

From Maxwell's equations to Electro-Magnetic Waves

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Abstract – This article shows how Electro-Magnetic source and wave are related and why the propagation velocity of light in vacuum is c, exclusively relative to its source.

1. Introduction

Starting with the explanation of the most basic concepts: static electric and magnetic fields, the Maxwell equations for the dynamic fields are shown to be hypotheses instead of laws. These hypotheses are used to show mathematically how an EM wave must look like. For this study reference [1] has been used as guidebook.

2. The static electric field

2.1 From force to voltage

About two and a half centuries ago Coulomb discovered the repulsive force between like electrical charges objects and the attractive force between unlike charged objects. Just like Newton discovered the attractive force between masses.

The last mentioned one has been mathematically expressed as $F_G = GMm/r^2$, with $[G]$ is Nm^2kg^{-2} . The distance between the centres of the objects is defined as r .

Coulomb's force has been mathematically expressed in basically exactly the same way by means of $F_C = CQq/r^2$, with $C = 1/4\pi\epsilon$ and ϵ the so-called dielectric permittivity of the medium in which the objects are located. As a result $[C]$ is Nm^2C^{-2} . In order to avoid confusion between both C 's, the first mentioned one will from now on be presented as C_G , so $[C_\epsilon]$ is Nm^2C^{-2} . The dimension of ϵ thus is: $N^{-1}m^{-2}C^2$. Let for the ease of the considerations Q be the main object and q , *being much smaller*, the object in the sphere of influence of Q . The charge q is supposed to be small enough to have negligible influence on the sphere of influence of Q .

Let us describe that sphere of influence of Q by means of the words 'electric field' of Q . Such a field has, according to Coulomb's experiences through his experiments, the possibility to attract or repulse an object with an electric charge of q . Let us call the strength of this field E_r at the distance r from Q and the related force F_{Cr} . So $F_{Cr} = qE_r$.

Given the relation $F_{Cr} = C_\epsilon Qq/r^2$, the electric field strength E_r has to be presented as $E_r = C_\epsilon Q/r^2$. Given the dimension of the variables on the right side of the equation, the dimension of E_r is N/C .

Remark: N/C can also be written as: $Nm/Cm = VAs/Cm = VC/Cm = V/m$, because Nm and VAs are both expressions for energy, so the dimension of E_r is also V/m , as normally used.

In order to move, in the electric field of Q , the object q from P_1 to P_2 , the integral $\int_s qE_s ds$ represents the work that has to be carried out in order to do so. The quantity qE_s represents at any place on that path the force in the direction of the movement. The total mentioned work thus equals the difference in *potential energy* in the two chosen points.

If the path is a closed curve, the result must be zero, so $\oint_s qE_s ds = 0$ and thus $\oint_s E_s ds = 0$. *

The quantity $E_s ds$ represents a voltage/potential, so $\int_s E_s ds$ from P_1 to P_2 results in $V_{P_2} - V_{P_1}$, the difference in voltage between the points P_1 and P_2 .

* The symbols \oint_s , resp. \int_s are used to express a line integral along a closed, resp. open curve s .

2.2 The static electric flux

The static electric flux Φ_E is also a measure of the electric field strength at any distance from the charge Q , in the sense of the amount of electric field *through a certain surface*.

The mathematical presentation is $\Phi_E = \iint_S E_r dS$ *, with E_r perpendicular to surface element dS . In case of an electric field equally spread over a sphere around object Q , the field strength E_r is the same at each element dS on that sphere.

So the surface integral at distance r from Q is $4\pi r^2 E_r = 4\pi r^2 \cdot C_\epsilon Q/r^2 = Q/\epsilon$ [Vm].

The result effectively shows that, whichever closed surface is chosen around Q , $\Phi_E = Q/\epsilon$.

2.3 The static dielectric flux

The expression in 2.2 for the static electric flux can also be written as: $Q = \iint_S \epsilon E_r dS = \iint_S D_r dS$

The variable D is normally used as symbol for dielectric displacement.

