Origin of the inertial mass (II): Vector gravitational theory

Wenceslao Segura González
e-mail: wenceslaoseguragonzalez@yahoo.es
Independent Researcher

Abstract. We deduce the inductive forces of a vectorial gravitational theory and we will study whether these forces can be identified with the forces of inertia that act on a body when it is accelerated.

1. Introduction

In a previous work we investigate the induction phenomena derived from a scalar gravitational theory [1]. We find that there are forces that have their origin in the movement of the field source, but they do not satisfactorily explain the origin of inertia

Now we propose a vector gravitational theory and we will also investigate its inductive effects, with the idea of explaining the phenomenon of inertia. An investigation of this type was made in a famous work by Sciama, but its deduction from centrifugal and Coriolis forces [2] is not satisfactory, which is what we will try to remedy in this paper.

When studying the effect of the Universe as a whole, we have to choose a cosmic model. As we are interested in the qualitative aspects, we consider the simplest cosmic model: static, with homogeneous density, large enough and of finite age. In later works we will examine the problem with more realistic cosmological models.

We advise the previous reading of the first part of this research: «Origin of the inertial mass (I): Scalar gravittional theory» [1].

2 Vector gravitational theory

To obtain a vector gravitational theory we have to adapt the electromagnetic theory to the gravity [3]. The gavitational potential is a tetravector that we define by

$$\phi^k = (\phi, c\mathbf{A})$$

ϕ is the scalar potential and **A** the vector potential. The source is the current tetradensity

$$j^k = \rho u^k = (j^o, \mathbf{j}) \tag{1}$$

ρ is the proper density of matter and u^k is the tetravelocity defined by

$$u^k = \frac{dx^k}{d\tau} \tag{2}$$

$d\tau$ is the proper time of the source particle.

The field equation is

$$\nabla^2\phi^k - \frac{1}{c^2}\frac{\partial^2\phi^k}{\partial t^2} = \frac{4\pi G}{c} j^k .$$

3 The proper time

We assume that movement is relative. In other words, it is equivalent to say that a body moves with respect to the Universe, that to say that it is the Universe that moves with respect to the body. Both situations are equivalent kinematically and dynamically. In our simplified cosmic model all the constituent bodies of the Universe are at rest among themselves, with the exception of the C body whose movement we study.

When we assume that the Universe as a whole is in motion, the proper time of its constituents coincides with the coordinate time t of the reference system with respect to which the Universe is at rest, which is naturally an inertial reference system, where all clocks that are at rest can be synchronized. Therefore, the tetravelocity of any fixed body to the whole of the Universe in any reference system is

$$u^k = \frac{dx^k}{d\tau} = \frac{dx^k}{dt},$$

therefore

$$\mathbf{u} = \left(u^\alpha\right) = \left(\frac{dx^\alpha}{dt}\right); \quad u^0 = c; \quad j^0 = \rho c; \quad \mathbf{j} = \rho\mathbf{u}.$$

Then the field equation is divided into a scalar field and a vector field

$$\nabla^2\phi - \frac{1}{c^2}\frac{\partial^2\phi}{\partial t^2} = 4\pi G\rho; \quad \nabla^2\mathbf{A} - \frac{1}{c^2}\frac{\partial^2\mathbf{A}}{\partial t^2} = \frac{4\pi G}{c^2}\mathbf{j}. \tag{3}$$

4 Retarded potentials

Equations (3) are solved with the retarded potentials technique

$$\phi(\mathbf{r},t) = -G\int\frac{[\rho]}{r'}dV'; \quad \mathbf{A}(\mathbf{r},t) = -\frac{G}{c^2}\int\frac{[\mathbf{j}]}{r'}dV' \tag{4}$$

$\mathbf{r}$ is the position vector of the field point, $\mathbf{r}'$ is the position of the field point with respect to the source at the moment of generating the force (or position retarded) and the brackets are values at the moment of emission or retarded values, that is in the instant $t' = t - r'/c$ .The vector $\boldsymbol{\sigma}$ is the position of a source point, therefore $\mathbf{r}' = \mathbf{r} - \boldsymbol{\sigma}$. From (4) we note that vector potential is of second order with respect to c.

