 10	1, 5	2	dipole	r^{-3}	-2	$SU(3)$	6
ΣG68	2, 14	1, 7	2	dipole	r^{-3}	-2	$SU(5)$	2
ΣG26	4, 8	2, 4	2	dipole	r^{-3}	-2	$SU(3)$	6
ΣG48	4, 12	2, 6	2	dipole	r^{-3}	-2	$SU(5)$	2
ΣG28	6, 10	3, 5	2	dipole	r^{-3}	-2	$SU(5)$	2
ΣG248	2, 6, 10, 14	1, 3, 5, 7	4	quadrupole	r^{-4}	-3	$SU(3)$	6
ΣG468	2, 6, 10, 18	1, 3, 5, 9	4	quadrupole	r^{-4}	-3	$SU(3)$	6
ΣG246	0	-	1	-	-	-	-	-
"	4, 8, 12	2, 4, 6	3	quadrupole	r^{-4}	-3	$SU(7)$	1
ΣG268	0	-	1	-	-	-	-	-
"	4, 12, 16	2, 6, 8	3	quadrupole	r^{-4}	-3	$SU(3)$	6
ΣG2468	0	-	1	-	-	-	-	-
"	4, 4, 8, 8,	2, 2, 4, 4,	4	octupole	r^{-5}	-4	$SU(7)$	1
"	12, 16, 20	6, 8, 10	3	"	"	"	"	"

(b) Solutions for which $2S \geq 2$, organized by spin

$\Sigma = 2S$	S	Monopole (SDF $= r^{-2}$)	Dipole (SDF $= r^{-3}$)	Quadrupole (SDF $= r^{-4}$)	Octupole (SDF $= r^{-5}$)	Number of solutions (The sum is 37.)
2	1	2G2	2G24, 2G46, 2G68	2G248, 2G468		6
4	2	4G4	4G26, 4G48	4G246, 4G268	4G2468a, 4G2468b	7
6	3	6G6	6G24, 6G28	6G248, 6G468		5
8	4	8G8	8G26	8G246	8G2468a, 8G2468b	5
10	5		10G28, 10G46	10G248, 10G468		4
12	6		12G48	12G246, 12G268	12G2468	4
14	7		14G68	14G248		2
16	8			16G268	16G2468	2
18	9			18G468		1
20	10				20G2468	1

chirality, and/or helicity. In the table, uses of g and α correlate with notation from Standard Model physics and with results regarding charged leptons. The symbol g correlates with the phrase nominal magnetic dipole moment. The symbol α denotes the fine-structure constant. Some measurements of the depletion of starlight emitted long ago are based on atomic hyperfine structure and may dovetail with observations correlating with the 2G68 solution. (Regarding measurements, see reference [1].) Solutions 4G4, 4G48, 4G246, 4G2468a, and 4G2468b correlate with gravity and dark energy forces. Measurements of increasing rates expansion of the universe, which pertain to the most recent few billion years of the evolution to date of the universe, may dovetail with observations correlating with the 4G48 solution and the notion that 4G48 correlates with at least net repulsion, if not some repulsion and never attraction. (Regarding measurements, see references [9] and [11].) Measurements of decreasing rates expansion of the universe, which pertain to a previous multi-billion years of evolution of the universe, may dovetail with observations correlating with the 4G246 solution and the notion that 4G246 correlates with at least net attraction, if not some attraction and never repulsion. (Regarding measurements, see references [3] and [12].) An earlier era of increasing rates of expansion may dovetail with the 4G2468a and 4G2468b solutions and the notion that (at least together) the 4G2468a and 4G2468b solutions correlate with net repulsion, if not some repulsion and never attraction. These tables do not address the topic of SDF for weak interaction forces. Discussion related to equation (44) correlates the range of a weak interaction boson inversely with the mass of the boson. The symbol $\gamma 2$ correlates with anomalous moment calculations. Our work offers the possibility of modeling anomalous moments via G-family aspects correlating with spins greater than one. The columns labeled span pertain for the models PR006INe, PR048INe, and PR288INe. For PR001INe modeling, each span is one.

Table VIII: Some modeling facets that correlate with electromagnetism and gravity

Aspect	Discussion	G-family solutions
Electromagnetism ...		
	• Regarding the earth, it could be appropriate to model at least three aspects of electromagnetism - one monopole aspect, one dipole aspect, and one quadrupole aspect.	
	∘ The earth might have a net charge and therefore a non-zero monopole effect.	2G2
	∘ The earth has a non-zero magnetic dipole moment, as evidenced by people's use of compasses and by the existence of van Allen belts.	2G24
	∘ The earth's axis of rotation does not equal the axis people associate with the magnetic dipole moment. An observer away from the earth can detect a quadrupole-like effect based on the rotation of the axis of dipole moment relative to a perceived-as-static axis of rotation for the earth. The word precession pertains.	2G248
	• Regarding an electron, it could be appropriate to model at least three aspects of electromagnetism - one monopole aspect, one dipole aspect, and one quadrupole aspect.	
	∘ An electron has charge as a monopole aspect.	2G2
	∘ An electron has magnetic moment as a dipole aspect.	2G24
	∘ For an electron, Larmor precession correlates with a quadrupole aspect.	2G248
	• Regarding any elementary fermion (including an electron), it can be appropriate to model yet other (beyond charge, nominal magnetic moment, and the possible quadrupole aspect) aspects of electromagnetism.	
	∘ Anomalous magnetic dipole moment provides an example.	$\gamma 2$ †
Gravitation ...		
	• Regarding almost any object, it could be appropriate to model at least the following two aspects of gravitation. We correlate 4G48 (along with 4G246, 4G2468a, and 4G2468b) with the phrase gravity and/or dark energy forces.	
	∘ A monopole aspect that people might correlate with mass.	4G4
	∘ A dipole aspect that people might correlate with rotation.	4G48
Relationships between electromagnetism and gravitation ...		
	• It might be difficult to develop comprehensive models that completely separate a concept of electromagnetism from a concept of gravitation. The term V_{-2} and its possible applicability to either electromagnetism or gravity hints at this difficulty. (See table II and related discussion about unification of forces.) The concept of anomalous moments supports notions of such difficulty.	
	∘ Anomalous magnetic dipole moment.	6G24$\in \gamma 2$ †
	∘ Anomalous gravitational dipole moment.	6G24$\in \gamma 4$ †

† Regarding $\gamma 2$ and $\gamma 4$, see table IX.

