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On $((r_1, r_2), m, (c_1, c_2))$-Regular Intuitionistic Fuzzy Graphs

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Abstract: In this paper, $((r_1, r_2), m, (c_1, c_2))$-regular intuitionistic fuzzy graph and totally $((r_1, r_2), m, (c_1, c_2))$-regular intuitionistic fuzzy graphs are introduced. A relation between $((r_1, r_2), m, (c_1, c_2))$-regularity and totally $((r_1, r_2), m, (c_1, c_2))$-regularity on Intuitionistic fuzzy graph is studied. A necessary and sufficient condition under which they are equivalent is provided. Also, $((r_1, r_2), m, (c_1, c_2))$-regularity on some intuitionistic fuzzy graphs whose underlying crisp graphs is a cycle is studied with some specific membership functions.

Key Words: Degree and total degree of a vertex in intuitionistic fuzzy graph, d_m-degree and total d_m-degree of a vertex in intuitionistic fuzzy graph, $(m, (c_1, c_2))$- intuitionistic regular fuzzy graphs, totally $(m, (c_1, c_2))$-intuitionistic regular fuzzy graphs.

AMS(2010): 05C12, 03E72, 05C72

§1. Introduction

In 1965, Lofti A. Zadeh [18] introduced the concept of fuzzy subset of a set as method of representing the phenomena of uncertainty in real life situation. K.T.Attanassov [1]introduced the concept of intuitionistic fuzzy sets as a generalization of fuzzy sets. K.T.Atanassov added a new component(which determines the degree of non-membership) in the definition of fuzzy set. The fuzzy sets give the degree of membership of an element in a given set(and the non-membership degree equals one minus the degree of membership), while intuitionistic fuzzy sets give both a degree of membership and a degree of non-membership which are more-or-less independent from each other, the only requirement is that the sum of these two degrees is not greater than one.

Intuitionistic fuzzy sets have been applied in a wide variety of fields including computer science, engineering, mathematics, medicine, chemistry and economics [1, 2]. Azriel Rosenfeld introduced the concept of fuzzy graphs in 1975 [5]. It has been growing fast and has numerous application in various fields. Bhattacharya [**?**] gave some remarks on fuzzy graphs, and some operations on fuzzy graphs were introduced by Morderson and Peng [9].

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Krassimir T Atanassov [2] introduced the intuitionistic fuzzy graph theory. R.Parvathi and M.G.Karunambigai [8] introduced intuitionistic fuzzy graphs as a special case of Atanassov's IFG and discussed some properties of regular intuitionistic fuzzy graphs [6]. M.G. Karunambigai and R.Parvathi and R.Buvaneswari introduced constant intuitionistic fuzzy graphs [7]. M. Akram, W. Dudek [3] introduced the regular intuitionistic fuzzy graphs. M.Akram and Bijan Davvaz [4] introduced the notion of strong intuitionistic fuzzy graphs and discussed some of their properties.

N.R.Santhi Maheswari and C.Sekar introduced d_2- degree of vertex in fuzzy graphs and introduced $(r, 2, k)$-regular fuzzy graphs and totally $(r, 2, k)$-regular fuzzy graphs [11]. S.Ravi Narayanan and N.R.Santhi Maheswari introduced $((2, (c_1, c_2)$-regular bipolar fuzzy graphs [13]. Also, they introduced d_m-degree, total d_m-degree, of a vertex in fuzzy graphs and introduced an m-neighbourly irregular fuzzy graphs [12, 15], (m, k)-regular fuzzy graphs [14, 15] and (r, m, k)-regular fuzzy graphs [15, 16].

N.R.Santhi Maheswari and C.Sekar introduced d_m- degree of a vertex in intuitionistic fuzzy graphs and introduced $(m, (c_1, c_2))$-regular fuzzy graphs and totally $(m, (c_1, c_2))$-regular fuzzy graphs [17]. These motivates us to introduce $((r_1, r_2), m, (c_1, c_2))$-regular intuitionistic fuzzy graphs and totally $((r_1, r_2), m, (c_1, c_2))$-regular intuitionistic fuzzy graphs.

§2. Preliminaries

We present some known definitions related to fuzzy graphs and intuitionistic fuzzy graphs for ready reference to go through the work presented in this paper.

Definition 2.1([9]) *A fuzzy graph $G : (\sigma, \mu)$ is a pair of functions (σ, μ), where $\sigma : V \to$[0,1] is a fuzzy subset of a non empty set V and $\mu : VXV \to$[0, 1] is a symmetric fuzzy relation on σ such that for all u, v in V, the relation $\mu(u, v) \leq \sigma(u) \wedge \sigma(v)$ is satisfied. A fuzzy graph G is called complete fuzzy graph if the relation $\mu(u, v) = \sigma(u) \wedge \sigma(v)$ is satisfied.*

Definition 2.2([12]) *Let $G : (\sigma, \mu)$ be a fuzzy graph. The d_m-degree of a vertex u in G is $d_m(u) = \sum \mu^m(uv)$, where $\mu^m(uv) = sup\{\mu(uu_1)\wedge\mu(u_1u_2)\wedge \ldots, \mu(u_{m-1}v) : u, u_1, u_2, \ldots, u_{m-1}, v$ is the shortest path connecting u and v of length $m\}$. Also, $\mu(uv) = 0$, for uv not in E.*

Definition 2.3([12]) *Let $G : (\sigma, \mu)$ be a fuzzy graph on $G^* : (V, E)$. The total d_m-degree of a vertex $u \in V$ is defined as $td_m(u) = \sum \mu^m(uv) + \sigma(u) = d_m(u) + \sigma(u)$.*