If the source moves with a velocity $\mathbf{u}$ then the potentials of Liénard-Wiechert have to be applied

$$d\phi = -G\frac{[dm]}{r' - \frac{\mathbf{u}\cdot\mathbf{r}'}{c}} = -G\frac{[dm]}{s}; \quad d\mathbf{A} = -\frac{G}{c^2}\frac{[dm]\mathbf{u}}{s}. \tag{5}$$

For the calculation of (5) we must use relationships [4]

$$\nabla\left(\frac{1}{s}\right) = -\frac{\mathbf{r}'}{s^2 r'} + \frac{\mathbf{u}}{cs^2} - \frac{\mathbf{r}'(\mathbf{r}'\cdot\mathbf{u})}{cs^3 r'} - \frac{\mathbf{r}'(\mathbf{r}'\cdot\mathbf{a})}{c^2 s^3} + \frac{u^2\mathbf{r}'}{c^2 s^3}$$
$$\frac{\partial}{\partial t}\left(\frac{\mathbf{u}}{s}\right) = \frac{r'}{s}\left[\frac{\mathbf{a}}{s} + \frac{\mathbf{u}(\mathbf{r}'\cdot\mathbf{u})}{s^2 r'} + \frac{\mathbf{u}(\mathbf{r}'\cdot\mathbf{a})}{cs^2} - \frac{\mathbf{u}u^2}{cs^2}\right]. \tag{6}$$

$\mathbf{u}$ is the retarded velocity of the source and $\mathbf{a}$ the aceleration, therefore

$$\mathbf{u} = \frac{d\boldsymbol{\sigma}}{dt'} = -\frac{d\mathbf{r}'}{dt'}.$$

The gravitoelectric field $\mathbf{E}$ and the gravitomagnetic field $\mathbf{B}$ are derived from the potentials ϕ and $\mathbf{A}$ in the same way as in electromagnetism

$$\mathbf{E} = -\nabla\phi - \frac{\partial\mathbf{A}}{\partial t}; \quad \mathbf{B} = \nabla\wedge\mathbf{A}, \tag{7}$$

the force acting on a particle of gravitational mass m_g that has a velocity $\mathbf{w}$ is

$$\mathbf{F} = m_g\mathbf{E} + m_g\mathbf{w}\wedge\mathbf{B}. \tag{8}$$

5 Induction of forces on a body that has a rectilinear acceleration

Suppose a body C that moves with accelerated and rectilinear movement with respect to the whole of the Universe. The situation is equivalent to saying that the Universe moves with respect to the body C. $\mathbf{u}$ and $\mathbf{a}$ are the velocity and acceleration of the Universe with respect to the body C. Therefore the body C has velocity $-\mathbf{u}$ and acceleration $-\mathbf{a}$ in relation to the Universe.

To integrate (5) we assume the Universe divided into concentric spherical shells of thickness $d\sigma$. dm is an infintesimal portion of mass of one of those shells, then the force exerted on body C is deduced from (5) and (8)

$$-m_g \nabla d\phi - \frac{\partial(d\mathbf{A})}{\partial t} = G[dm]m_g \nabla\left(\frac{1}{s}\right) + \frac{G}{c^2}[dm]m_g \frac{\partial}{\partial t}\left(\frac{\mathbf{u}}{s}\right).$$

Comparing (4) and (5)

$$[dm] = \left(1 - \frac{\mathbf{r}' \cdot \mathbf{u}}{r'c}\right)[\rho]dV' \approx [\rho]dV',$$

therefore

$$-m_g \nabla d\phi - \frac{\partial(d\mathbf{A})}{\partial t} = G[\rho]m_g \nabla\left(\frac{1}{s}\right)dV' + \frac{G}{c^2}[\rho]m_g \frac{\partial}{\partial t}\left(\frac{\mathbf{u}}{s}\right)dV'.$$