Tables IX and X point to the following concepts. Solutions $\Sigma \mathrm{G}\Gamma$ for which $\Sigma \in \Gamma$ correlate with concepts of nominal long-range forces correlating with, for example, electromagnetism, gravitation, and dark energy forces. Solutions $\Sigma \mathrm{G}\Gamma$ for which $\Sigma \notin \Gamma$ and $\Sigma \neq 0$ correlate with anomalous moments with regard to each $\gamma \in \Gamma$. Some solutions, such as 2G68, $\Sigma \mathrm{G}\Gamma$ for which $\Sigma \notin \Gamma$ and $\Sigma \neq 0$ correlate only with interactions involving transitions within multi-component objects.

Perhaps, regarding elementary long-range forces, a good use of the word photon correlates with all $2\mathrm{G}\Gamma$ for which $2 \in \Gamma$. If so, in PRnnnINe models other than PR001INe, photons interact with DM|ENS|ST. In PRnnnINe models other than PR001INe, the 2G68 solution correlates with a means for DM|ENS|ST to interact with photons emitted by OM|ENS|ST. Perhaps, a good use of the word graviton correlates with all $4\mathrm{G}\Gamma$ for which $4 \in \Gamma$. If so, gravitons correlate with both monopole gravity and dark energy forces.

Table IX: G-family monopole and dipole solutions, organized by SDF

Known			ΣΦΓ	S	SDF	Span
Phenomena (In effect, the solution correlates or interacts with ...)	Example symbol	Use other than ΣG	($\Sigma = 2S$)			(PRj.., $j \geq 006$)
(Strong interaction forces)		(2U)	("0G0")	(1)	(r^0)	(6)
CHAR {or, charge}	q		2G2	1	r^{-2}	1
Gravity, rest energy	m		4G4	2	r^{-2}	6
BNUM {or, baryon number}	B		6G6	3	r^{-2}	2
WHCH {or, weak hypercharge}	Y_{W}		8G8	4	r^{-2}	1
Nominal magnetic dipole moment	$g \approx 2$		2G24	1	r^{-3}	1
Anomalous magnetic dipole moment	$\propto \alpha^2$	$\gamma 2$	6G24	3	r^{-3}	1
			2G46	1	r^{-3}	6
			10G46	5	r^{-3}	6
Hyperfine structure {atomic states}			2G68	1	r^{-3}	2
			14G68	7	r^{-3}	2
Anomalous magnetic dipole moment	$\propto \alpha^1$	$\gamma 2$	4G26	2	r^{-3}	6
Anomalous magnetic dipole moment	$\propto \alpha^3$	$\gamma 2$	8G26	4	r^{-3}	6
Gravity and/or dark energy forces			4G48	2	r^{-3}	2
			12G48	6	r^{-3}	2
Anomalous magnetic dipole moment	$\propto \alpha^2$	$\gamma 2$	6G28	3	r^{-3}	2
Anomalous magnetic dipole moment	$\propto \alpha^4$	$\gamma 2$	10G28	5	r^{-3}	2

Table X: G-family quadrupole and octupole solutions, organized by SDF

Known			ΣΦΓ	S	SDF	Span
Phenomena (In effect, the solution correlates or interacts with ...)	Example symbol	Use other than ΣG	($\Sigma = 2S$)			(PRj.., $j \geq 006$)
Precessing magnetic dipole moment			2G248	1	r^{-4}	6
			6G248	3	r^{-4}	6
			10G248	5	r^{-4}	6
			14G248	7	r^{-4}	6
Precessing dipole moment {?}			2G468	1	r^{-4}	6
			6G468	3	r^{-4}	6
			10G468	5	r^{-4}	6
			18G468	9	r^{-4}	6
(Weak interaction forces)		(Z, ∈2W)	(0G246)	(1)	-	(6)
(Weak interaction forces)		(T^0, ∈2T)	"	"	"	"
Gravity and/or dark energy forces			4G246	2	r^{-4}	1
			8G246	4	r^{-4}	1
			12G246	6	r^{-4}	1
(Weak interaction forces)		(W, ∈2W)	(0G268)	(1)	-	(1)
(Weak interaction forces)		($\mathrm{T}^{\pm}$, ∈2T)	"	"	"	"
			4G268	2	r^{-4}	6
			12G268	6	r^{-4}	6
			16G268	8	r^{-4}	6
(Weak interaction forces)		(H^0, ∈0H)	(0G2468)	(0)	-	(1)
Gravity and/or dark energy forces			4G2468a	2	r^{-5}	1
Gravity and/or dark energy forces			4G2468b	2	r^{-5}	1
			8G2468a	4	r^{-5}	1
			8G2468b	4	r^{-5}	1
			12G2468	6	r^{-5}	1
			16G2468	8	r^{-5}	1
			20G2468	10	r^{-5}	1

Table XI: Explanations for inferred ratios of density of dark matter to density of ordinary matter or inferred ratios of impact of dark matter to impact of ordinary matter

---

The ratio .. of amount or effects of dark matter to amount or effects of ordinary matter pertains regarding .. .

1. People infer the ratio based on measurements of .. .
2. We offer an explanation of .. .

Five-plus to one ($\gtrapprox 5:1$), regarding stuff in the observable universe.

1. CMB (or, cosmic microwave background) radiation. [4]
2. The ratio correlates with the ratio of five DM|ENS to one OM|ENS, plus the existence of OM|ENS|ST-DM|DI.

Five-plus to one ($\gtrapprox 5:1$), regarding stuff in some galaxy clusters.

1. Gravitational lensing. [7] and [10]
2. The ratio correlates with the ratio of five DM|ENS to one OM|ENS, plus the existence of OM|ENS|ST-DM|DI.

Zero to one or zero-plus to one ($\gtrapprox 0:1$), regarding long-ago states of some then newly formed galaxies.

1. Velocities of motion of stars within galaxies (or, galaxy rotation curves). [5]
2. The ratio correlates with a scenario for the formation and early evolution of some galaxies. (See tables XII and XIII.)

Between zero to one ($\gtrapprox 0:1$) and one to one ($< 1:1$), regarding a galaxy that has at less dark matter stuff than ordinary matter stuff and possibly is nearly entirely ordinary matter ($\gtrapprox 0:1$).

1. Velocities of motion of stars within the galaxy (or, galaxy rotation curve). [14]
2. The ratio correlates with a lack of accumulation of stuff correlating with dark matter ensembles. (See tables XII and XIII.)

Somewhat less than four to one ($\lessapprox 4:1$), regarding some galaxies.

1. Gravitational lensing, by such galaxies, of light that passes near each galaxy. [6]
2. The ratio correlates with the ratio of five DM|ENS to one OM|ENS; effects on one DM|ENS|ST, of 4G48 early during galaxy formation; and eventual accumulation of DM|DI|ST correlating with the other four DM|ENS. (See tables XII and XIII.)

One to one ($\approxeq 1:1$), regarding the absorption (by stuff in the observable universe) of one frequency of light, mostly emitted long ago by stars.

