Origin of the inertial mass (I): scalar gravitational theory

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Abstract. We will deduce the induction forces obtained from the field equation of Nordstrom's scalar gravitational theory and investigate whether they can explain the origin of the forces of inertia acting on a body when it is accelerated.

1. Introduction

The General Theory of Relativity is a relativistic field theory, whose potential is the second-order metric tensor. The theory is expressed with a system of nonlinear differential equations of second degree. The source of this field is the second-order energy-momentum tensor.

But we can investigate simpler gravitational theories, such as a scalar theory (whose potential is the ecalar ϕ) or a vector theory (whose potential is the tetravector ϕ^k). In the first theory the field source is a scalar (the density of mass ρ) and in the second theory it will be a vector (the current density j^k). Both theories have to be developed within the framework of the special theory of relativity, that is, the space-time is pseudo-euclidian.

These theories are linear, which means that it is necessary to add the equations of motion. This is not necessary in a non-linear theory such as General Relativity.

In this paper, we will study the inductive effect of scalar gravity and investigate whether the induction produced by the whole universe can explain the origin of inertia.

In the year 1952 Dennis Sciama wrote a famous paper entitled "On the origin of inertia" [1,2], where he developed a simplified vector gravitational theory and affirmed that a scalar theory of gravity would not produced inductive effects. As we will see later this statement is wrong.

Mach's principle affirms that the inertia (or the inertial mass) of a body is originated by the whole of the Universe. We understand that the force of inertia is $-m_i\mathbf{a}$ where m_i is the inertial mass and $\mathbf{a}$ the acceleration of the body C, that is, a force that is opposite to the accelerated movement of C. Then the Mach's principle affirms that this force of inertia is produced by the inductive forces that have their origin in the relative movement of the Universe with respect to the body C. We try to prove whether this statement is true.

2 Newton's gravitational theory as a field theory. Nordström theory

Although in its original form Newton's theory of gravitation is not a field theory, we can formulate it as if it were. In effect, if ϕ is the gravitational potential, the field equation would be the Poisson equation

$$\nabla^2\phi = 4\pi G\rho \tag{1}$$

with the equation of movement

$$m_i\mathbf{a} = -m_g\nabla\phi \tag{2}$$

m_i and m_g are the inertial and gravitational masses of the body on which gravity acts and $\mathbf{a}$ is its acceleration.

But the equation (1) is not invariant under Lorentz transformations, which means that the field equation depends on the reference system, what is not admissible. The field derived from (1) is transmitted at infinite velocity, as corresponds to the action at a distance of the theory of Newton.

However, if we modify (1) as

$$\nabla^2\phi-\frac{1}{c^2}\frac{\partial^2\phi}{\partial t^2}=4\pi G\rho \tag{3}$$

then we have a field that is transmitted at the velocity of light c. Since ϕ and ρ are scalars, the field equation (3) is a relativistic invariant. We therefore have a scalar gravitational theory compatible with the special theory of relativity. (3) is the field equation of Nordström's scalar theory

The equation (2) must also be generalized. As the linear momentum is defined by

$$\mathbf{p}=\frac{m_i\mathbf{w}}{\sqrt{1-w^2/c^2}}=m_i\gamma\mathbf{w}$$

then the force acting on the particle of mass m_i that has a velocity **w** is

$$\mathbf{F}=\frac{d}{dt}\left(m_i\gamma\mathbf{w}\right)=-m_g\nabla\phi$$

for nonrelativistic velocities coincides with (2).