From (6) we find that the only inductive terms (which are those that depend on the movement of the source) are

$$\nabla\left(\frac{1}{s}\right) = -\frac{\mathbf{r}'(\mathbf{r}' \cdot \mathbf{a})}{c^2 s^3}; \quad \frac{\partial}{\partial t}\left(\frac{\mathbf{u}}{s}\right) = \frac{\mathbf{a}}{s} + \frac{\mathbf{u}(\mathbf{r}' \cdot \mathbf{a})}{cs^2}$$

we have put $r' \approx \sigma$ and we neglect the terms that depend on $1/\sigma^2$ because they are negligible compared to those that depend on $1/\sigma$.

For simplicity we assume that the acceleration $-\mathbf{a}$ of the Universe is parallel to the x axis $\mathbf{a} = a\mathbf{i}$ and that in polar coordinates the vector $\boldsymbol{\sigma}$ is

$$\boldsymbol{\sigma} = \sigma \sin\theta \cos\varphi \mathbf{i} + \sigma \sin\theta \sin\varphi \mathbf{j} + \sigma \cos\theta \mathbf{k} \tag{9}$$

using the results found in the reference 1 we arrived after the integration to

$$-m_g \nabla \phi - \frac{\partial \mathbf{A}}{\partial t} = -\frac{2\pi}{3}\frac{G}{c^2}\rho m_g \sigma^2 \mathbf{a} + 2\pi \frac{G}{c^2}\rho m_g \sigma^2 \mathbf{a} = \frac{4\pi}{3} G\rho t^2 m_g \mathbf{a} \tag{10}$$

the limit of spatial integration is the furthest point from the Universe that has a causal relationship with the body C, that is $\sigma = ct$, where t is the age of the Universe.

The force (10) acts on the body C when it has the rectilinear acceleration $\mathbf{a}$, and is caused by the action of the whole of the Universe. We identify this force with the force of inertia, therefore

$$-m_i(-\mathbf{a}) = \frac{4\pi}{3} G\rho t^2 m_g \mathbf{a} \quad \Rightarrow \quad m_i = \frac{4\pi}{3} G\rho t^2 m_g, \tag{11}$$

m_i is the inertial mass. As in the present moment the inertial mass is assumed equal to the gravitational mass, then by (11)

$$\frac{4\pi}{3} G\rho t_0^2 = 1 \quad \Rightarrow \quad \phi_0 = c^2 \tag{12}$$

t_0 is the current age of the Universe and ϕ_0 is the current gravitational potential in the position occupied by the body C. Naturally, these results are valid in the framework of the cosmic model considered.

(11) means that the inertial mass is the result of the action of the whole of the Universe on the body, it is not an innate magnitude of the matter, but it is acquired by the cosmic action. The numerical value (12) is approximately correct. Therefore we conclude, within the limitations of our cosmic model, that the Mach's principle is fulfilled, which states that the inertia of a body is produced by the inductive forces of the Universe.

6 Centrifugal force induction

We will investigate if the relative rotation of the Universe induces centrifugal force on a body C. For this we use (5) and (6) and follow the same technique as in the previous section.

We will consider that body C is near the origin of coordinates and on the z-axis

$$\mathbf{r} = r\mathbf{k}$$

we assume that the rotation of the Universe is around the y axis

$$\boldsymbol{\omega} = \omega \mathbf{j},$$

which means that the body C rotates with respect to the Universe with the angular velocity $-\boldsymbol{\omega}$.

The position vector of a source point is (9).