1. Depletion of starlight. [1]
2. The ratio correlates with absorption via interactions mediated by 2G68. 2G68 has a span of 2. Dark matter hydrogen atom analogs provide for half of the absorption. (See discussion related to table IX.)

One to somewhat more than zero ($1 :> 0$), regarding some dark matter galaxies.

1. Amount of light emitted by some galaxies with few visible stars. [13]
2. The ratio correlates with a galaxy formation scenario for DM|ENS-centric galaxies that parallels a galaxy formation scenario for OM|ENS-centric galaxies. (See tables XII and XIII.)

D. Explanations for ratios of dark matter effects to ordinary matter effects

PR006INe explains each of the DM|DI-to-OM|DI (or, dark matter density or impact to ordinary matter density or impact) ratios that table XI shows. (Regarding symbols such as DM|DI, see table Vb.) To the extent the first two ratios that table XI shows exceed five to one, arc-based or tweak-based hadron-like particles provide for the excess. PR048INe models and PR288INe models comport with PR006INe results regarding DM|DI-to-OM|DI ratios.

E. Dark energy forces, the cosmology timeline, the rate of expansion of the universe, and a scenario for galaxy formation and evolution

We think that the events and eras that tables XII and XIII show comport with known data; reasonably verified or likely aspects of traditional physics theory; and each of PR006INe models, PR048INe models, and PR288INe models. Some aspects do not necessarily dovetail well with PR001INe models. For the sake of simplicity, we discuss PR006INe modeling. For the sake of simplicity, we assume that a concept of big bang pertains. For the sake of simplicity, we assume that a concept of an inflationary epoch pertains. This discussion de-emphasizes various astrophysics objects and phenomena. De-emphasized objects include quasars and black hole jets. De-emphasized phenomena include collisions between galaxies. Some items in tables XII and XIII correlate with known or hypothesized happenings that traditional physics models discuss. Regarding each of some of the items tables XII and XIII show, discussion in the tables features notions that our complementary physics theory suggests and traditional physics theory does not include.

Table XIII suggests key steps in the early evolution of galaxies that form early in the evolution of the universe. People might assume that some traditional physics theory does not comport with the scenario we suggest. An example of pushback might feature concerns that some traditional models suggest that a significant role for SIDM (or, self-interacting dark matter) in galaxy formation would lead to galaxies that are smaller than actual galaxies. Perhaps, people might view such pushback cautiously. Perhaps, such models include assumptions not supported by observations. Likely, such models do not correlate with range of G-family phenomena we suggest. Possibly, observations support some aspects of such G-family phenomena. The concept that, within an ordinary matter intensive galaxy, dark matter can form a halo may constitute observation of effects correlating with the 2G248 solution and/or the 2G468 solution. For example, a precessing magnetic dipole moment (correlating with the quadrupole 2G248 solution) could correlate with the ordinary matter centric core of the galaxy.

F. Dark energy densities

Equation (35) shows an inferred ratio of present density of the universe of DE (or, dark energy stuff) to present density of the universe of OMDM (or, ordinary matter plus dark matter). (See table Vb and reference [4].) We know of no inferences that would not comport with a steady increase, regarding such DE|DI:OMDM|DI density ratios, from approximately zero, with time since somewhat after the big bang.

$$\text{inferred } \Omega_\Lambda/(\Omega_\mathrm{b}+\Omega_\mathrm{c}) \approx 2.2 \tag{35}$$

For PR001INe modeling and PR006INe modeling, traditional physics thinking might correlate with such ratios. (See table Va.) For PR048INe modeling and PR288INe modeling, such ratios might correlate with effects of interactions between OMDM|ENS|ST and DE|ENS|ST. These interactions would involve, in effect, transfers of information via elementary particles with PR048INe spans of eight or 48. Here, eight equals 48/6 and correlates with six-instance instance symmetry that we correlate with $SU(2)\times U(1)$ and with elementary fermions. We suggest that 1N particles and 1R particles have PR048INe spans of eight. We suggest the possibility that 0H particles (or, Higgs bosons) have a span of 48. Thus, we suggest interactions that might correlate with inferred values of $\Omega_\Lambda/(\Omega_\mathrm{b}+\Omega_\mathrm{c})$.

G. Motion, kinematics conservation laws, and approximate conservation of fermion generation

We now include theory regarding kinematic conservation laws and regarding conservation of generation.

We consider G-family solutions and results that table VIIa shows. In traditional physics, Poincare-group symmetries correlate with modeling via special relativity. Poincare-group symmetries feature one S1G (or, one-generator) TA-side symmetry and three $SU(2)$ three-generator SA-side symmetries. The S1G symmetry correlates with conservation of energy and the $SU(2)$ symmetries correlate with conservation of momentum, conservation of angular momentum, and boost symmetry. We correlate the S1G symmetry with the oscillator TA0. We correlate conservation of energy with the TA-side symmetries to which tables VIIa and VI allude. For example, for 2G2, 2G24, and 8G8, conservation of energy correlates with the three TA-side oscillator pairs TA6-and-TA5, TA4-and-TA3, and TA2-and-TA1 and the oscillator TA0. For kinematics modeling and in conjunction with $A^{ALG}=0$, we consider three SA-side oscillator pairs and the notion that each pair correlates with an $SU(2)$ symmetry. Regarding 4G4 (or, monopole gravity) and other G-family solutions for which a TA-side symmetry of $SU(3)$ pertains, based on the notion that conservation of energy correlates with a TA-side $SU(7)$ symmetry, we consider for

Table XII: Some elements for a cosmology timeline, assuming PR006INe modeling and some other assumptions (first of two tables)

Timeline elements (Elements are not necessarily mutually exclusive regarding happenings. Elements do not necessarily occur strictly in the order we show.)