Definition 2.4([12]) *If each vertex of G has the same d_m - degree k, then G is said to be an (m, k)-regular fuzzy graph.*

Definition 2.5([12]) *If each vertex of G has the same total d_m - degree k, then G is said to be totally (m, k)-regular fuzzy graph.*

Definition 2.6([15, 16]) *If each vertex of G has the same degree r and has the same d_m-degree k, then G is said to be (r, m, k)-regular fuzzy graph.*

Definition 2.7([15, 16]) *If each vertex of G has the same total degree r and has the same total d_m-degree k, then G is said to be totally (r, m, k)-regular fuzzy.*

Definition 2.8([7]) *An intuitionistic fuzzy graph with underlying set V is defined to be a pair $G = (V, E)$ where*

(1) $V = \{v_1, v_2, v_3, \cdots, v_n\}$ such that $\mu_1 : V \to [0, 1]$ and $\gamma_1 : V \to [0, 1]$ denote the degree of membership and nonmembership of the element $v_i \in V, (i = 1, 2, 3, \cdots, n)$, such that $0 \le \mu_1(v_i) + \gamma_1(v_i) \le 1$;

(2) $E \subseteq V \times V$ where $\mu_2 : V \times V \to [0, 1]$ and $\gamma_2 : V \times V \to [0, 1]$ are such that $\mu_2(v_i, v_j) \le min\{\mu_1(v_i), \mu_1(v_j)\}$ and $\gamma_2(v_i, v_j) \le max\{\gamma_1(v_i), \gamma_1(v_j)\}$ and $0 \le \mu_2(v_i, v_j) + \gamma_2(v_i, v_j) \le 1$ for every $(v_i, v_j) \in E, (i, j = 1, 2, \cdots, n)$.

Definition 2.9([7]) *If $v_i, v_j \in V \subseteq G$, the μ-strength of connectedness between two vertices v_i and v_j is defined as $\mu_2^{\infty}(v_i, v_j) = sup\{\mu_2^k(v_i, v_j) : k = 1, 2, \cdots, n\}$ and γ-strength of connectedness between two vertices v_i and v_j is defined as $\gamma_2^{\infty}(v_i, v_j) = inf\{\gamma_2^k(v_i, v_j) : k = 1, 2, \cdots, n\}$.*

If u and v are connected by means of paths of length k then $\mu_2^k(u, v)$ is defined as sup $\{\mu_2(u, v_1) \wedge \mu_2(v_1, v_2) \wedge \cdots \wedge \mu_2(v_{k-1}, v)$: $(u, v_1, v_2, \cdots, v_{k-1}, v) \in V\}$ and $\gamma_2^k(u, v)$ is defined as inf $\{\gamma_2(u, v_1) \vee \gamma_2(v_1, v_2) \vee \cdots \vee \gamma_2(v_{k-1}, v)$: $(u, v_1, v_2, \cdots, v_{k-1}, v) \in V\}$.

Definition 2.10([7]) *Let $G = (V, E)$ be an intuitionistic fuzzy graph on $G^*(V, E)$. Then the degree of a vertex $v_i \in G$ is defined by $d(v_i) = (d_{\mu_1}(v_i), d_{\gamma_1}(v_i))$, where $d_{\mu_1}(v_i) = \sum \mu_2(v_i, v_j)$ and $d_{\gamma_1}(v_i) = \sum \gamma_2(v_i, v_j)$ for $(v_i, v_j) \in E$ and $\mu_2(v_i, v_j) = 0$ and $\gamma_2(v_i, v_j) = 0$ for $(v_i, v_j) \notin E$.*

Definition 2.11([7]) *Let $G = (V, E)$ be an Intuitionistic fuzzy graph on $G^*(V, E)$. Then the total degree of a vertex $v_i \in G$ is defined by $td(v_i) = (td_{\mu_1}(v_i), td_{\gamma_1}(v_i))$, where $td_{\mu_1}(v_i) = d\mu_1(v_i) + \mu_1(v_i)$ and $td_{\gamma_1}(v_i) = d\gamma_1(v_i) + \gamma_1(v_i)$.*

Definition 2.12([17]) *Let $G = (V, E)$ be an intuitionistic fuzzy graph on $G^*(V, E)$. Then the d_m - degree of a vertex $v \in G$ is defined by $d_{(m)}(v) = (d_{(m)\mu_1}(v), d_{(m)\gamma_1}(v))$, where $d_{(m)\mu_1}(v) = \sum \mu_2^{(m)}(u, v)$ where $\mu_2^{(m)}(u, v) = sup\{\mu_2(u, u_1) \wedge \mu_2(u_1, u_2) \wedge \cdots \wedge \mu_2(u_{m-1}, v) : u, u_1, u_2, \cdots, u_{m-1}, v$ is the shortest path connecting u and v of length $m)\}$ and $d_{(m)\gamma_1}(v) = \sum \gamma_2^{(m)}(u, v)$, where $\gamma_2^{(m)}(u, v)$=inf $\{\gamma_2(u, u_1) \vee \gamma_2(u_1, u_2) \vee \cdots \vee \gamma_2(u_{m-1}, v) : u, u_1, u_2, \cdots, u_{m-1}, v$ is the shortest path connecting u and v of length $m\}$. The minimum d_m-degree of G is $\delta_m(G)$ $=\wedge\{(d_{(m)\mu_1}(v), d_{(m)\gamma_1}(v)) : v \in V\}$.*

The maximum d_m-degree ofG is $\Delta_m(G)$=$\vee\{(d_{(m)\mu_1}(v), d_{(m)\gamma_1}(v)) : v \in V\}$..