From $\mathbf{r}' = \mathbf{r} - \boldsymbol{\sigma}$ it follows

$$r' = \sigma\sqrt{1+\left(\frac{r}{\sigma}\right)^2 - 2\frac{r}{\sigma}\cos\theta} \approx \sigma - r\cos\theta \quad \Rightarrow \quad \frac{r'}{\sigma} \approx 1 - \frac{r}{\sigma}\cos\theta. \tag{13}$$

The velocity and acceleration of a source point is

$$\mathbf{u} = \boldsymbol{\omega} \wedge \boldsymbol{\sigma} = \sigma\omega\cos\theta\mathbf{i} - \sigma\omega\sin\theta\cos\varphi\mathbf{k}; \quad \mathbf{a} = \boldsymbol{\omega} \wedge (\boldsymbol{\omega} \wedge \boldsymbol{\sigma}) = -\sigma\omega^2\sin\theta\cos\varphi\mathbf{i} - \sigma\omega^2\cos\theta\mathbf{k}$$

and we assume that $s \approx \sigma$.

The terms that interest us are the fourth and fifth of the first equation (6), which were integrated in reference 1, and the first and second of the second equation (6), whose integral is

$$-m_g\frac{\partial\mathbf{A}}{\partial t} = \frac{G}{c^2}[\rho]m_g\iiint\left[\frac{\mathbf{a}}{\sigma}\left(1-\frac{r}{\sigma}\cos\theta\right) + \frac{\mathbf{u}(\mathbf{r}\cdot\mathbf{u})}{\sigma^3}\right]\sigma^2\sin\theta d\theta d\varphi d\sigma$$

we have used the equation (13). Adding the results, the induction force is

$$-m_g\nabla\phi - m_g\frac{\partial\mathbf{A}}{\partial t} = -\frac{2\pi}{3}G\rho m_g\omega^2\mathbf{r}t^2 + \frac{4\pi}{3}G\rho m_g\omega^2\mathbf{r}t^2 = \frac{2\pi}{3}G\rho m_g\omega^2\mathbf{r}t^2.$$

therefore

$$m_i = \frac{2\pi}{3}G\rho t^2 m_g$$

which means that the force of induction produced by the relative rotation of the whole of the Universe produces a force that is identified in sense and in magnitude with the centrifugal force.

7 Coriolis force induction

We try to know if the force of Coriolis acting on a body that has a velocity **w** can be identified with an induction force produced by the relative rotation of the Universe. This force can only be originated by the gravitomagnetic term of the force or second term of the equation (8).

We need to know the strength of gravitomagnetic field **B** and for this we must calculate the vector potential from (4)

$$d\mathbf{A} = -\frac{G}{c^2}\frac{[\mathbf{j}]}{r'}dV' = -\frac{G}{c^2}\frac{\rho\boldsymbol{\omega}\wedge\boldsymbol{\sigma}}{\sigma}\left(1+\frac{r}{\sigma}\cos\theta\right)dV',$$

suppose that $\mathbf{r} = r\mathbf{k}$ and that the relative rotation of the Universe has any orientation, then the potential produced by a spherical shell of thickness $d\sigma$ and located at distance σ is

$$\delta\mathbf{A} = -\frac{G}{c^2}\frac{\rho}{\sigma}\sigma^2 d\sigma\boldsymbol{\omega} \wedge \iint\boldsymbol{\sigma}\left(1+\frac{r}{\sigma}\cos\theta\right)\sin\theta d\theta d\varphi$$

by (9)

$$\delta\mathbf{A} = -\frac{4\pi}{3}\frac{G}{c^2}\rho\,\boldsymbol{\omega}\wedge\mathbf{r}\sigma d\sigma \quad \Rightarrow \quad \mathbf{A} = -\frac{2\pi}{3}G\rho t^2\boldsymbol{\omega}\wedge\mathbf{r},$$

again we consider the simplified cosmic model. Now we use another coordinate system where

$$\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}; \quad \boldsymbol{\omega} = \omega\mathbf{j}$$

then

$$\nabla \wedge (\boldsymbol{\omega}\wedge\mathbf{r}) = 2\boldsymbol{\omega}$$

therefore the gravitomagnetic field strength is

$$\mathbf{B} = \nabla\wedge\mathbf{A} = -\frac{4\pi}{3}G\rho t^2\boldsymbol{\omega},$$

when applying equation (8) we find the induction force acting on particle C

$$\mathbf{F} = m_g\mathbf{w}\wedge\mathbf{B} = \frac{2\pi}{3}G\rho t^2 2\boldsymbol{\omega}\wedge\mathbf{w}$$

