Visualizing the distributions of the escape paths of quaternion fractals

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Abstract

The length and displacement distributions of the escape paths of the points in some quaternion fractal sets are visualized.

1 Escape path length and displacement histograms

As discussed in [1], a 3D scalar field of quaternion magnitudes (e.g. $|Z|$) results from calculating a quaternion fractal set when using a finite 3D lattice of regularly spaced points as input. Here we will visualize the distributions of the escape paths' lengths, as well as the escape paths' displacements, for those points within the set. As in [1], the 'max iteration count' is 8.

The histograms in Figs. 3 and 4 together show how the maximum length is generally greater than the maximum displacement. This is also generally the case for the length and displacement per individual escape path, which is generally indicative of curved escape paths – the escape paths generally *meander* because there are *bends*. As for the histograms in Figs. 5 - 8, the maximum length and displacement aren't given, but this does not stop us from using the histograms to visualize the basic properties of the iterative function in question – the histograms serve as unmistakable (and beautiful) signatures.

In a lot of the cases (but not all cases) a curved escape path forms a loop (see pages 7, 12, and 13 in [2]), which gives rise to the commonly-used name 'orbit' (see [3]). However, most of the time the loop is not quite exact, and so all 'max iteration count'+1 = 9 points per escape path end up being distinct. This means that when a curved escape path forms an orbit, the orbit is generally not quite perfect – the curved escape path is likely jittery, or precessing, or spiral-shaped, or all three.

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References

[1] Halayka S. *Some visually interesting non-standard quaternion fractal sets* Chaos, Solitons & Fractals Vol. 41, Issue 5

[2] `https://iquilezles.org/www/articles/arquimedes/arquimedes.pdf`

[3] `http://math.bu.edu/DYSYS/FRACGEOM/FRACGEOM.html`

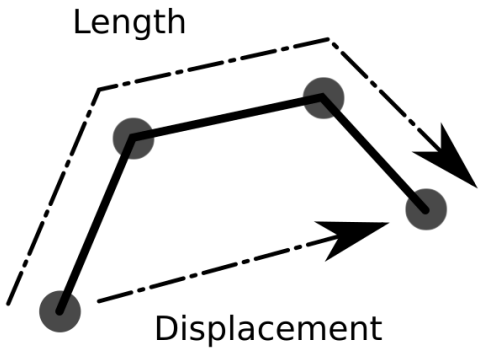

Figure 1: Length and displacement per escape path, in 2D, where the point count is 4.

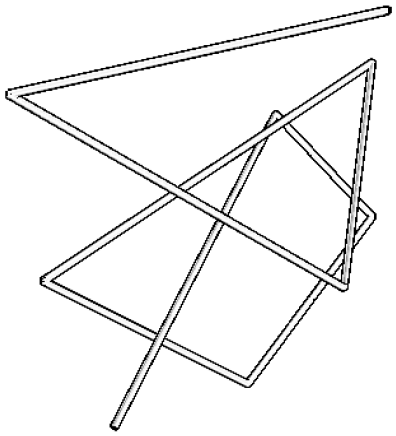

Figure 2: An escape path, where the point count is 'max iteration count'+1 = 9. The points have been projected down from 4D to 3D using Hopf mapping.

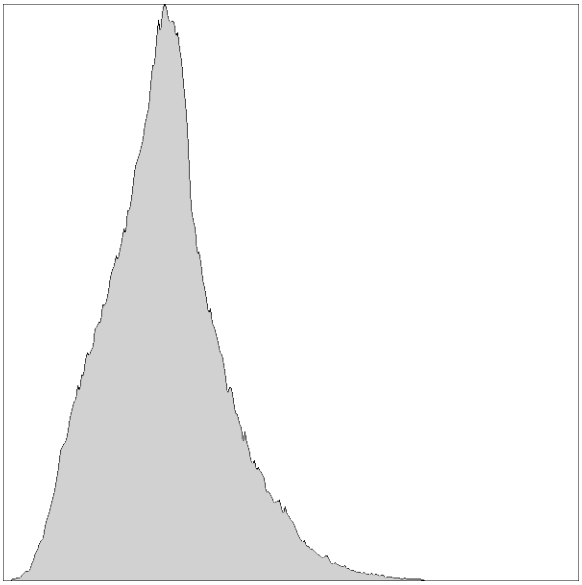

Figure 3: Lengths of $Z' = Z^2 + C$, where $C_{xyzw} = 0.3, 0.5, 0.4, 0.2$. For this histogram the maximum length is 21.2391.

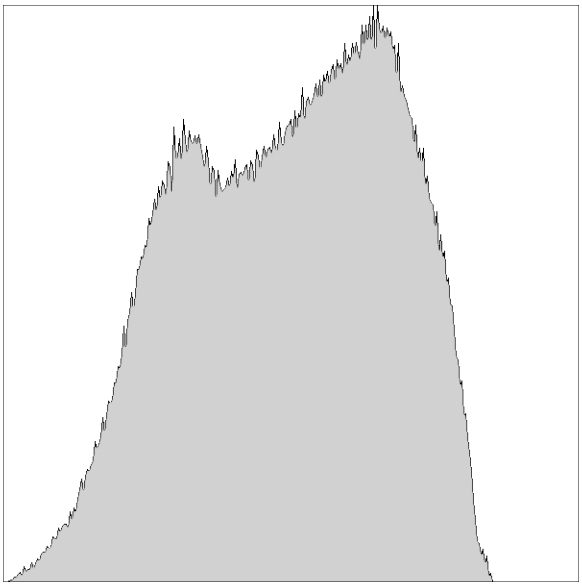

Figure 4: Displacements of $Z' = Z^2 + C$, where $C_{xyzw} = 0.3, 0.5, 0.4, 0.2$. For this histogram the maximum displacement is 2.36506.