This name will be commented in chapter 3.

The equation shows that the dimension of D_r (from now on written as D) equals the dimension of ϵE_r (E_r from now on written as E) and Q/S , being Cm^{-2} .

Differentiating to time, the equation $D=Q/S$ results in: $dD/dt = dQ/dt/S = I_D/S$, with I_D used as symbol for dielectric current. But dQ/dt in a *static* situation is zero.

Therefore the situation has to be transferred to a dynamic one.

2.4 The dynamic dielectric flux

In order to obtain a meaningful concept of an electric charge that changes with time, without applying solid conductors in which electrons operate as moving electric charges, we can imagine a moving charge Q in empty space or tangible medium. In empty space the electric permittivity has to be chosen as ϵ_0 , in a tangible medium as $\epsilon_0 \epsilon_r$, shortly written as ϵ , like up to now.

Imagine a charge Q in a reference system relative to which Q moves with constant velocity v , say along the x -axis. During time t until $t+\Delta t$, Q moves from x to $x+v\Delta t = x+\Delta r$. At time t the electric field strength at distance r from x can be expressed as in 2.1: $E(t) = C_\epsilon Q/r^2$. At time $t+\Delta t$ the field strength, at this same position relative to x , is $E(t+\Delta t) = C_\epsilon Q/(r-\Delta r)^2$. So $E(t+\Delta t)-E(t) = 2\Delta r C_\epsilon Q/r^3$.

Multiplying both sides with ϵ , results in: $\Delta D/\Delta t = \epsilon 2\Delta r C_\epsilon Q/r^3/\Delta t = vQ/r^3/2\pi$ ($\epsilon C_\epsilon = 1/4\pi$).

Multiplying both sides with an arbitrary small surface element dS results in $I_D = dSvQ/2\pi r^3$ [A].

This is not a surprising result, because what is the fundamental difference between moving charges in an empty space/tangible medium, respectively in a conductor?

This concept has been used in [2]! In the model applied there, orbiting electrons around an atomic nucleus are considered as circular shaped electric currents, generating magnetic fields through the, by these orbits, enclosed surfaces.

In order to look for another approach too, a changing Q as function of time will be considered from the point of view that Q does not change its position, but its value.

Suppose at time t the charge is Q and at time $t+\Delta t$ it is $Q+\Delta Q$. At distance r from this changing electric charge, $D(t)$ is $\epsilon C_\epsilon Q/r^2 = Q/4\pi r^2$ while $D(t+\Delta t) = (Q+\Delta Q)/4\pi r^2$.

So $\Delta D/\Delta t = (\Delta Q/\Delta t)/4\pi r^2$ [$\text{Cm}^{-2}\text{s}^{-1} = \text{Am}^{-2}$].

Multiplying both sides with an arbitrary small surface element dS shows: $I_D = dS\Delta Q/\Delta t/4\pi r^2$ [A].

This is the moment to start the investigation of magnetic fields, because electric as well as dielectric currents create magnetic fields.

From now on a current can be either an electric or a dielectric current.

* The symbols \iint_S , resp. \iiint_S represent the surface integral over a closed, resp. open surface S .

3. The magnetic field

3.1 The static magnetic field

Two types of static magnetic fields will be considered: the one created by a current through an infinite long straight conductor, the other by a circular shaped current.

A straight line current creates a circular shaped magnetic field with this current as centre and in a plane perpendicular to this line. Its strength H at distance r from this current is $I/2\pi r$ [A/m].

A circular shaped current creates a magnetic field through the surface enclosed by this current, perpendicular to this surface. Its strength H in the centre of this circle is equal to $I/2r$ [A/m].

Remark about the similarity between electric and magnetic fields:

A fundamental difference between a static electric field and a static magnetic field is that the electric field is an open one, leading to the results: $\oint_S E ds = 0$ and $\iint_S E dS = Q/\epsilon = \Phi_E$, while the magnetic field is a closed one, leading to the results: $\oint_S H ds = I$ and $\iint_S H dS = 0$.