According to the Mach's principle, this induction force is the force of inertia associated with the Coriolis acceleration

$$\frac{2\pi}{3}G\rho t^2 m_g 2\boldsymbol{\omega}\wedge\mathbf{w} = -m_i\left(-2\boldsymbol{\omega}\wedge\mathbf{w}\right) \quad\Rightarrow\quad m_i = \frac{2\pi}{3}G\rho t^2 m_g$$

which means that the force of inertia of Coriolis can be interpreted as the force of induction of the rotation of the Universe with respect to the body C.

8 Inertial force produced by angular acceleration

Suppose that a body C is near the origin of a reference system, with respect to which the Universe has an angular velocity $\boldsymbol{\omega}$ and an angular acceleration $\dot{\boldsymbol{\omega}}$. We want to find the induction force exerted on the body C by the angular acceleration.

We assume that

$$\mathbf{r} = r\mathbf{k}; \quad \boldsymbol{\omega} = \omega\mathbf{j}$$

and $\boldsymbol{\sigma}$ is given by (9), then the velocity **u** and the acceleration **a** resulting from the variation of the angular acceleration magnitude are

$$\mathbf{u} = \sigma\,\omega cos\theta\,\mathbf{i} - \sigma\omega\sin\theta\cos\varphi\mathbf{k}; \quad \mathbf{a} = \dot{\boldsymbol{\omega}}\wedge\boldsymbol{\sigma} = \sigma\,\dot{\omega}cos\theta\,\mathbf{i} - \sigma\dot{\omega}\sin\theta\cos\varphi\mathbf{k}.$$

The only terms that interest us are the fourth of the first equation (6) and the first of the second equation (6). The gravitomagnetic part of the induction force is

$$-m_g\frac{\partial\mathbf{A}}{\partial t} = \frac{G}{c^2}[\rho]m_g\iiint\frac{\mathbf{a}}{\sigma}\left(-\frac{r}{\sigma}\cos\theta\right)\sigma^2\sin\theta d\theta d\varphi d\sigma = \frac{2\pi}{3}G\rho m_g t^2\dot{\boldsymbol{\omega}}\wedge\mathbf{r}$$

which, when adding to the gravitoelectric part calculated in reference 1, we get a zero induction force. Therefore the inertial force produced by the acceleration of the rotation can not be deduced from the vector gravitational theory.

At this point we would like to warn that the equation (8) is an adaptation of the electromagnetic theory, but in the General Theory of Relativity, the first terms of the corresponding force are [6]

$$\mathbf{F} = -m_g\nabla\phi - 4m_g\frac{\partial\mathbf{A}}{\partial t} + 4\mathbf{w}\wedge\mathbf{B}$$

where the two magnetic terms are multiplied by the coefficient 4. This makes us doubt the equation (8).

9 Induction force on a body in uniform motion

Suppose a body C that has a uniform and rectilinear movement with respect to the Universe. **u** is the velocity of the Universe with respect to C. Next we calculate the induction force that is exerted on C.

The only terms of (6) that we have to use in this problem is the second and third of the first equation, the other terms not null can be neglected for non-relativistic velocities, then

$$\nabla\left(\frac{1}{s}\right) = \frac{\mathbf{u}}{cs^2} - \frac{\mathbf{r}'\left(\mathbf{r}'\cdot\mathbf{u}\right)}{cs^3r'}$$

and the induction force produced by an element of the spherical shell is

$$-m_g\nabla d\phi - \frac{\partial\left(d\mathbf{A}\right)}{\partial t} = G\rho m_g\left[\frac{\mathbf{u}}{cs^2} - \frac{\mathbf{r}'\left(\mathbf{r}'\cdot\mathbf{u}\right)}{cs^3r'}\right]\sigma^2\sin\theta d\theta d\varphi d\sigma.$$