Definition 2.13([17]) *Let G : (V, E) be an intuitionistic fuzzy graph on $G^*(V, E)$. If all the vertices of G have same d_m- degree c_1,c_2, then G is said to be a $(m, (c_1, c_2))$- regular intuitionistic fuzzy graph.*

Definition 2.14([17]) *Let $G = (V, E)$ be an intuitionistic fuzzy graph on $G^*(V, E)$. Then the total d_m-degree of a vertex $v \in G$ is defined by $td_{(m)}(v)$= $(td_{(m)\mu_1}(v), td_{(m)\gamma_1}(v))$, where $td_{(m)\mu_1}(v) = d_{(m)\mu_1}(v) + \mu_1(v)$ and $td_{(m)\gamma_1}(v) = d_{(m)\gamma_1}(v) + \gamma_1(v)$. The minimum td_m-degree of G is $t\delta_m(G) = \wedge\{(td_{(m)\mu_1}(v),\ td_{(m)\gamma_1}(v)) : v \in V\}$. The maximum td_m-degree of G is $t\Delta_m(G) = \vee\{(td_{(m)\mu_1}(v), td_{(m)\gamma_1}(v)) : v \in V\}$.*

Definition 2.15([17]) *Let $G = (V, E)$ be an intuitionistic fuzzy graph on $G^*(V, E)$. If each vertex of G has same total d_m - degree c_1,c_2, then G is said to be totally $(m,(c_1,c_2))$ - regular intuitionistic fuzzy graph.*

§3. $((r_1, r_2), m, (c_1, c_2))$- Regular intuitionistic Fuzzy Graphs

Definition 3.1 *Let $G : (V, E)$ be an intuitionistic fuzzy graph on $G^*(V, E)$. If $d(v) = (r_1, r_2)$ and $d_{(m)}(v) = (c_1, c_2)$ for all $v \in V$, then G is said to be $((r_1, r_2), m, (c_1, c_2))$-regular intuitionistic fuzzy graph. That is, if each vertex of G has the same degree (r_1, r_2) and has the same d_m-degree (c_1, c_2), then G is said to be $((r_1, r_2), m, (c_1, c_2))$-regular intuitionistic fuzzy graph.*

Example 3.2 Consider an intuitionistic fuzzy graph on $G^*(V, E)$, a cycle of length 7.

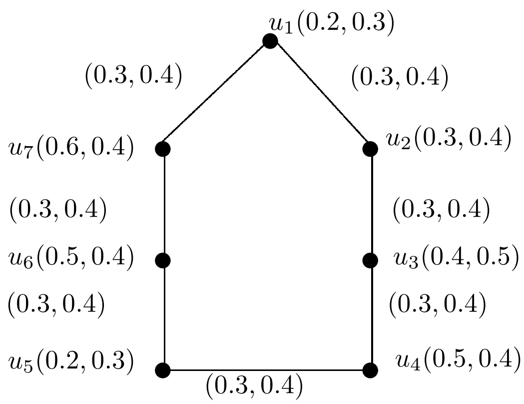

Figure 1

Here, $d_{\mu_1}(u) = 0.6$; $d_{\gamma_1}(u) = 0.8$; $d(u) = (0.6, 0.8)$ for all $u \in V$.
$d_{(3)\mu_1}(u_1) = (0.3 \wedge 0.3 \wedge 0.3) + (0.3 \wedge 0.3 \wedge 0.3) = 0.3 + 0.3 = 0.6$;
$d_{(3)\gamma_1}(u_1) = (0.4 \vee 0.4 \vee 0.4) + (0.4 \vee 0.4 \vee 0.4) = (0.4) + (0.4) = 0.8$;
$d_{(3)}(u_1) = (0.6, 0.8)$; $d_{(3)}(u_2) = (0.6, 0.8)$; $d_{(3)}(u_3) = (0.6, 0.8)$;
$d_{(3)}(u_4) = (0.6, 0.8)$; $d_{(3)}(u_5) = (0.6, 0.8)$; $d_{(3)}(u_6) = (0.6, 0.8)$; $d_{(3)}(u_7) = (0.6, 0.8)$.

Hence G is $((0.6, 0.8), 3, (0.6, 0.8))$-regular intuitionistic fuzzy graph.

Example 3.3 Consider an intuitionistic fuzzy graph on $G^*(V, E)$, a cycle of length 6.

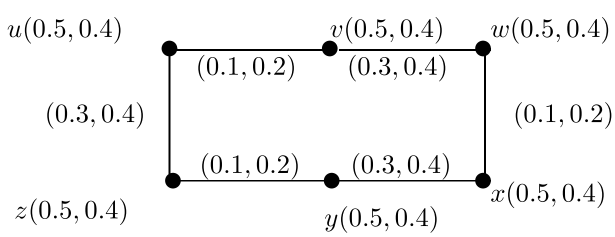

Figure 2

$d_{\mu_1}(u) = 0.4$; $d_{\gamma_1}(u) = 0.6$; $d(u) = (0.4, 0.6)$;

$d_{(3)\mu_1}(u) = \sup\{(0.1 \wedge 0.3 \wedge 0.1), (0.3 \wedge 0.1 \wedge 0.3)\} = \sup\{0.1, 0.1\} = 0.1$;

$d_{(3)\gamma_1}(u) = \inf\{(0.2 \vee 0.4 \vee 0.2), (0.4 \vee 0.2 \vee 0.4)\} = \inf\{0.4, 0.4\} = 0.4$;

$d_{(3)}(u) = (0.1, 0.4)$, $d_{(3)}(u) = (0.1, 0.4)$, for all $u \in V$.