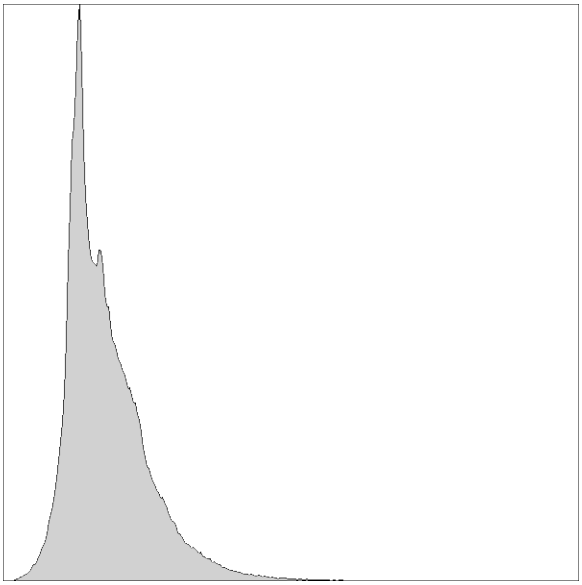

Figure 5: Lengths of $Z' = Z^5 + C$, where $C_{xyzw} = 0.3, 0.5, 0.4, 0.2$.

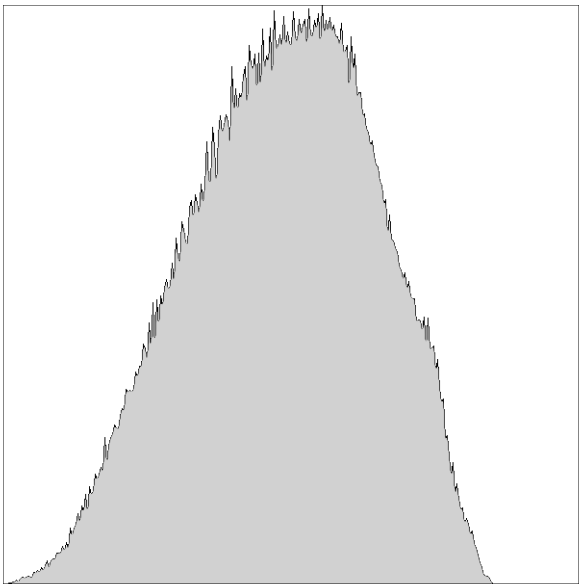

Figure 6: Displacements of $Z' = Z^5 + C$, where $C_{xyzw} = 0.3, 0.5, 0.4, 0.2$.

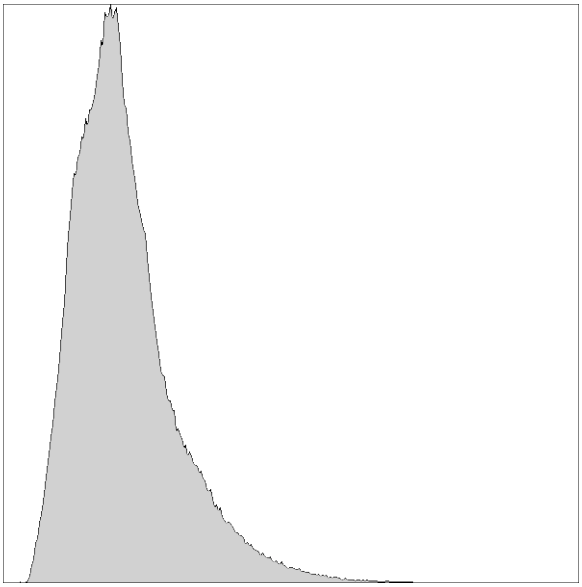

Figure 7: Lengths of $Z' = \sin(Z) + C \cdot \sin(Z)$, where $C_{xyzw} = 0.3, 0.5, 0.4, 0.2$.

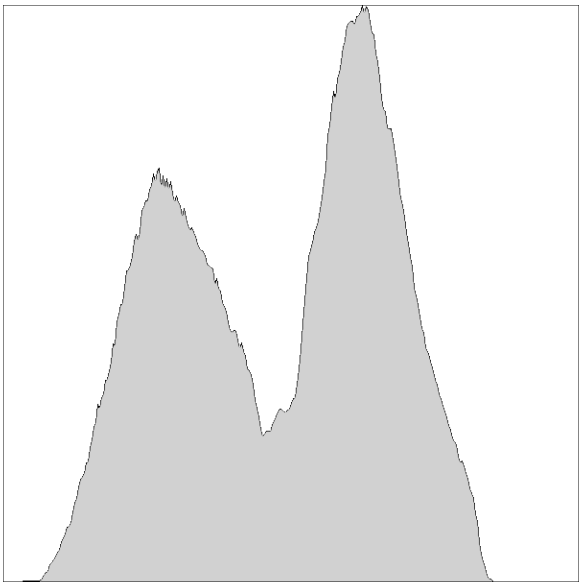

Figure 8: Displacements of $Z' = \sin(Z) + C \cdot \sin(Z)$, where $C_{xyzw} = 0.3, 0.5, 0.4, 0.2$.