Integrating for the whole of the Universe and neglecting terms, the induction force acting on the body C is

$$\mathbf{F} = \frac{8\pi}{3}G\rho t m_g\mathbf{u}, \tag{14}$$

by the principle of dynamic equilibrium the sum of all the forces acting on the body is zero. We assume that on the body acts in addition to the force (14), the force of inertia, which for a body in rectilinear motion is the equation (10), then when we apply the principle of dynamic equilibrium is found

$$\mathbf{F} + \mathbf{F}_i = 0 \quad\Rightarrow\quad \frac{8\pi}{3}G\rho t m_g\mathbf{u} = \frac{4\pi}{3}G\rho t^2 m_g\mathbf{a} \quad\Rightarrow\quad \mathbf{a} = \frac{2\mathbf{u}}{t} \tag{15}$$

t is the age of the Universe and **a** is the acceleration of the body C. The equation (15) tells us that when a body is in uniform motion with respect to the Universe, an induction force appears that produces an acceleration given by (15), of magnitude so extraordinarily small that we can consider it null. Therefore there is no force of inertia on a body in uniform motion and therefore remains with equal velocity, a statement that corresponds to the principle of inertia.

10 Participation of the Universe in the formation of the inertial mass

According to Mach's principle, the inertial mass of a body is generated by the action of the whole Universe. On a body C act forces that are produced in all parts of the observable Universe. Some of these forces originate in very distant objects, that is, they were produced a long time ago; while the forces exerted by nearby objects were produced a short time ago.

Now we study the participation in the generation of the inertial mass of the different epochs of the Universe. We consider the Universe formed by N spherical shells in whose center is the body C. To the furthest shell we give the numeration $n = 0$ and in it are the objects that produce the force at the beginning of the Universe, whose effects reach the body C in the present moment, when the age of the Universe is t_0.

The thickness of the spherical shells is $c\tau$ where $\tau = t_0/N$. The inertial mass produced by shell $t = t_0/N$ is by (11)

$$\delta m_i = \frac{8\pi}{3} G\rho m_g t\tau \tag{16}$$

t is the time takes the gravitational interaction to travel from the spherical shell n to the body C, therefore

$$t = t_0 - n\tau$$

of (12) and (16) we get

$$\frac{\delta m_i}{m_i} = \frac{2}{N}\left(1 - \frac{t'}{t_0}\right) \tag{17}$$

$t' = n\tau$ is the time retarded.

We must remember that (17) is applicable to the cosmic model that we are considering which has a constant and uniform density of matter in the Universe. But independently of the cosmic model, (17) shows us that each epoch of the Universe contributes differently to the formation of the inertial mass of a body. (17) shows that the first moments of the Universe (when $t' = 0$) have a greater contribution to the inertial mass; while the present Universe ($t' = t_0$) the contribution is minimal. This result can be more pronounced in a realistic cosmic model.

11 Mach's principle and the Big Bang

Now we will use a more realistic cosmic model with the exclusive idea of studying the influence that the Big Bang has on the formation of the inertial mass, and considering mainly the qualitative aspects.

Now we suppose, for example, a Universe where the density depends on the cosmic age according to the law

$$\rho(t') = \rho_0 \frac{t_0}{t'} \tag{18}$$

ρ_0 is the density at the present moment and t' is the age of the Universe at the time of generating the force or retarded time. By (18) the density of the Universe would be very high when its age is small, a situation similar to what happens in the Big Bang, therefore (18) will inform us about how the inertial mass is generated in a Universe that has a high density at its beginning.