Here, G is $((0.4, 0.6), 3, (0.1, 0.4))$- regular intuitionistic fuzzy graph.

Example 3.4 Non regular intuitionistic fuzzy graphs which is $(m, (c_1, c_2))$-regular intuitionistic fuzzy graph.

Let $G : (V, E)$ be an intuitionistic fuzzy graph such that $G^*(V, E)$, a path on $2m$ vertices. Let all the edges of G have the same membership value (c_1, c_2). Then,

For $i = 1, 2, \cdots, m$,

$$\begin{aligned} d_{(m)\mu_1}(v_i) &= \{\mu(e_i) \wedge \mu(e_{i+1}) \wedge \cdots \wedge \mu(e_{m-2+i}) \wedge \mu(e_{m-1+i})\} \\ &= \{c_1 \wedge c_1 \wedge \cdots \wedge c_1\}, \end{aligned}$$

$$d_{(m)\mu_1}(v_i) = c_1,$$

$$\begin{aligned} d_{(m)\gamma_1}(v_i) &= \{\gamma(e_i) \vee \gamma(e_{i+1}) \vee \cdots \vee \gamma(e_{m-2+i}) \vee \gamma(e_{m-1+i})\} \\ &= \{c_2 \vee c_2 \vee \cdots \vee c_2\} \end{aligned}$$

$$d_{(m)\gamma_1}(v_i) = c_2,$$

$$\begin{aligned} d_{(m)\mu_1}(v_{m+i}) &= \{\mu(e_i) \wedge \mu(e_{i+1}) \wedge \cdots \wedge \mu(e_{m-2+i}) \wedge \mu(e_{m-1+i})\} \\ &= \{c_1 \wedge c_1 \wedge \cdots \wedge c_1\}, \end{aligned}$$

$$d_{(m)\mu_1}(v_{m+i}) = c_1,$$

$$\begin{aligned} d_{(m)\gamma_1}(v_{m+i}) &= \{\gamma(e_i) \vee \gamma(e_{i+1}) \vee \cdots \vee \gamma(e_{m-2+i}) \vee \gamma(e_{m-1+i})\} \\ &= \{c_2 \vee c_2 \vee \cdots \vee c_2\}, \end{aligned}$$

$$d_{(m)\gamma_1}(v_{m+i}) = c_2.$$

Hence G is $(m, (c_1, c_2))$ - regular intuitionistic fuzzy graph.

For $i = 2, 3, \cdots, 2m - 1$,

$d_\mu(v_i) = \mu(e_{i-1}) + \mu(e_i) = c_1 + c_1 = 2c_1$;

$d_\gamma(v_i) = \gamma(e_{i-1}) + \gamma(e_i) = c_2 + c_2 = 2c_2$;

$d(v_i) = (2c_1, 2c_2) = (k_1, k_2)$ where $k_1 = 2c_1$ and $k_2 = 2c_2$;

$d_\mu(v_1) = \mu(e_1) = c_1$ and $d_\gamma(v_1) = \gamma(e_1) = c_2$,

So, $d(v_1) = (c_1, c_2)$, $d_\mu(v_{2m}) = \mu(e_{2m-1}) = c_1$ and $d\gamma(v_{2m}) = \gamma(e_{2m-1}) = c_2$.

So, $d(v_{2m}) = (c_1, c_2)$. Therefore, $d(v_1) \neq d(v_i) \neq d(v_{2m})$ for $i = 2, 3, \cdots, 2m - 1$.

Hence G is non regular intuitionistic fuzzy graph which is $(m, (c_1, c_2))$-regular intuitionistic fuzzy graph.

Example 3.5 Let $G : (V, E)$ be an intuitionistic fuzzy graph such that $G^*(V, E)$, an even cycle of length $\geq 2m + 1$.

Let

$$\mu(e_i) = \begin{cases} k_{1l} & \text{if } i \text{ is odd} \\ \text{membership value x} \geq k_1 & \text{if i is even} \end{cases}$$

and

$$\gamma(e_i) = \begin{cases} k_2 & \text{if } i \text{ is odd} \\ \text{membership value } y \leq k_2 & \text{if } i \text{ is even} \end{cases}$$

where x, y are not constant functions. Then,

$$d_{(m)\mu_1}(v) = min\{k_1, x\} + min\{x, k_1\} = k_1 + k_1 = 2k_1 = c_1, \text{ where } c_1 = 2k_1$$

$$d_{(m)\gamma_1}(v) = \max\{k_2, y\} + max\{y, k_2\} = k_2 + k_2 = 2k_2 = c_2, \text{ where } c_2 = 2k_2.$$

So, $d_{(m)}(v) = (c_1, c_2)$, for all $v \in V$.

Case 1. Let $G : (A, B)$ be an intuitionistic fuzzy graph such that $G^*(V, E)$, an even cycle of length $\leq 2m+2$. Then $d(v_i)$=$(x+c_1, y+c_2)$, for all $i = 1, 2, \cdots, 2m+1$. Hence G is non-regular $(m, (c_1, c_2))$-regular intuitionistic fuzzy graph.