1. A big bang occurs. For an instant (with respect to modeling based on space-time coordinates), conservation of energy does not pertain. (See reference [2].) Energy populates G-family states 2G2, ΣG24, ΣG246, ΣG2468a, and/or ΣG2468b. Population occurs somewhat equally with respect to relevant instances.
2. For each relevant instance and for each type of elementary fermion, fermion pair production occurs. Such pair production converts some energy correlating with G-family states into equal amounts of matter elementary fermions and antimatter elementary fermions. Elementary fermions for which $\sigma = -1$, that is quarks and arcs, exist in seas.
3. The following occur. (We are not certain regarding temporal overlaps and/or ordering regarding these happenings.)
   (a) Seas undergo phase transitions, leading to predominance of 1Q⊗2U hadron-like particles and possibly other hadron-like particles such as 1R⊗2U particles. Each relevant ensemble correlates with a set of such particles.
   (b) Interactions involving charged 2W particles convert, for each relevant ensemble, most antimatter charged leptons into antimatter neutrinos and convert a similar amount of matter neutrinos into matter charged leptons. (For ensembles other than the ordinary matter ensemble, this statement defines the two one-word terms matter and antimatter.) Interactions involving charged 2T particles convert, for each relevant instance, most antimatter quarks into matter quarks. People correlate with the result of this process, the term baryon asymmetry.
      i. For PR006INe models (but not necessarily for PR048INe models or PR288INe models), to the extent neutrinos behave as Dirac fermions, neutrino asymmetry occurs and antimatter neutrinos dominate the directly observable cosmic neutrino background.
   (c) An inflationary epoch occurs.
4. Expansion, within each relevant ensemble and based on repulsive SDF r^{-5} G-family forces, pertains regarding then current and later extant objects. Such objects range from at least as big as hadron-like particles to at least as big as galaxy filaments.
5. Clumping, within each relevant ensemble and based on attractive SDF r^{-4} G-family forces, pertains regarding then current and later extant objects. Smaller objects clump before larger objects clump.
6. Hadron-like particles - singly and/or in small clumps - become potential bases for and/or form atomic nuclei.
7. Atoms form. CMB (cosmic microwave background radiation) begins to propagate freely. The combination of atomic physics and electromagnetism (or, G-family spin-1 phenomena) has entered an era which SDF r^{-2} phenomena dominate. Ordinary matter CMB exhibits little evidence regarding the possible (regarding PR048INe models and PR288INe models) existence dark energy stuff.
8. (See table XIII.)

kinematics two SA-side oscillator pairs, with each pair correlating with $SU(2)$ symmetry. These two pairs correlate with conservation of momentum and conservation of angular momentum, but not with boost symmetry. Regarding the solutions 6G6, ΣG28, ΣG48, and ΣG68, tables VI and VIIa show TA-side $SU(5)$ symmetries. An $SU(7)$ conservation of energy symmetry implies, in effect, the applicability of just one of conservation of momentum and conservation of angular momentum. We think that (somewhat parallel to Mossbauer effect), for 2G68 and the other G-family solutions for which TA-side $SU(5)$ symmetry pertains, the solutions correlate with interactions involving multi-elementary-particle objects. Correlating 2G68

Table XIII: Some elements for a cosmology timeline, assuming PR006INe modeling and some other assumptions (second of two tables)

Timeline elements (Elements are not necessarily mutually exclusive regarding happenings. Elements do not necessarily occur strictly in the order we show.)
1. (See table XII.)
2. Within each relevant ensemble, SDF r^{-4} 4G-solution attractive forces have catalyzed clumping that eventually leads to stars and galaxies that feature stuff from just that ensemble. Regarding a potentially galaxy-sized clump, SDF r^{-3} 4G-solution repulsive forces have repelled stuff correlating with one ensemble.
3. Some stars form. Objects the size of stars have entered an era that correlates with SDF r^{-2} 4G-solution attractive forces. A star features stuff correlating with one ensemble.
4. Some galaxies form. Objects the size of galaxies have entered an era that correlates with SDF r^{-2} 4G-solution attractive forces. A galaxy features stuff correlating with one ensemble.
5. Galaxies evolve. A galaxy can attract, via SDF r^{-2} 4G-solution attractive forces, stuff correlating primarily with its own ensemble and with four other ensembles. Within a galaxy, early black holes correlate primarily with the originally dominant (within the galaxy) ensemble and can accrue stuff correlating with other ensembles.
6. Material that does or will correlate with a galaxy cluster can be equally balanced with respect to six ensembles.
7. The dominant 4G-solution force within a galaxy cluster and between neighboring galaxy clusters becomes the SDF r^{-2} 4G-solution attractive force.
8. The dominant 4G-solution force between galaxy filaments can remain (for some time) SDF r^{-3} 4G-solution repulsive force. People observe continuing acceleration in the rate of expansion of the universe. Overtime, the dominant force between each of two neighboring galaxy filaments becomes the SDF r^{-2} 4G-solution attractive force. Overall, transitions from SDF r^{-3} 4G-solution repulsive force to SDF r^{-2} 4G-solution attractive force occur sooner for pairs of smaller neighboring galaxy filaments than for pairs of larger galaxy filaments.

with hyperfine transitions in hydrogen atoms provides an example. (See tables IX and XI.)

We discuss one way to model aspects of hadron-like particles. For 2U and 2T solutions, oscillators SA0, SA1, and SA2 provide a relevant S1G symmetry (via SA0) and a relevant $SU(2)$ symmetry (via SA1-and-SA2). (See table III.) Paralleling work regarding the G-family and, for example, 4G4, kinematics modeling features two TA-side $SU(2)$ symmetries. For 1Q and 1R, the TA4-and-TA3 oscillator pair correlates with a TA-side $SU(2)$ symmetry. For hadron-like particles, together, the three TA-side $SU(2)$ symmetries, correlate with conservation of momentum, conservation of angular momentum, and boost symmetries. In our work, complementary modeling for a (hypothetically) lone 1Q or 1R particle does not include an ability for that particle to both emit and absorb one (that is, the same) elementary boson. Such modeling correlates with a lack of free-ranging quarks. Emission of a gluon by one quark and absorption of the gluon by another quark correlates with converting a combination of one SA-side S1G symmetry and three TA-side $SU(2)$ symmetries into Poincare-group symmetry (or, one TA-side S1G symmetry and three SA-side $SU(2)$ symmetries).

The above discussion about hadron-like particles suggests possibilities for using models for which neither the dynamics of gluons nor the dynamics of quarks correlates with Poincare-group symmetry (or, special relativity).

We discuss a notion of an approximate symmetry correlating with the term conservation of fermion generation. A sufficiently isolated interaction between an elementary fermion and a W boson conserves the generation of the fermion. For example, emission by an electron (which is a generation-one lepton) of a W^- boson produces a generation-one neutrino. Within a hadron, pairs of interactions mediated by W bosons can lead to a change in generation for a quark. This change in quark generation runs counter to the notion of sufficient isolation. Possibly, sufficiently isolated interactions mediated by bosons that have span-1 in all four PRnnnINe models exhibit conservation of fermion generation. Possibly, interactions mediated by bosons that correlate with span-2 or span-6 in PR006INe models do not exhibit conservation of fermion generation. We note that monopole gravity (or, 4G4) has a span of six in all PRnnnINe models except PR001INe.