By (17) the contribution to the inertial mass of a spherical shell located at a distance ct and thickness cdt is

$$dm_i = \frac{8\pi}{3} G\rho_0 \frac{t_0}{t'} m_g t dt = \frac{8\pi}{3} G\rho_0 t_0 m_g \frac{t}{t_0 - t} dt$$

when we integrate for the whole Universe we find a singularity in $t = t_0$. To avoid this singularity we calculate the action of the Universe from a time t_c (instead of t_0)

$$\frac{m_i}{m_g} = \frac{8\pi}{3} G\rho_0 t_0^2 \left[\frac{t_c}{t_0} + \ln\left(1 - \frac{t_c}{t_0}\right) \right], \tag{19}$$

with the current values of ρ_0 and t_0 is found

$$\frac{8\pi}{3} G\rho_0 t_0^2 \approx 0{,}92.$$

we verified that the inertial mass observed in the current epoch would be generated by the forces produced by the Universe from $t_c = 0{,}46t_0$ to $t_c = t_0$. But taking into account ancient cosmic actions, the relationship (18) increases, which means that the inertial mass becomes much greater than the gravitational mass.

For example, if we take the limit at $t_c = 0{,}999t_0$, which corresponds to an age of the Universe of about fourteen million years, the ratio between the inertial mass and the gravitational mass is approximately 8.5.

The conclusion is that if the density in the past was greater than at present (as it must happen in a cosmic model with Big Bang), the participation in the inertial mass of the primitive Universe increases, until being determinant. That is to say, that the inertial mass would be generated mainly by the first stages of the Universe, until being very superior to the mass observed.

We conclude that if the Universe had a Big Bang, the Mach's principle would not be fulfilled and vice versa.

12 Force induced by cosmic expansion

Next, we calculate the induction force produced by a movement of cosmic expansion. We assume that the observer is at the center of the Universe, therefore $r' = \sigma$. And that the expansion velocity is radial

$$\mathbf{u} = u\frac{\boldsymbol{\sigma}}{\sigma} = u\sin\theta\cos\varphi\mathbf{i} + u\sin\theta\sin\varphi\mathbf{j} + u\cos\theta\mathbf{k}$$

where u is any function of σ.

We follow the same procedure as in the previous sections. So when calculating the angular integrals that are derived from the two equations (6) we find that the result is zero. That is, a cosmic expansion does not produce induction force, which is logical for reasons of symmetry.

13 Conclusions

We have used a vector gravitational theory, from which we deduce induction forces. The results show that it is possible to identify the induction force produced by the Universe with the force of inertia acting on an accelerated body. At least we find the right sense for these forces and a magnitude that is in line with observations.

However, we have not been able to deduce a force of inertia for the case in which the Universe has angular acceleration. Another defect found is that the coefficients of proportionality between the inertial mass and the gravitational mass varies according to the type of acceleration considered.

The conclusion is that although the vector gravitational theory is an approximation to the correct theory, and the cosmic model that we have used is very simple, we find that the induction forces produced by the whole Universe are comparable in sense and magnitude to the forces of inertia that are observed in accelerated bodies, which is a strong support for Mach's principle.

In the third part of this research we will deal with a tensor theory, which will allow us to improve the results obtained so far. In later research we will apply the results to more realistic cosmic models.

Bibliography

(1) SEGURA GONZÁLEZ, Wenceslao: «Origin of the inertial mass (I): scalar gravitational theory», viXra.org, 2018, http://vixra.org/abs/1809.0524.

(2) SCIAMA, D. W.: «On the origin of inertia», *Monthly Notices of the Royal Astronomical Society*, Vol. 113, 1953, pp.34-42.

(3) SEGURA GONZÁLEZ, Wenceslao: *Teoría relativista de campo*, eWT Ediciones, 2014, pp. 76-93.
(4) SEGURA GONZÁLEZ, Wenceslao: «Mach's principle: the cosmic origin fo the inertial mass», 2017, viXra: 1706.0145.
(5) PANOFSKY W.; PHILLIPS, M.: *Classical Electricity and Magnetism*, Addison-Wesley, 1972. pp. 354-356.
(6) SEGURA GONZÁLEZ, Wenceslao: *Gravitoelectromagnetismo y principio de Mach*, eWT Ediciones, 2013, pp. 47-50.