3 Retarded potential. Potential of Liénard-Wiechert

The solution of equation (3) is obtained by the theory of retarded potentials[3]

$$\phi(\mathbf{r},t)=-G\int\frac{\rho(t-r'/c)}{r'}dV'=-G\int\frac{[\rho]}{r'}dV' \tag{4}$$

the vector **r** is the position of the point of the field, $\mathbf{r}'$ is the retarded position of the source, t is the moment in which the gravitational signal arrives at the point of the field, dV' is the element of volume occupied by the source and the brackets are values at the moment of emission or retarded values, that is in the instant $t'=t-r'/c$

If the source of the field has a velocity **u** and an acceleration **a**, equation (4) must be adapted. The gravitational field $d\phi$ produced in the position **r** and at time t by a mass element dm that has velocity **u**, acceleration **a** and is in the position $\mathbf{r}'$ is given by the potential of Liénard-Wiechert[3]

$$d\phi=-G\frac{[dm]}{r'-\dfrac{\mathbf{u}\cdot\mathbf{r}'}{c}}=-G\frac{[dm]}{s}. \tag{5}$$

To apply the equation (5) to the scalar gravitational theory we need to know that[4]

$$\nabla\left(\frac{1}{s}\right)=-\frac{\mathbf{r}'}{s^2r'}+\frac{\mathbf{u}}{cs^2}-\frac{\mathbf{r}'(\mathbf{r}'\cdot\mathbf{u})}{cs^3r'}-\frac{\mathbf{r}'(\mathbf{r}'\cdot\mathbf{a})}{c^2s^3}+\frac{u^2\mathbf{r}'}{c^2s^3} \tag{6}$$

∇ is defined by

$$\nabla=\frac{\partial}{\partial x}\mathbf{i}+\frac{\partial}{\partial y}\mathbf{j}+\frac{\partial}{\partial z}\mathbf{k}$$

x, y, z are the Cartesian coordinates of the field.

In equation (6) appears the velocity and aceleration of the source, then the potential ϕ will also depend on this velocity and aceleration, therefore in the scalar gravitational theory there are phenomena of induction, that is forces produced by the movement of the source.

4 Relative movement

In the study that we will do next we accept that the movement is relative. Only the movement of one body with respect to another body has meaning, and all the magnitudes that are used (position, velocity and acceleration) are relative to other bodies. Therefore we deny the existence of a Newtonian absolute space.

It is kinematic and dynamically equivalent to affirm that a body moves with respect to the whole of the Universe or to say that it is the Universe that moves with respect to the body. Now, the relative motion of the Universe produces gravitational inductive effects as verified by (5) and (6), forces that act on any body that moves with respect to the Universe. According to Mach's principle, these forces are the forces of inertia.

We will study the inductive forces that are generated in the scalar gravitational theory for several different situations.

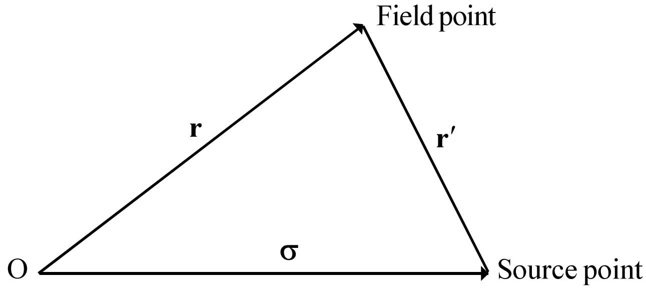
5 Induction of forces on a body that has a rectilinear acceleration

Let's consider a simplified model of the Universe, which will give us an acceptable view of the problem. We will assume that the Universe is static, of uniform density, of sufficiently large size and limited age. All the bodies of this Universe are at rest. A body C of gravitational mass m_g has an accelerated rectilinear movement with respect to the whole of the Universe.

As we consider that the movement is relative, the situation is equivalent to suppose that the body C is at rest and it is the Universe that moves in the opposite sense of the body C. **a** and **u** are the acceleration and velocity of the Universe with respect to the body C. Which means that C moves with acceleration $-\mathbf{a}$ and velocity $-\mathbf{u}$ with respect to the Universe.