IV. DISCUSSION

A. Possible complements to aspects of traditional physics QED, QCD, and QFT; plus anomalous magnetic dipole moments

Results regarding $\gamma 2$ suggest that our work points to a complement to traditional physics QED (or, quantum electrodynamics). (See table IX.) Regarding anomalous magnetic dipole moments, we suggest, for each power of α, a finite number of terms, whereas, for each power of α, traditional QED features a conditionally convergent sum. As yet, our work needs to rely on observations and/or traditional QED to provide information sufficient to determine the strength for each power. However, reference [2] suggests that, given results regarding electrons and muons, people can use our complementary QED to calculate, for a tauon, an α^2 contribution (to the tauon anomalous magnetic dipole moment) that matches a first-order contribution people estimate via traditional QED.

Per discussion above, possibly PDE modeling provides for a complement to QCD (or, quantum chromodynamics) and possibly PDE modeling points to a way to extend QFT (or, quantum field theory) to correlate with existence and properties of elementary particles.

Regarding our complementary QFT, the following statements pertain. An interaction vertex correlates with a point-like value of ν_{SA}. That value can be either $-3/2$ or -1. Across incoming fields, the sum of the values of the field-like ν_{SA} equals the point-like ν_{SA} for the interaction vertex. Across outgoing fields, the sum of the values of the field-like ν_{SA} equals the point-like ν_{SA} for the interaction vertex. For the case of a vertex with point-like $\nu_{SA} = -3/2$, one elementary boson field enters, one elementary fermion field enters, one elementary boson field exits, one elementary fermion field exits, and no more than one of the elementary boson fields correlates with a ground (or, empty) state. For the case of a vertex with point-like $\nu_{SA} = -1$, either one elementary boson enters and one elementary fermion matter-and-antimatter pair exits or one elementary fermion matter-and-antimatter pair enters and one elementary boson exits. Throughout our complementary QFT, an elementary fermion (or one of its direct successor elementary fermions) does not absorb an elementary boson that the elementary fermion created.

Regarding our complementary QFT, such simplicity correlates with the results that table IX shows via the use of the symbol $\gamma 2$ and the appearance of various powers of α. Also, whereas traditional QFT includes interactions in which one boson enters and two bosons leave, our complementary QFT handles such interactions via production of virtual fermion matter-and-antimatter pairs. For example, a gluon can produce a virtual matter-and-antimatter pair of arcs (or, 1R particles) and the pair of arcs can produce one gluon before annihilation of the pair and one gluon correlating with annihilation of the pair. Or, a Higgs boson can decay by first producing a matter-and-antimatter pair of virtual charged leptons.

B. Roles for PDE solutions, regarding suggesting new elementary particles

Table XIV shows PDE solutions germane to the topic of the elementary particles that nature includes.

Table XIVa shows solutions for which $2\nu_{SA}$ is an odd negative integer and $D^*_{SA} = 3$ pertains. For each row in the table, we use the radial component of a D-dimensional solution as the radial solution of a D^*_{SA}-dimensional solution and then add angular coordinates correlating with $D^*_{SA} = 3$ dimensions. For each row in the table, Ω_{SA} and σ pertain to the D-dimensional solution. For each row in the table, the D-dimensional radial solution and the D^*_{SA}-dimensional radial solution share a common value of S and share a common value of $|\Omega_{SA}|$. D^*_{SA}-dimensional solutions for which $\nu = -1/2$ are volume-like. D^*_{SA}-dimensional solutions for which $\nu_{SA} = -3/2$ are point-like. We assume that volume-like solutions correlate with field-like constructs and that point-like solutions correlate with particle-like constructs. The right-most two columns propose correlating families of elementary particles with solutions. The symbol m denotes mass. The C family includes the known charged leptons and no other particles. The N family includes the known neutrinos and no other particles. The 1Q particles include the known quarks and no other particles. We point to the possibility that nature includes 1R particles. We assume that the lack of a volume-like solution that would correlate with 3C and 3N correlates with the notion that the point-like solution that correlates with 3C and 3N is not physics-relevant. Reference [2] discusses reasons correlating with the notion that nature might not include elementary fermions for which $\Sigma \geq 3$. Below,

Table XIV: PDE solutions for all elementary particles other than gluons and G-family entities

(a) Fermion-centric PDE solutions

D^*_{SA}	ν_{SA}	$D^*_{SA}+2\nu_{SA}$	S	Ω_{SA}	σ	D	D	$D+2\nu_{SA}$	$2S+1$	$\Sigma\Phi$	
										$(m \neq 0)$	$(m \approx 0)$
3	$-1/2$	2	$1/2$	$3/4$	$+1$	$(5-4\Omega)/2$	1	0	2	1C	1N
3	$-1/2$	2	$1/2$	$-3/4$	-1	$(5-4\Omega)/2$	4	3	2	1Q	1R
3	$-1/2$	2	$3/2$	$-15/4$	-1	$(5-4\Omega)/2$	10	9	4	(3Q)	(3R)
3	$-1/2$	2	...								
3	$-3/2$	0	$3/2$	$15/4$	$+1$	$(21-4\Omega)/6$	1	-2	4	(3C)	(3N)
3	$-3/2$	0	$1/2$	$3/4$	$+1$	$(21-4\Omega)/6$	3	0	2	1C	1N
3	$-3/2$	0	$1/2$	$-3/4$	-1	$(21-4\Omega)/6$	4	1	2	1Q	1R
3	$-3/2$	0	$3/2$	$-15/4$	-1	$(21-4\Omega)/6$	6	3	4	(3Q)	(3R)
3	$-3/2$	0	...								

(b) Relationships between some PDE parameters for ΣW, ΣH, and ΣT solutions

D^*_{SA}	ν_{SA}	$D^*_{SA}+2\nu_{SA}$	S	Ω_{SA}	σ	D	D	$D+2\nu_{SA}$	$2S+1$	$\Sigma\Phi$
3	-1	1	1	2	$+1$	$3-\Omega$	1	-1	3	2W
3	-1	1	0	0	$+1$	$3-\Omega$	3	1	1	0H
3	-1	1	0	0	-1	$3-\Omega$	3	1	1	(0T)
3	-1	1	1	-2	-1	$3-\Omega$	5	3	3	2T
3	-1	1	2	-6	-1	$3-\Omega$	9	7	5	(4T)
3	-1	1	...							

we discuss the topic of the extent to which neutrinos have non-zero-mass. (See discussion of neutrino oscillations, neutrino masses, and Majorana neutrinos.)