Case 2. Let $G : (A, B)$ be an intuitionistic fuzzy graph such that $G^*(V, E)$, an odd cycle of length $\leq 2m + 1$. Then $d(v_1) = (c_1, c_2) + (c_1, c_2) = (2c_1, 2c_2)$ and $d(v_i) = (x + c_1, y + c_2)$, for all $i = 2, 3, \cdots, 2m$. Hence G is non-regular $(m, (c_1, c_2))$-regular intuitionistic fuzzy graph.

§4. Totally $((r_1, r_2), m, (c_1, c_2))$- Regular Intuitionistic Fuzzy Graphs

Definition 4.1 *Let $G : (A, B)$ be an intuitionistic fuzzy graph on $G^*(V, E)$. If each vertex of G has the same total degree (r_1, r_2) and same total d_m-degree (c_1, c_2) , then G is said to be totally $((r_1, r_2), m, (c_1, c_2))$- regular intuitionistic fuzzy graph.*

Example 4.2 In Figure 2, $td_{(3)}(v) = d_{(3)}(v) + A(v)$ = (0.1,0.4)+(0.5,0.4)=(0.6,0.8) for all $v \in V$. $td(v) = d(v) + A(v) = (0.4, 0.6) + (0.5, 0.4) = (0.9, 1.0)$ for all $v \in V$ Hence G is ((0.9,1.0),3,(0.6,0.8))- regular intuitionistic fuzzy graph.

Example 4.3 A $((r_1, r_2), m, (c_1, c_2))$- regular intuitionistic fuzzy graph need not be totally $((r_1, r_2), m, (c_1, c_2))$- regular intuitionistic fuzzy graph. Consider $G : (A, B)$ be a intuitionistic fuzzy graph on $G^*(V, E)$, a cycle of length 7.

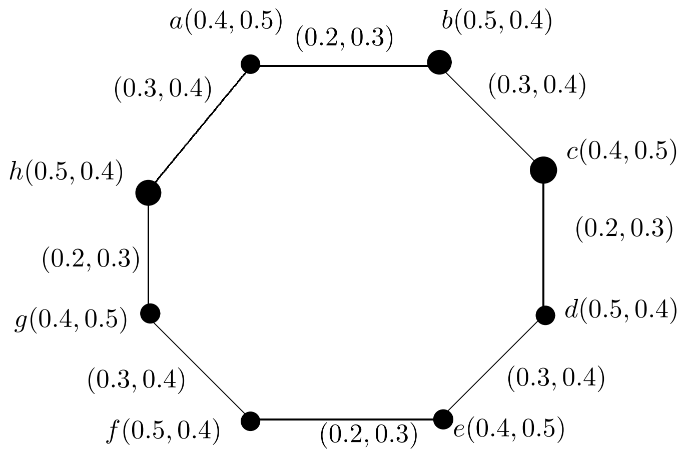

Figure 3

Here $d(v) = (0.5, 0.7)$ for all $v \in V$ and $d_{(3)}(v) = (0.4, 0.8)$, for all $v \in V$. But $td_{(3)}(a) = (0.4, 0.8) + (0.4, 0.5) = (0.8, 1.3)$, $td_{(3)}(b) = (0.4, 0.8) + (0.5, 0.4) = (0.9, 1.2)$. Hence G is ((0.5,0.7), 3 , (0.4,0.8)) - regular intuitionistic fuzzy graph.

But $td_3(a) \neq td_3(b)$. Hence G is not totally $((r_1, r_2)$, 3, $(c_1, c_2))$ - regular intuitionistic fuzzy graph.

Example 4.4 A $((r_1, r_2), m, (c_1, c_2))$- regular intuitionistic fuzzy graph which is totally $((r_1, r_2), m, (c_1, c_2))$-regular intuitionistic fuzzy graph.

Consider $G : (A, B)$ be an intuitionistic fuzzy graph on $G^*(V, E)$, a cycle of length 6. For m = 3,

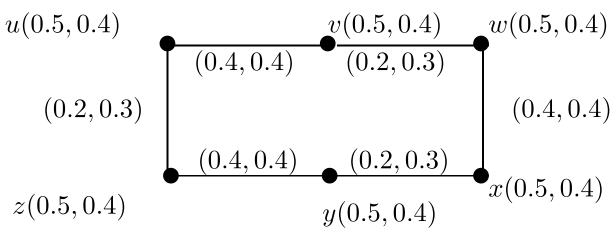

Figure 4

Here, $d(v) = (0.6, 0.8)$ and $d_{(3)}(v) = (0.2,\ 0.3)$, for all $v \in V$. Also, $td(v) = (1.1, 1.4)$ and $td_{(3)}(v) = ((0.7, 0.9)$ for all $v \in V$. Hence G is $((0.6, 0.8), 3, (0.2, 0.3))$ regular intuitionistic fuzzy graph and totally $((1.1, 1.4), 3, (0.7, 0.9))$ - regular intuitionistic fuzzy graph.

Theorem 4.5 *Let $G : (A, B)$ be an intuitionistic fuzzy graph.on $G^*(V, E)$. Then $A(u) = (k_1, k_2)$, for all $u \in V$ if and only if the following are equivalent:*

(i) $G : (V, E)$ is $((r_1, r_2), m, (c_1, c_2))$- regular intuitionistic fuzzy graph;

(ii) G : (V, E) is totally $((r_1 + k_1, r_2 + k_2), m, (c_1 + k_1, c_2 + k_2))$- regular intuitionistic fuzzy graph.