K is a coordinate system whose origin O is close to body C and is at rest in relation to the Universe. The position vectors are those of drawing 1, therefore

$$\mathbf{r}' = \mathbf{r} - \boldsymbol{\sigma}. \tag{7}$$



Drawing 1.

We want to obtain classical effects, therefore we will suppose that the velocity u is very small with respect to c, that is, we neglect the relativistic effects. Then we consider the cosmic time t, the same for the whole Universe.

To make the integration of all the inductive forces produced by the Universe we will consider that it is formed by concentric spherical shells of "infinitesimal" thickness $d\sigma$. dm is an element of mass belonging to one of those shells, then the force that this mass exerts on the body C is by (5)

$$-m_g \nabla d\phi = G[dm] m_g \nabla\left(\frac{1}{s}\right). \tag{8}$$

Since the velocity u is small with respect to c, then

$$s = r' - \frac{\mathbf{r}' \cdot \mathbf{u}}{c} \approx r'.$$

From (6) we find that the only of its terms that we have to consider is the fourth, that has the dependence $1/\sigma$; all the others terms have the dependence $1/\sigma^2$ and since σ has a cosmic dimension these terms can be discarded. The first term of (6) produces the Newtonian gravitational force that we know vanishes when it is integrated for the whole Universe.

Comparing (4) with (5)

$$[dm] = \left(1 - \frac{\mathbf{r}' \cdot \mathbf{u}}{r'c}\right)[\rho] dV' \approx [\rho] dV',$$

using spherical coordinates we have of (8)

$$-m_g \nabla d\phi = G[\rho] m_g \nabla\left(\frac{1}{s}\right) dV' \approx -G[\rho] m_g \frac{\boldsymbol{\sigma}(\boldsymbol{\sigma} \cdot \mathbf{a})}{c^2 \sigma^3} \sigma^2 \sin\theta d\theta d\varphi d\sigma.$$

The induction force exerted by the spherical shell of thickness $d\sigma$ on the body C is

$$-m_g \nabla \delta\phi = \iint -m_g \nabla d\phi = -\frac{G}{c^2} \rho m_g \iint \frac{\boldsymbol{\sigma}(\boldsymbol{\sigma} \cdot \mathbf{a})}{\sigma} \sin\theta d\theta d\varphi d\sigma \tag{9}$$

ρ is the density of the Universe that is constant and homogeneous. First we will do this integration and then integrate for all the spherical shells of the Universe.

If, for sake of simplicity we assume that the acceleration **a** of the Universe is parallel to the *x* axis $\mathbf{a}=a\mathbf{i}$ and that in polar coordinates the vector $\boldsymbol{\sigma}$ is

$$\boldsymbol{\sigma}=\sigma\sin\theta\cos\varphi\mathbf{i}+\sigma\sin\theta\sin\varphi\mathbf{j}+\sigma\cos\theta\mathbf{k} \tag{10}$$

it is easy to find from (9)

$$-m_g\nabla\delta\phi=-\frac{4\pi}{3}\frac{G}{c^2}\rho m_g\mathbf{a}\sigma d\sigma. \tag{11}$$

Finally we do the integration for the whole Universe, taking into account that the furthest spherical shell that affects the body C is at a distance $\sigma=ct$, being *t* the age of the Universe, since the forces coming from more distant bodies have not had time to reach the body C. The integral of (10) results

$$-m_g\nabla\phi=-\frac{2\pi}{3}G\rho m_g\mathbf{a}t^2. \tag{12}$$

(12) means that if the body C of inertial mass m_i has an acceleration $-\mathbf{a}$ with respect to the Universe, on it acts the induction force (12), which can be understood as the force of inertia, namely

$$-m_i(-\mathbf{a})=-\frac{2\pi}{3}G\rho m_g\mathbf{a}t^2 \quad\Rightarrow\quad m_i=-\frac{2\pi}{3}G\rho t^2 m_g \tag{13}$$

which is absurd, not only because the inertial mass would be negative, but because the force of inertia would have the same sense as the acceleration of the body C, which does not correspond to the observed.