Table XIVb shows solutions for which $2\nu_{SA}$ is an even negative integer and and $D^*_{SA}=3$ pertains. Solutions for which $D^*_{SA}=3$ and $\nu_{SA}=-1$ are volume-like. The right-most column proposes correlating families of elementary particles with solutions. The W family includes the known Z and W bosons and no other particles. The H family includes the known Higgs boson and no other particles. We think that, for the purposes of elementary-particle physics, the 0T solution is the same as the 0H solution. We point to the possibility that nature includes 2T particles. Reference [2] discusses reasons correlating with the notion the nature might not include non-zero-mass elementary bosons for which $\Sigma \geq 4$. One reason correlates with the notion that squares of masses for some 4T elementary particles would likely be negative. Assuming that one counts antiparticles as being distinct from particles and that one ignores results for rows that correlate with parenthesized $\Sigma\Phi$, the column labeled $2S+1$ provides the number of elementary particles correlating with $\Sigma\Phi$.

C. Lack of elementary particle magnetic monopoles, elementary particle electric dipole moments, and a neutron electric dipole moment

Table VII points to no G-family solutions that would correlate with interactions with a magnetic monopole elementary particle or that would correlate with a non-zero electric dipole moment for an elementary particle. Possibly, the lacks of such G-family solutions correlate with nature not including a magnetic monopole elementary particle and with nature not including elementary particles that have non-zero electric dipole moments.

Perhaps, for each hadron for which modeling based on PDE techniques pertains and for which all the quarks occupy one state with respect to spatial characteristics, the electric dipole moment is zero. (See discussion above regarding PDE-based modeling that correlates with some aspects of the strong, electromagnetic, and weak interactions.) We suggest that the neutron and proton are such hadrons.

D. A prediction for the tauon mass, based on a ratio of the strength of electromagnetism to the strength of gravity

Equation (38) possibly pertains. Here, m denotes mass, τ denotes tauon, e denotes electron, q denotes charge, ε_0 denotes the vacuum permittivity, and G_N denotes the gravitational constant. Based on 2016 data, equation (38) predicts a tauon mass with a standard deviation of less than one quarter of the standard deviation correlating with the experimental result. (For data, see reference [4].) Possibly, a more accurate experimental determination of either G_N or m_τ could predict a more accurate, than experimental results, value for, respectively, m_τ or G_N.

$$\beta' = m_\tau / m_e \tag{36}$$

$$(4/3) \times \beta^{12} = ((q_e)^2/(4\pi\varepsilon_0))/(G_N(m_e)^2) \tag{37}$$

$$\beta' = \beta \tag{38}$$

$$m_{\tau,\ \text{calculated}} \approx (1776.8445 \pm 0.024)\ MeV/c^2 \tag{39}$$

$$m_{\tau,\ \text{experimental}} \approx (1776.86 \pm 0.12)\ MeV/c^2 \tag{40}$$

The factor of 4/3 in equation (37) correlates with notions that 2G2 correlates with four so-called channels and 4G4 correlates with three channels. For a 2G2 interaction between two electrons, the strength for each channel is $((q_e)^2/(4\pi\varepsilon_0))/4$ and four channels pertain. For a 4G4 interaction between two electrons, the strength for each channel is $G_N(m_e)^2/3$ and three channels pertain. By extrapolation, for $\Sigma = 10$ and $\Gamma = \Sigma$, ΣGΓ would correlate with zero channels and no interactions. Regarding equation (37), the exponent 12 factors to be 2×6. The factor of two correlates with two interaction vertices - one that excites the carrier of an interaction and one that de-excites the carrier. We note as a pointer to a possibly useful research opportunity the possibility that the factor of six correlates with the meaning of the six in the name PR006INe.

E. Other relationships regarding masses of elementary particles

We discuss approximate ratios for the squares of masses of the Higgs, Z, and W bosons. The most accurately known of the three masses is the mass of the Z boson. Based on the ratios (of squares of masses) that equation (41) shows, the possibly least accurately suggested mass is that of the W boson. Equation (41) correlates with a number that is within three standard deviations of the nominal mass of the W boson. (For data, see reference [4].) Reference [2] correlates the numbers in equation (41) with, respectively, $17 = 17$, $9 = 10 - 1$, and $7 = 10 - 1 - 2$. Each of zero, one, two, five, 10, and 17 correlates with a PDE solution for which $D'' = 2$.

$$(m_{H^0})^2 : (m_Z)^2 : (m_W)^2 :: 17 : 9 : 7 \tag{41}$$

Reference [2] suggests, but does not require, equation (42) as an extrapolation from equation (41). Reference [2] also suggests that a threshold energy for producing tweaks may considerably exceed the rest energy of a tweak.

$$(m_{H^0})^2 : (m_Z)^2 : (m_W)^2 : (m_{T^0})^2 : (m_{T^\pm})^2 :: \ 51 : 27 : 21 : 9 : 7 \tag{42}$$

Reference [2] discusses a formula that approximately fits the masses of the six quarks and three charged leptons. The formula includes two integer variables (one of which correlates somewhat with generation and the other of which correlates somewhat with charge) and six parameters. The six parameters can be m_e, m_μ (or, the mass of a muon), β , α, and two other numbers.

Table XV: C, P, and T transformations

Swap (for each odd j' and with $j'' = j' + 1$)	Swap	Swap pertains for the transformation		
		T	C	P
$n_{TAj''}$ and $n_{TAj'}$	-	Yes	Yes	No
-	n_{TA0} and n_{SA0}	No	No	No
$n_{SAj'}$ and $n_{SAj''}$	-	No	Yes	Yes

F. Neutrino oscillations, neutrino masses, and Majorana neutrinos

Remarks above suggest that each G-family solution that correlates, per table VIIa, with a TA-side $SU(3)$ symmetry and with some interaction with neutrinos can catalyze neutrino oscillations. Monopole gravity (or, 4G4) provides one example. Monopole gravity interacts with the property of generation.

Possibly, neutrinos have zero mass. Possibly, interactions correlating with (for example) 8G8 produce observed astrophysical effects that people interpret as implying non-zero masses for at least one generation of neutrino. (The 8G8 solution correlates with properties such as handedness. The 6G6 solution correlates with baryon number and likely with zero interaction with individual leptons. We do not suggest the 6G6 correlates with perceptions of non-zero mass for neutrinos.)

Work above correlates with the notion that neutrinos behave like Dirac fermions. Reference [2] proposes a means to model Majorana neutrinos. We are uncertain as to whether people will need to model neutrinos as being Majorana fermions.

G. $SU(3) \times SU(2) \times U(1)$ boson symmetries

Regarding the 2G2⊕2G24 solution (or, traditional physics photon), 2W solutions, and 2U solutions, work above suggests an instance-related $SU(3) \times SU(2) \times U(1)$ symmetry, as well as a traditional physics interaction-related $SU(3) \times SU(2) \times U(1)$ symmetry. Regarding the instance-related $SU(3) \times SU(2) \times U(1)$ symmetry, the 2G2⊕2G24 solution contributes no symmetry.

H. CPT-related symmetries

Our work suggests that, tor ALG models, table XV pertains and extends, from traditional physics, concepts of CPT-related symmetries. For example, based on table XV, TA-side symmetry includes aspects related to color charge.