Proof Suppose $A(u) = (k_1, k_2)$, for all $u \in V$. Assume that G is $((r_1, r_2), m, (c_1, c_2))$-regular intuitionistic fuzzy graph. Then $d(u) = (r_1, r_2)$ and $d_{(m)}(u) = (c_1, c_2)$, for all $u \in V$.

So, $td(u) = d(u) + A(u)$ and $td_{(m)}(u) = d_{(m)}(u) + A(u) \Rightarrow td(u) = (r_1, r_2) + (k_1, k_2)$ and $td_{(m)}(u) = (c_1, c_2) + (k_1, k_2)$. So, $td(u) = (r_1 + k_1, r_2 + k_2), td_{(m)}(u) = (c_1 + k_1, c_2 + k_2)$. Hence G is totally $((r_1 + k_1, r_2 + k_2), m, (c_1 + k_1, c_2 + k_2))$ - regular intuitionistic fuzzy graph. Thus (i) $\Rightarrow$ (ii) is proved.

Now, suppose G is totally $((r_1 + k_1, r_2 + k_2), m, (c_1 + k_1, c_2 + k_2))$- regular intuitionistic fuzzy graph. Then $td(u) = (r_1 + k_1, r_2 + k_2)$ and $td_{(m)}(u) = (c_1 + k_1, c_2 + k_2)$, for all $u \in V \Rightarrow d(u) + A(u) = (r_1 + k_1, r_2 + k_2)$ and $d_{(m)}(u) + A(u) = (c_1 + k_1, c_2 + k_2)$, for all $u \in V \Rightarrow d(u) + (k_1, k_2) = (r_1, r_2) + (k_1, k_2)$ and $d_{(m)}(u) + (k_1, k_2) = (c_1, c_2) + (k_1, k_2)$, for all $u \in V \Rightarrow$d(u)$= (r_1, r_2)$ and $d_{(m)}(u) = (c_1, c_2)$, for all $u \in V$. Hence G is $((r_1, r_2), m, (c_1, c_2))$-regular intuitionistic fuzzy graph. Thus (ii) $\Rightarrow$ (i) is proved. Hence (i) and (ii) are equivalent.

Conversely, suppose (i) and (ii) are equivalent. Suppose $A(u)$ is not constant function, then $A(u) \neq A(w)$, for atleast one pair $u, v \in V$. Let G be a $((r_1, r_2), m, (c_1, c_2))$ - regular intuitionistic fuzzy graph and totally $((r_1+k_1, r_2+k_2), m, (c_1+k_1, c_2+k_2))$- regular intuitionistic fuzzy graph. Then $d_{(m)}(u) = d_{(m)}(w) = (c_1, c_2)$ and $d(u) = d(w) = (r_1, r_2)$. Also, $td_{(m)}(u) = d_{(m)}(u) + A(u) = (c_1, c_2) + A(u)$ and $td_{(m)}(w) = d_{(m)}(w) + A(w) = (c_1, c_2) + A(w), td(u) = d(u) + A(u) = (r_1, r_2) + A(u)$ and $td(w) = d(w) + A(w) = (r_1, r_2) + A(w)$. Since $A(u) \neq A(w)$, $(c_1, c_2) + A(u) \neq (c_1, c_2) + A(w)$ and $(r_1, r_2) + A(u) \neq (r_1, r_2) + A(w) \Rightarrow td_{(m)}(u) \neq td_{(m)}(w)$ and $td(u) \neq td(w)$. So, G is not totally $((r_1+k_1, r_2+k_2), m, (c_1+k_1, c_2+k_2))$- regular intuitionistic fuzzy graph. Which is a contradiction.

Now let G be a totally $((r_1+k_1, r_2+k_2), m, (c_1+k_1, c_2+k_2))$ - regular intuitionistic fuzzy graph. Then $td_{(m)}(u) = td_{(m)}(w)$ and $td(u) = td(w) \Rightarrow d_{(m)}(u) + A(u) = d_{(m)}(w) + A(w)$ and $d(u) + A(u) = d(w) + A(w) \Rightarrow d_{(m)}(u) - d_{(m)}(w) = A(w) - A(u) \neq 0$ and $d(u) - d(w) = A(w) - A(u) \neq 0 \Rightarrow d_{(m)}(u) \neq d_{(m)}(w)$ and $d(u) \neq d(w)$. So, G is not $((r_1, r_2), m, (c_1, c_2))$ - regular intuitionistic fuzzy graph. Which is a contradiction. Hence $A(u) = (k_1, k_2)$, for all $u \in V$. □

Theorem 4.6 *If an intuitionistic fuzzy graph $G : (A, B)$ is both $((r_1, r_2), m, (c_1, c_2))$ - regular intuitionistic fuzzy graph and totally $((r_1, r_2), m, (c_1, c_2))$ - regular intuitionistic fuzzy graph then A is constant function.*

