Disproof of the Riemann Hypothesis

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Occupy a Nice Place in the Future
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We present a disproof by direct contradiction. We use an elementary representation of the Riemann ζ function to show that there are infinitely many non-trivial zeros of ζ off the critical line. All of these zeros are in the neighborhood of infinity and we define that neighborhood.

Let $\mathbb{R}$ be separated between real numbers in the neighborhood of the origin $\widehat{0}$ and real numbers in the neighborhood of infinity $\pm\widehat{\infty}$. Call real numbers in the neighborhood of zero $\mathbb{R}_0$ and call real numbers in the neighborhood of infinity $\widehat{\mathbb{R}}$ so that for $\mathbb{R}\sim\mathbb{R}_0$ we have

$$\forall x \in \mathbb{R}_0 \quad \exists \quad b \in \mathbb{R} \quad : \quad x = \widehat{0} + b \ , \tag{1}$$

and

$$\forall x \in \widehat{\mathbb{R}} \quad \exists \quad b \in \mathbb{R} \ , \ b > 0 \quad : \quad x = \pm\big(\widehat{\infty} - b\big) \ . \tag{2}$$

The symbol $\widehat{\infty}$ inherits all canonically non-standard analytical properties of ∞ except for additive absorption [1]. Therefore, every number in the neighborhood of infinity is distinct and there are as many real numbers in the neighborhood of infinity as there are non-zero real numbers in the neighborhood of the origin. Every $\widehat{\mathbb{R}}$ number is incomparably large in absolute value compared to every $\mathbb{R}_0$ number so

$$x \in \mathbb{R}_0 \ , \ y \in \widehat{\mathbb{R}} \qquad \Longrightarrow \qquad \frac{x}{y} = 0 \ . \tag{3}$$

Riemann has continued the domain of the Dirichlet function to $\mathbb{C}$ so that we have the Euler product

$$\zeta(z) = \prod_{p\in\text{primes}} \frac{1}{1-p^{-z}} \ , \qquad \text{for} \qquad z \in \mathbb{C} \ . \tag{4}$$

We will consider the further analytic continuation to the exterior neighborhood of infinity around $\mathbb{C}$. Complex numbers including numbers in the neighborhood of infinity have the form

$$z = x + iy \ , \qquad \text{where} \qquad x, y \ \in \ \mathbb{R}_0 \cup \widehat{\mathbb{R}} \ . \tag{5}$$

To disprove Riemann's hypothesis, let z_0 be a number in the neighborhood of negative real infinity in the form

$$z_0 = -\big(\widehat{\infty} - b\big) + iy_0 \ , \qquad \text{with} \qquad b, y_0 \in \mathbb{R} \ , \ b > 0 \ . \tag{6}$$

Therefore,

$$\zeta(z_0) = \prod_{p\in\text{primes}} \frac{1}{1 - p^{\widehat{\infty}}\, p^{-b}\, p^{-iy_0}} \ . \tag{7}$$

Using the formula

$$a^{b+ic} = a^b \big[\cos(c\ln a) + i\sin(c\ln a) \big] \ , \tag{8}$$

and choosing $\big(y_0 \ln p'\big) = 2n\pi$ for some prime p' we get

$$\zeta(z_0) = \frac{1}{1-\widehat{\infty}} \left(\prod_{\substack{p\in\text{primes} \\ p\neq p'}} \frac{1}{1-p^{-z_0}} \right) \ . \tag{9}$$

By equation (3) we get

$$\zeta(z_0) = 0 \ . \tag{10}$$

Since $-\widehat{\infty} + b \neq 1/2$, we have disproven Riemann's hypothesis. The Riemann ζ function has a zero at every point in the neighborhood of minus real infinity which has the form of z_0 with appropriate y_0. In other work [2], we have shown that ζ will have zeros off the critical line in the neighborhood of infinity.

[1] Jonathan W. Tooker. Proof of the Limits of Sine and Cosine at Infinity. *viXra:1809.0234*, (2018).
[2] Jonathan W. Tooker. On the Riemann Zeta Function. *viXra:1703.0073*, (2017).