Smooth Unique Periodic Solutions in the absence of external Force for Navier_Stokes Three Dimensional Equation

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Abstract

Due to the existence of huge number of different information on Navier_Stokes equation on internet, introduction and method used to come to the following solution is less important than the solution its self. As a result the paper shows the periodic solution for Navier_Stokes equations. All conditions for physically reasonable solution as posted by clay mathematics institute is fulfilled. The following solution is counter example for existence of smooth unique periodic solution.

1. **Navier_Stokes equation**

$$\frac{\partial \vec{U}}{\partial t} + \left(u_x \frac{\partial}{\partial x} + u_y \frac{\partial}{\partial y} + u_z \frac{\partial}{\partial z}\right)\vec{U} = \nu \nabla^2 \vec{U} - \vec{\nabla} \mathrm{p}$$

Where

- $\vec{U}$: is velocity vector field where components are function of space and time variables.

$$\vec{U} = \hat{\imath} u_x + \hat{\jmath} u_y + \hat{k} u_z$$

- p: is scalar pressure function of space and time variables.
- Periodic initial velocity vector field, for a given constants a, b, c element of non zero real numbers.

$$\vec{U}^0(x, y, z) = \vec{U}^0(x + a, y + b, z + c)$$

- Scalar function of three space variables.

$$g = ax + by + cz$$

- Gradient of a function as period vector.

$$\vec{\nabla} g = \hat{\imath} a + \hat{\jmath} b + \hat{k} c$$

- Position vector.

$$\vec{R} = \hat{\imath} x + \hat{\jmath} y + \hat{k} z$$

- Scalar function as the dot product of gradient of the function and position vector.

$$g = \vec{\nabla} g \cdot \vec{R}$$

2. **Initial velocity vector field as the sum of infinite cosine and sine series**

$$\vec{U}^0 = \vec{a}_0 + \sum_{n=1}^{\infty} \vec{a}_n \cos\left(\frac{2n\pi}{\left|\vec{\nabla} g\right|^2} g\right) + \vec{b}_n \sin\left(\frac{2n\pi}{\left|\vec{\nabla} g\right|^2} g\right)$$

$$\vec{a}_0 = \frac{abc}{8} \int_{\frac{-1}{c}}^{\frac{1}{c}} \int_{\frac{-1}{b}}^{\frac{1}{b}} \int_{\frac{-1}{a}}^{\frac{1}{a}} \vec{U}^0 dxdydz$$

$$\vec{a}_n = \frac{abc}{4} \int_{\frac{-1}{c}}^{\frac{1}{c}} \int_{\frac{-1}{b}}^{\frac{1}{b}} \int_{\frac{-1}{a}}^{\frac{1}{a}} \vec{U}^0 \cos\left(\frac{2n\pi}{\left|\vec{\nabla} g\right|^2} g\right) dxdydz$$

$$\vec{b}_n = \frac{abc}{4} \int_{\frac{-1}{c}}^{\frac{1}{c}} \int_{\frac{-1}{b}}^{\frac{1}{b}} \int_{\frac{-1}{a}}^{\frac{1}{a}} \vec{U}^0 \sin\left(\frac{2n\pi}{\left|\vec{\nabla} g\right|^2} g\right) dxdydz$$

- Consequence of divergence free initial velocity vector field.

$$\vec{a}_n \cdot \vec{\nabla} g = 0$$

$$\vec{b}_n \cdot \vec{\nabla} g = 0$$

- Scalar function of time variable.

$$h(t) = \left(1 - \frac{\left(\vec{a}_0 \cdot \vec{\nabla} g\right)^2}{|\vec{a}_0|^2 \left|\vec{\nabla} g\right|^2}\right) + \left(\frac{\left(\vec{a}_0 \cdot \vec{\nabla} g\right)^2}{|\vec{a}_0|^2 \left|\vec{\nabla} g\right|^2}\right) e^{-\nu t\left(\frac{24}{\left|\vec{\nabla} g\right|^2}\right)}$$

$$H_n(t) = e^{-\nu t\left(\frac{2n\pi}{\left|\vec{\nabla} g\right|}\right)^2}$$

- Scalar function of three space and time variables.

$$l = g - \left(\vec{a}_0 \cdot \vec{\nabla} g\right) \int_0^t h(\tau) d\tau$$

3. **Periodic Velocity vector field solution with three space variable and one time variables components**

$$\vec{U} = \vec{a}_0 h(t) + \sum_{n=1}^{\infty} \left(\vec{a}_n \cos\left(\frac{2n\pi}{\left|\vec{\nabla} g\right|^2} l\right) + \vec{b}_n \sin\left(\frac{2n\pi}{|\vec{\nabla} g|^2} l\right)\right) H_n(t)$$

4. **Scalar pressure vector field solution**

$$-p = \frac{\vec{a}_0 \cdot \vec{\nabla} g}{\left|\vec{\nabla} g\right|^2} \left(\frac{\partial}{\partial t} h(t)\right) l$$

References

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