Proof Let G be a $((s_1, s_2), m, (k_1, k_2))$ - regular intuitionistic fuzzy graph and totally $((s_3, s_4), m, (k_3, k_4))$ - regular intuitionistic fuzzy graph. Then, let $d_{(m)}(u) = (k_1, k_2)$, $td_{(m)}(u) = (k_3, k_4)$, $d(u) = (s_1, s_2)$, $td(u) = (s_3, s_4)$ for all $u \in v$. Now, $td_{(m)}(u) = (k_3, k_4)$ and $td(u) = (s_3, s_4)$ for all $u \in v \Rightarrow d_{(m)}(u) + A(u) = (k_3, k_4)$ and $d(u) + A(u) = (s_3, s_4)$ for all $u \in v \Rightarrow (k_1, k_2) + A(u) = (k_3, k_4)$ and $(s_1, s_2) + A(u) = (s_3, s_4)$ for all $u \in v \Rightarrow A(u) = (k_3, k_4) - (k_1, k_2)$ and $A(u) = (s_3, s_4) - (s_1, s_2)$ for all $u \in v \Rightarrow A(u) = (k_3 - k_1, k_4 - k_2)$ and $A(u) = (s_3 - s_1), (s_4 - s_2)$ for all $u \in v$. Hence $A(u)$ is constant function. □

§5. $((r_1, r_2), m, (c_1, c_2))$- Regularity on a Cycle with Some Specific Membership Functions

Theorem 5.1 *For any $m \geq 1$, Let $G : (A, B)$ be an intuitionistic fuzzy graph such that $G^*(V, E)$, a cycle of length $\geq 2m$. If B is constant function then G is $((r_1, r_2), m, (c_1, c_2))$ -*

regular intuitionistic fuzzy graph, where $(k_1, k_2) = 2B(uv)$.

Proof Suppose B is a constant function say $B(uv) = (c_1, c_2)$, for all $uv \in E$. Then $d_\mu(u)$= sup $\{(c_1 \wedge c_1 \wedge \cdots \wedge c_1), (c_1 \wedge c_1 \wedge \cdots \wedge c_1)\} = c_1$ for all $v \in V$. $d_\gamma(u) = \inf\{(c_2 \vee c_2 \vee \cdots \vee c_2), (c_2 \vee c_2 \vee \cdots \vee c_2)\} = c_2$ for all $v \in V$. $d_{(m)}(v) = (c_1, c_2)$ and $d(v) = (c_1, c_2) + (c_1, c_2) = 2(c_1, c_2)$. Hence G is $((r_1, r_2), m, (c_1, c_2))$ - regular intuitionistic fuzzy graph. □

Remark 5.2 The Converse of above theorem need not be true.

Example 5.3 Consider an intuitionistic fuzzy graph on $G^*(V, E)$.

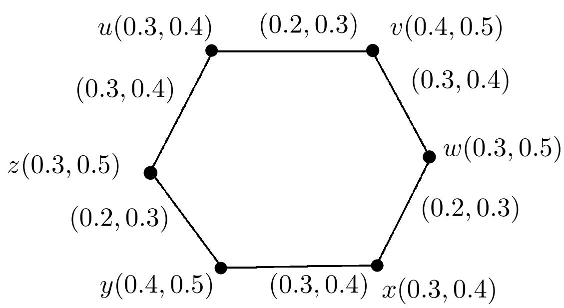

Figure 5

Here, $d(u) = (0.5, 0.7)$ and $d_{(3)}(u) = (0.3, 0.4)$, for all $u \in V$. Hence G is $((0.5, 0.7), 3, (0.3, 0.4))$-regular intuitionistic fuzzy graph. But B is not a constant function.

Theorem 5.4 *For any* $m \geq 1$, *let* $G : (A, B)$ *be an intuitionistic fuzzy graph such that* $G^*(V, E)$, *a cycle of length* $\geq 2m + 1$. *If* B *is constant function, then* G *is* $((r_1, r_2), m, (c_1, c_2))$ *- regular intuitionistic fuzzy graph, where* $(r_1, r_2) = 2B(uv)$ *and* $(c_1, c_2) = 2B(uv)$.

Proof Suppose B is constant function say $B(uv) = (c_1, c_2)$, for all $uv \in E$. Then, $d_{(m)\mu_1}(v)$= $\{c_1 \wedge c_1 \wedge \cdots \wedge c_1\} + \{c_1 \wedge c_1 \wedge \cdots \wedge c_1\} = c_1 + c_1 = 2c_1$, $d_{(m)\gamma_1} = \{c_2 \vee c_2 \vee \cdots \vee c_2\} + \{c_2 \vee c_2 \vee \cdots \vee c_2\} = c_2 + c_2 = 2c_2$, for all $v \in V$. So,$d_{(m)}(v) = 2(c_1, c_2)$, for all $u \in V$. Also, $d(v) = (c_1, c_2) + (c_1, c_2) = 2(c_1, c_2)$ Hence G is $(2(c_1, c_2), m, 2(c_1, c_2))$ - regular intuitionistic fuzzy graph. □

Theorem 5.5 *For any* $m \geq 1$, *let* $G : (A, B)$ *be an intuitionistic fuzzy graph such that* $G^*(V, E)$, *a cycle of length* $\geq 2m + 1$. *If the alternate edges have the same membership values and same non membership values, then* G *is* $((r_1, r_2), m, (c_1, c_2))$ *- regular intuitionistic fuzzy graph.*

Proof If the alternate edges have same membership and same non membership values then, let

$$\mu(e_i) = \begin{cases} k_1 & \text{if } i \text{ is odd} \\ k_2 & \text{if } i \text{ is even} \end{cases} \qquad \gamma(e_i) = \begin{cases} k_3 & \text{if } i \text{ is odd} \\ k_4 & \text{if } i \text{ is even} \end{cases}$$

Here, we have 4 possible cases.

Case 1. Suppose $k_1 \leq k_2$ and $k_3 \geq k_4$.