I. The Planck length and a series of formulas for lengths

We suggest a series of formulas for lengths. One formula includes a factor of m^1 and correlates with the notion of Schwarzschild radius. The next formula includes a factor of m^0 and correlates with the Planck length. The next formula includes a factor of m^{-1} and correlates (when applied to 2W bosons) with the range of the weak interaction and (when applied to a proton) with the charge radius and somewhat with a range for the strong interaction. The formula for the Planck length is the geometric mean of the formulas for the other two lengths. Equation (43) shows the ratio between successive formulas. Equation (44) shows, for the electron, that ratio.

$$(G_N)^{-1/2} m^{-1} \hbar^{1/2} c^{1/2} 2^{-1} \tag{43}$$

$$(G_N)^{-1/2} (m_e)^{-1} \hbar^{1/2} c^{1/2} 2^{-1} \approx 1.1945 \times 10^{22} \tag{44}$$

Possibly, our work points does not yet point to physics-relevance for the Planck length.

J. Fermion handedness and weak interaction parity violation

Table III indicates, via the symbol $\pi^L_{0,-1}$, the notion that known elementary fermions correlate with left handedness. Regarding PRnnnINe models for which nnn is not 001, we are uncertain as to whether half or none of the ensembles correlate with right-handed elementary fermions. Assuming the scenario we discuss regarding producing baryon asymmetry pertains to nature, we are uncertain as to the extent fermion handedness and the related weak interaction parity violation correlate with whether matter non-neutrino fermions dominate or antimatter non-neutrino fermions dominate. (See table XII.)

K. General relativity, flatness, and geodesic motion

Equation (45) summarizes aspects of observations that people correlate with the topic of curvature of the universe. (See reference [4].) Possibly, $\Omega_K = 0$ correlates with the term flat.

$$\Omega_K \approx -0.005^{+0.016}_{-0.017} \tag{45}$$

Our work suggests that forces correlating with 4G2468a and 4G2468b correlate with boost symmetry. To the extent forces correlating with 4G2468a and 4G2468b dominated the early evolution of the universe, flatness might have pertained for the stuff correlating with each ensemble. Similarly, 4G246 correlates with boost symmetry and flatness might have pertained for each ensemble for some time after 4G246 attraction became dominant. Regarding PRnnnINe models other than PR001INe models, attraction correlating with 4G4 might have started to provide for clumping between stuff correlating with each set of six 4G4-connected ensembles. Regardless, effects correlating with 4G246 might have preserved large-scale flatness. Possibly, 4G48 does not correlate with boost symmetry. (Our work suggests that 4G48 does not mediate interactions involving individual elementary particles. That suggestion need not correlate with the extent to which boost symmetry pertains regarding interactions with multi-particle objects.) Significantly observable deviation from large-scale flatness might pertain for only after effects correlating with 4G48 became dominant (that is, only for at most a recent few billion years). Discussion above correlates the 4G4 solution (or, monopole gravity) with TA-side $SU(3)$ symmetry and with a lack of boost symmetry. Possibly, for purposes of modeling, significant deviations from any scale flatness pertain only for situations in which 4G4 dominates 4G48.

For PR048INe models and PR288INe models, the general relativity concept of geodesic motion can pertain within PR006INe subsets but not for the entirety of modeling. For example, the sun can deflect, via 4G4, a photon emitted by ordinary matter, but the sun would not deflect, via 4G4, a photon emitted by dark energy stuff.

Our work suggests nominal elementary long-range forces correlating with $\Sigma \geq 6$ {or, $S \geq 3$}. But, possibly, under all circumstances, nominal elementary long-range forces for which $\Sigma = 4$ or $\Sigma = 2$ are more significant than nominal elementary long-range forces for which $\Sigma \geq 6$.

Possibly, concepts such as those we just mentioned point to opportunities for observational and theoretical research regarding each of the following topics and regarding relationships between each of the following topics - the domain of applicability of the Einstein field equations; the notion that (within those equations) the cosmological constant is a constant; the notion and applicability of the concept of the Hubble constant; notions regarding geodesic motion; the strengths of forces correlating with the 4G48, 4G246, 4G2468a, and 4G2468b solutions; and so forth.

L. The Standard Model

Reference [2] suggests that, to the extent that satisfying symmetries such as $SU(3) \times SU(2) \times U(1)$ boson symmetries suffices, people might be able to add, to the Standard Model, elementary particles and elementary long-range forces that our work suggests. Beyond our use of Poincare-group symmetries our work does not, as yet, explore Lagrangian aspects of the Standard Model.

M. Arrow of time

Reference [2] suggests a $\Psi(t_0, r_0)$ that correlates with the TA0-and-SA0 oscillator pair and has similarities to equation (22). Reference [2] shows that such a $\Psi(t_0, r_0)$ normalizes for exactly one of incoming radial momentum or outgoing radial momentum. We might expect that people would choose, for modeling a boson that enters an interaction vertex, normalization for incoming radial momentum. We might expect that people would choose, for modeling a boson that exits an interaction vertex, normalization for outgoing radial momentum. Possibly, the lack of dual normalization provides insight regarding the topic of arrow of time.

N. Evolution of theories; plus, opportunities regarding observations, experiments, and theories

Much physics theory has roots in the principle of stationary action and Lagrangian mathematics. Such roots correlate with notions of the motion of objects.

This article shows theory that has roots in the notion of the existence of objects and in Hamiltonian mathematics. This article also presents and uses a basis for merging modeling correlating with existence of objects and modeling correlating with motion of objects.

This article suggests, for elementary particles and elementary forces, results that have some parallels to the periodic table for chemical elements.

This article suggests that PR006INe modeling explains observations, about ratios of effects of dark matter to effects of ordinary matter, that PR001INe modeling likely cannot not explain. We suggest that people explore the extent to which PR048INe modeling is physics-relevant.

We suggest that opportunities exist to develop more sophisticated theory and modeling than the theory and modeling we present. Hopefully, such a new level of work would provide more insight than we provide.

We are not aware of observations that our work contradicts. We are aware of candidate theories and models with which our work does not comport. Possibly, such candidate theories and models feature assumptions that observations do not necessarily support.

Our work (including this article and reference [2]) suggests possible opportunities for observational research, experimental research, development of precision measuring techniques and data analysis techniques, numerical simulations, and theoretical research regarding elementary particle physics, nuclear physics, atomic physics, astrophysics, and cosmology. Our work suggests applied mathematics techniques that may have uses other than uses we make. Possibly, our work regarding harmonic oscillator math and/or relationships between that math and group theory points to under-utilized or new mathematics.