In the following sections we will investigate whether within the framework of the scalar gravitational theory inertial forces are generated that we can identify with the centrifugal and Coriolis forces.

6 Centrifugal force induction

We will investigate if the relative rotation of the Universe induces centrifugal force on a body C. For this we use (5) and (6) and follow the same technique as in the previous section.

We assume that body C is on the *z*-axis

$$\mathbf{r}=r\mathbf{k}$$

we assume that the rotation of the Universe is around the *y* axis

$$\boldsymbol{\omega}=\omega\,\mathbf{j},$$

which means that the body C rotates with respect to the Universe with the angular velocity $-\boldsymbol{\omega}$. The vector of position of a point of the Universe is given by (10).

The velocity of a point in the Universe with respect to the coordinate system for which the Universe is rotating is

$$\mathbf{u}=\boldsymbol{\omega}\wedge\boldsymbol{\sigma}=\sigma\omega\cos\theta\mathbf{i}-\sigma\omega\sin\theta\cos\varphi\mathbf{k};\quad \mathbf{a}=\boldsymbol{\omega}\wedge(\boldsymbol{\omega}\wedge\boldsymbol{\sigma})=-\sigma\omega^2\sin\theta\cos\varphi\mathbf{i}-\sigma\omega^2\cos\theta\mathbf{k}$$

As $\boldsymbol{\sigma}\cdot\mathbf{u}=0$ then [3]

$$\frac{\partial t'}{\partial t}=\frac{1}{1-\mathbf{u}\cdot\mathbf{r}'/cr'}\approx\frac{1}{1-\mathbf{r}\cdot(\boldsymbol{\omega}\wedge\boldsymbol{\sigma})/c\sigma}\approx 1 \quad\Rightarrow\quad t'\approx t.$$

being *t* the proper time at the origin of the coordinate system, or the temporal coordinate in the inertial system with respect to which the Universe is at rest. *r* and σ are proper distances, which coincide with the coordinates distances since radial distances are not affected by relativistic effects.

$\boldsymbol{\sigma}$ is perpendicular to **u** then by (7)

$$s=r'-\frac{\mathbf{r}'\cdot\mathbf{u}}{c}=r'+\frac{r\sigma\omega\sin\theta\cos\varphi}{c}\approx\sigma\left(1+\frac{\omega r\sin\theta\cos\varphi}{c}\right)\approx\sigma.$$

The only terms in (6) that are not canceled when the integration is done are the fourth and the fifth. Then the force induced by an infinitesimal spherical shell is at a distance σ

$$-m_g\nabla d\phi=G[\rho]m_g\nabla\left(\frac{1}{s}\right)dV'\approx G[\rho]m_g\left\{-\frac{\mathbf{r}'(\mathbf{r}'\cdot\mathbf{a})}{c^2s^3}+\frac{u^2\mathbf{r}'}{c^2s^3}\right\}\sigma^2\sin\theta d\theta d\varphi d\sigma.$$

Integrating for the whole Universe

$$-m_g \nabla \phi = \left(-2\pi + \frac{4\pi}{3} \right) G \rho m_g \omega^2 \mathbf{r} t^2 = -\frac{2\pi}{3} G \rho m_g \omega^2 \mathbf{r} t^2$$

then it follows

$$m_i = -\frac{2\pi}{3} G \rho t^2 m_g , \tag{14}$$

we find again a force of inertia contrary to the observed, which is always opposite to the acceleration of the body. However, the numerical value of proportionality between the inertial and the gravitational mass is roughly in accordance with the cosmic measurements.

If the Universe is supposed to be rotating it must be admitted that its constituents, except the closest bodies, have a velocity greater than that of light. But these superluminal velocities are not a problem because the whole Universe moves in unison and therefore the principle of causality remains valid. There are no singularities in expressions like $\sqrt{1 - u^2/c^2}$ since the proper time of all the constituents of the Universe is the proper time of the origin of the reference system or the coordinate time of the reference system with respect to which the whole Universe is at rest, which is obviously an inertial reference system.

