Refutation of modal logic with the difference modality of topological T_0-spaces

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated *proof* value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). Results are a 16-valued truth table in row-major and horizontal, or repeating fragments of 128-tables for more variables.

LET p, q, r, s;
~ Not, ¬; + Or, ∨; & And, ∧; = Equivalent; > Imply;
# necessity, for all or every, □, ∀; % possibility, for one or some, ◊;
[≠] ~#; ⟨≠⟩ ~%; ◊A = ¬□¬A; ⟨≠⟩A = ¬[≠]¬A; [≠]A∧A = [∀]A.

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In this paper we will use the following axioms:

($D_\Box$) $[\forall]p \rightarrow \Box p$, (2.4.1)

(~#p&p)>#p ; TNTN TNTN TNTN TNTN (2.4.2)

(B_D) $p \rightarrow [\neq]\langle\neq\rangle p$, (2.5.1)

p>~(#(~#~p=(p=p))=(p=p)); TCTC TCTC TCTC TCTC (2.5.2)

(AT_0) $(p \wedge [\neq]\neg p \wedge \langle\neq\rangle(q \wedge [\neq]\neg q)) \rightarrow (\Box\neg q \vee \langle\neq\rangle(q \wedge \Box\neg p))$ (2.7.1)

((p&~#~p)&~(#(~(q&~#~q)=(p=p))=(p=p)))>
(#~q+~(~(#(~(q&#~p)=(p=p))=(p=p))=(p=p))) ;
TNTN TNTN TNTN TNTN (2.7.2)

We introduce the notation for the following logics:

$\mathbf{S4D} = \mathbf{K_2} + T_\Box + 4_\Box + D_\Box + B_D + 4_D$
$\mathbf{S4DT_0} = \mathbf{S4D} + AT_0$

Eqs. 2.4.2, 2.5.2, and 2.7.2 as rendered are *not* tautologous. This means logics **S4D** and $\mathbf{S4DT_0}$ are also *not* tautologous. Hence, modal logic with the difference modality of topological T_0-spaces is refuted.