---

[1] Judd D. Bowman, Alan E. E. Rogers, Raul A. Monsalve, Thomas J. Mozdzen, and Nivedita Mahesh. An absorption profile centred at 78 megahertz in the sky-averaged spectrum. *Nature*, 555(7694):67–70, February 2018.

[2] Thomas J. Buckholtz. *About Much Physics: United Models and Specific Predictions*. T. J. Buckholtz & Associates, May 2018.

[3] N. G. Busca, T. Delubac, J. Rich, S. Bailey, A. Font-Ribera, D. Kirkby, J.-M. Le Goff, M. M. Pieri, A. Slosar, E. Aubourg, J. E. Bautista, D. Bizyaev, M. Blomqvist, A. S. Bolton, J. Bovy, H. Brewington, A. Borde, J. Brinkmann, B. Carithers, R. A. C. Croft, K. S. Dawson, G. Ebelke, D. J. Eisenstein, J.-C. Hamilton, S. Ho, D. W. Hogg, K. Honscheid, K.-G. Lee, B. Lundgren, E. Malanushenko, V. Malanushenko, D. Margala, C. Maraston, K. Mehta, J. Miralda-Escude, A. D. Myers, R. C. Nichol, P. Noterdaeme, M. D. Olmstead, D. Oravetz, N. Palanque-Delabrouille, K. Pan, I. Paris, W. J. Percival, P. Petitjean, N. A. Roe, E. Rollinde, N. P. Ross, G. Rossi, D. J. Schlegel, D. P. Schneider, A. Shelden, E. S. Sheldon, A. Simmons, S. Snedden, J. L. Tinker, M. Viel, B. A. Weaver, D. H. Weinberg, M. White, C. Yeche, and D. G. York. Baryon acoustic oscillations in the ly$a$ forest of boss quasars. *Astronomy & Astrophysics*, 552(A96), April 2013.

[4] C. Patrignani et. al. (Particle Data Group). *Chin. Phys. C*, 40, 100001, 2016.

[5] R. Genzel, N. M. Forster Schreiber, H. Ubler, P. Lang, T. Naab, R. Bender, L. J. Tacconi, E. Wisnioski, S. Wuyts, T. Alexander, A. Beifiori, S. Belli, G. Brammer, A. Burkert, C.M. Carollo, J. Chan, R. Davies, M. Fossati, A. Galametz, S. Genel, O. Gerhard, D. Lutz, J. T. Mendel, I. Momcheva, E. J. Nelson, A. Renzini, R. Saglia, A. Sternberg, S. Tacchella, K. Tadaki, and D. Wilman. Strongly baryon-dominated disk galaxies at the peak of galaxy formation ten billion years ago. *Nature*, 543(7645):397–401, March 2017.

[6] J. Jimenez-Vicente, E. Mediavilla, C. S. Kochanek, and J. A. Munoz. Dark matter mass fraction in lens galaxies: New estimates from microlensing. *The Astrophysical Journal*, 799(2):149, 2015.

[7] Ewa L. Lokas and Gary A. Mamon. Dark matter distribution in the coma cluster from galaxy kinematics: breaking the mass-anisotropy degeneracy. *Monthly Notices of the Royal Astronomical Society*, 343(2):401–412, August 2003.

[8] Eric Weisstein (Wolfram MathWorld). Delta function. Link: http://mathworld.wolfram.com/DeltaFunction.html.

[9] S. Perlmutter, G. Aldering, G. Goldhaber, R. A. Knop, P. Nugent, P. G. Castro, S. Deustua, S. Fabbro, A. Goobar, D. E. Groom, I. M. Hook, A. G. Kim, M. Y. Kim, J. C. Lee, N. J. Nunes, R. Pain, C. R.

Pennypacker, R. Quimby, C. Lidman, R. S. Ellis, M. Irwin, R. G. McMahon, P. Ruiz-Lapuente, N. Walton, B. Schaefer, B. J. Boyle, A. V. Filippenko, T. Matheson, A. S. Fruchter, N. Panagia, H. J. M. Newberg, and W. J. Couch. Measurements of [[uppercase omega]] and [[uppercase lambda]] from 42 high-redshift supernovae. *The Astrophysical Journal*, 517(2):565–586, June 1999. Note: [[uppercase omega]] denotes Ω; also, [[uppercase lambda]] denotes Λ.

[10] Elena Rasia, Giuseppe Tormen, and Lauro Moscardini. A dynamical model for the distribution of dark matter and gas in galaxy clusters. *Monthly Notices of the Royal Astronomical Society*, 351(1):237–252, June 2004.

[11] Adam G. Riess, Alexei V. Filippenko, Peter Challis, Alejandro Clocchiatti, Alan Diercks, Peter M. Garnavich, Ron L. Gilliland, Craig J. Hogan, Saurabh Jha, Robert P. Kirshner, B. Leibundgut, M. M. Phillips, David Reiss, Brian P. Schmidt, Robert A. Schommer, R. Chris Smith, J. Spyromilio, Christopher Stubbs, Nicholas B. Suntzeff, and John Tonry. Observational evidence from supernovae for an accelerating universe and a cosmological constant. *The Astronomical Journal*, 116(3):1009–1038, September 1998.

[12] Adam G. Riess, Louis-Gregory Strolger, John Tonry, Stefano Casertano, Henry C. Ferguson, Bahram Mobasher, Peter Challis, Alexei V. Filippenko, Saurabh Jha, Weidong Li, Ryan Chornock, Robert P. Kirshner, Bruno Leibundgut, Mark Dickinson, Mario Livio, Mauro Giavalisco, Charles C. Steidel, Txitxo BenÃtez, and Zlatan Tsvetanov. Type ia supernova discoveries at z $>$ 1 from the hubble space telescope: Evidence for past deceleration and constraints on dark energy evolution. *The Astrophysical Journal*, 607:665–687, June 2004.

[13] Pieter van Dokkum, Roberto Abraham, Jean Brodie, Charlie Conroy, Shany Danieli, Allison Merritt, Lamiya Mowla, Aaron Romanowsky, and Jielai Zhang. A high stellar velocity dispersion and ˜100 globular clusters for the ultra-diffuse galaxy dragonfly 44. *The Astrophysical Journal Letters*, 828(1):L6, 2016. http://iopscience.iop.org/article/10.3847/2041-8205/828/1/L6.

[14] Pieter van Dokkum, Shany Danieli, Yotam Cohen, Allison Merritt, Aaron J. Romanowsky, Roberto Abraham, Jean Brodie, Charlie Conroy, Deborah Lokhorst, Lamiya Mowla, Ewan O'Sullivan, and Jielai Zhang. A galaxy lacking dark matter. *Nature*, 555(7698):629–632, March 2018.