7 Coriolis force and force produced by angular acceleration

The Coriolis force depends on the velocity **w** of the particle that moves with respect to the Universe. As in (6) this velocity does not appear, it is impossible for the Coriolis force to be deduced from the induction produced by the potential ϕ.

Suppose that the body C is located in the position **r** near the origin of coordinates and that it has a rotation acceleration $-\dot{\boldsymbol{\omega}}$. This situation is equivalent to the fact that body C is at rest and the Universe has an angular acceleration $\dot{\boldsymbol{\omega}}$. Let's investigate the inductive force of this movement.

The only term that can generate the force of inertia produced by the angular acceleration is the fourth term of (6), therefore

$$-m_g \nabla \phi = -\frac{G}{c^2} \rho m_g \iiint \frac{\mathbf{r}'(\mathbf{r}' \cdot \mathbf{a})}{\sigma} \sin\theta d\theta d\varphi d\sigma . \tag{15}$$

It is easy to check

$$\mathbf{r}'(\mathbf{r}' \cdot \mathbf{a}) \equiv -\boldsymbol{\sigma}(\mathbf{r} \cdot \mathbf{a}) = r \sigma^2 \dot{\omega} \sin^2\theta \cos^2\varphi \, \mathbf{i}$$

and the integral (15) gives

$$-m_g \nabla \phi = -\frac{2\pi}{3} G \rho m_g t^2 r \dot{\omega} \mathbf{i} \tag{16}$$

the acceleration of C with respect to the Universe is $-\dot{\boldsymbol{\omega}} \wedge \mathbf{r} = -\dot{\omega} r \mathbf{i}$. Now we identify this force (16) with the inertial, resulting

$$m_i = -\frac{2\pi}{3} G \rho t^2 m_g$$

this is an absurd result because it would mean that the force of inertia of the body C would have the same sense as its acceleration. Therefore the scalar gravitational theory is unable to explain the force of inertia on a body that has an angular acceleration.

8 Conclusions

We use a scalar gravitational theory that is relativist invariant. The field equation is solved by the technique of retarded potential. We also assume that movement is relative, that is, the bodies move in relation to other bodies not in relation to a hypothetical space.

We consider a very simple cosmic model, where the density of matter is constant and homogeneous. We are not interested in the exact numerical results, we are more interested in the qualitative aspects of the problem.

In the previous investigation we have verified whether the forces of gravitational induction produced by the relative movement of the whole of the Universe generate the inertial forces that are observed, or in other words if the phenomenon of inertia has an origin in the cosmos.

The results obtained are the following:
1) Scalar gravitational theory produces inductive effects, that is, it generates forces dependent on the acceleration of the gravitational field source, contrary to what Sciama had predicted.
2) The induction forces that have been deduced are of opposite sense to the forces of inertia, therefore contrary to observation.
3) The numerical values of the induction forces derived from the scalar gravitational theory are approximately correct

$$\frac{2\pi}{3} G \rho t_0^2 \approx 1.$$

where t_0 is the current age of the Universe
4) The gravitational mass of a body has an inalterable value, but the inertial mass is variable and determined by the gravitational mass and by the cosmic properties of the Universe. This variability with time of the inertial mass has measurable effects.
5) The inertial mass is proportional to the gravitational mass, as experimentally observed.
6) There is no force of inertia when the body has an uniform movement, as the law of inertia affirms.
7) It is found that the induction force that we identify with the force of inertia depends linearly on the acceleration, at least in the classical approximation, as indicated by Newton's second law.

In the second part of this study we will calculate the inductive forces derived from a vector theory of gravitation, we will see that the results will be much better than those found for the scalar theory.

9 Bibliography

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