$d_{(m)\mu_1}(v) = min\{k_1, k_2\} + min\{k_1, k_2\} = k_1 + k_1 = 2k_1;$

$d_{(m)\gamma_1}(v) = max\{k_3, k_4\} + max\{k_3, k_4\} = k_3 + k_3 = 2k_3;$

$d_{(m)}(v) = (2k_1, 2k_3)$ and $d(v) = (k_1, k_3) + (k_2, k_4) = (k_1 + k_2, k_3 + k_4)$.

Case 2. Suppose $k_1 \leq k_2$ and $k_3 \leq k_4$.

$d_{(m)\mu_1}(v) = \min\{k_1, k_2\} + min\{k_1, k_2\} = k_1 + k_1 = 2k_1;$

$d_{(m)\gamma_1}(v) = \max\{k_3, k_4\} + max\{k_3, k_4\} = k_4 + k_4 = 2k_4;$

$d_{(m)}(v) = (2k_1, 2k_4)$ and $d(v) = (k_1, k_3) + (k_2, k_4) = (k_1 + k_2, k_3 + k_4)$.

Case 3. Suppose $k_1 \geq k_2$ and $k_3 \leq k_4$.

$d_{(m)\mu_1}(v) = \min\{k_1, k_2\} + \min\{k_1, k_2\} = k_2 + k_2 = 2k_2;$

$d_{(m)\gamma_1}(v) = \max\{k_3, k_4\} + \max\{k_3, k_4\} = k_4 + k_4 = 2k_4;$

$d_{(m)}(v) = (2k_2, 2k_4)$ and $d(v) = (k_1, k_3) + (k_2, k_4) = (k_1 + k_2, k_3 + k_4)$.

Case 4. Suppose $k_1 \geq k_2$ and $k_3 \geq k_4$.

$d_{(m)\mu_1}(v) = min\{k_1, k_2\} + min\{k_1, k_2\} = k_2 + k_2 = 2k_2;$

$d_{(m)\gamma_1}(v) = max\{k_3, k_4\} + max\{k_3, k_4\} = k_3 + k_3 = 2k_3;$

$d_{(m)}(v) = (2k_2, 2k_3)$ and $d(v) = (k_1, k_3) + (k_2, k_4) = (k_1 + k_2, k_3 + k_4)$.

Thus, $d(v) = (r_1, r_2)$ and $d_{(m)}(v) = (c_1, c_2)$ for all $v \in V$. Hence G is $((r_1, r_2), m, (c_1, c_2))$-regular intuitionistic fuzzy graph. □

Remark 5.6 Even if the alternate edges of an intuitionistic fuzzy graph whose underlying graph is an even cycle of length $\geq 2m + 2$ having same membership values and same non membership values, then G need not be totally $((r_1, r_2), m, (c_1, c_2))$- regular intuitionistic fuzzy graph, since if A $= (\mu_1, \gamma_1)$ is not a constant function, G is not totally $((r_1, r_2), m, (c_1, c_2))$ - regular intuitionistic fuzzy graph, for any $m \geq 1$.

Theorem 5.7 *For any $m \geq 1$, let $G : (A, B)$ be an intuitionistic fuzzy graph on $G^* : (V, E)$, a cycle of length $\geq 2m + 1$. Let $k_2 \geq k_1$, $k_4 \geq k_3$ and let*

$$\mu(e_i) = \begin{cases} k_1 & \text{if } i \text{ is odd} \\ k_2 & \text{if } i \text{ is even} \end{cases} \qquad \gamma(e_i) = \begin{cases} k_3 & \text{if } i \text{ is odd} \\ k_4 & \text{if } i \text{ is even} \end{cases}$$

Then, G is $((r_1, r_2), m, (c_1, c_2))$- regular intuitionistic fuzzy graph.

Proof We consider cases following.

Case 1. Let $G : (A, B)$ be an intuitionistic fuzzy graph on $G^*(V, E)$, an even cycle of length $\leq 2m + 2$. Then by theorem 5.3, G is $((r_1, r_2), m, (c_1, c_2))$ - regular intuitionistic fuzzy graph.

Case 2. Let $G = (A, B)$ be an intuitionistic fuzzy graph on $G^*(V, E)$ an odd cycle of length $\geq 2m + 1$. For any $m \geq 1$, $d_{(m)}(v) = (2k_1, 2k_4)$, for all $v \in V$. But $d(v_1) = (k_1, k_3) + (k_1, k_3) = 2(k_1, k_3)$ and $d(v_i) = (k_1, k_3) + (k_2, k_4) = ((k_1 + k_2), (k_3 + k_4))$ So, $d(v_i) \neq d(v_1)$ for $i = 2, 3, \ldots, m$

Hence G is not $((r_1, r_2), m, (c_1, c_2))$-regular intuitionistic fuzzy graph. □

Remark 5.8 Let $G : (A, B)$ be an intuitionistic fuzzy graph such that $G^*(V, E)$ is a cycle of length $\geq 2m + 1$. Even if let

$$\mu(e_i) = \begin{cases} k_1 & \text{if } i \text{ is odd} \\ k_2 & \text{if } i \text{ is even} \end{cases} \qquad \gamma(e_i) = \begin{cases} k_3 & \text{if } i \text{ is odd} \\ k_4 & \text{if } i \text{ is even} \end{cases}$$

Then G need not be totally $((r_1, r_2), m, (c_1, c_2))$- regular intuitionistic fuzzy graph, since if $A = (\mu_1, \gamma_1)$ is not a constant function, G is not totally $((r_1, r_2), m, (c_1, c_2))$- regular intuitionistic fuzzy graph.

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