The last mentioned expression is normally written as $\Phi_B = \iint_S \mu H dS = \iint_S B dS = 0$, with Φ_B the so-called magnetic flux [Vs] and B the magnetic flux density [Vs/m²].

From the point of view of similarity with the magnetic field it is strange that the electric flux has not been defined as $\Phi_D = \iint_S \epsilon E dS = \iint_S D dS = Q$. The dimension of Φ_D is As (or C). And the dimension of D , as an electric flux density, still As m⁻², or Cm⁻².

Such a convention would lead to the following table for static fields:

	Electric field		Magnetic field	
Field strength	E	V/m	H	A/m
Medium properties	ϵ	As/Vm	μ	Vs/Am
Flux density	$D = \epsilon E$	As/m ²	$B = \mu H$	Vs/m ²
Flux	Φ_D	As	Φ_B	Vs
Physical laws				
Closed curve integrals	$\oint_S E ds = 0$		$\oint_S H ds = I$	
Closed surface integrals	$\iint_S D dS = Q$		$\iint_S B dS = 0$	

3.2 The dynamic magnetic field

The just mentioned equation: $\oint_S H ds = I$ can simply be proven in case of a straight line current, Because $H = I/2\pi r$ at each point around the current at distance r , and H and ds do have the same direction in each point too, the closed integral along this circle is $I/2\pi r * 2\pi r = I$.

Reference [1] at this place declares at page 256: "Investigation of other magnetic fields finally leads to the conclusion that $\oint_S H_s ds = I$, with I the sum of all currents enclosed in s , is a general property of the magnetic field." Seemingly a theoretical substantiation of this general property is too complex, maybe even impossible. The expression thus has to be qualified as hypothesis.

Finally the closed curve integral is presented as: $\oint_S H ds = d/dt \iint_S D dS + \iint_S \gamma E dS$, with $\iint_S \gamma E dS$ defined as conduction current.

However the expression is not helpful trying to understand how an EM wave is generated, because an EM wave is normally not generated in fields including conduction currents.

The complete equation is officially called: "Ampère's circuital law (with Maxwell's addition)", or shortly Maxwell's nth equation, with n now-a-days undefined!

In this article only $\oint_S H ds = d/dt \iint_S D dS$ will be used and called Maxwell-Ampère *hypothesis*.

Remark:

Before we continue with this equation we have to take care of the fact that the equation $\Phi_D = \iint_S D \, dS$ concerns a closed surface integral over the charge Q that creates the electric flux density D in a static situation, while in $\oint_s H \, ds = d/dt \iint_S D \, dS$ the electric flux density is supposed to go through an open surface enclosed by the curve s meant in the left side of the equation. We thus have to accept even more penetrating that the expression $\oint_s H \, ds = d/dt \iint_S D \, dS$ is not a law but a hypothesis: valid as long as it has not been proven to be invalid.

The second difference between $\iint_S D \, dS$ and $\iint_S D \, dS$ is that the first mentioned one equals Q in the static situation, while the second one is supposed to be applied in a dynamic situation, given the fact that the differentiation, applied to it, is supposed to be meaningful. The same kind of remark is applicable to the following consideration.

It is generally accepted that a voltage can be generated in a closed wire by changing a magnetic flux through the *open* surface enclosed by the wire, mathematically expressed by $V = d\Phi_B/dt$. This Φ_B thus is not the same magnetic flux as in $\Phi_B = \iint_S B \, dS$. So just like in the electric situation the relation $\int_s E \, ds = d/dt \iint_S B \, dS$ has to be qualified as a hypothesis.

This relation is presented as "Maxwell-Faraday law", but will be qualified here as hypothesis.

Summarized:

Maxwell-Ampère hypothesis

$$\oint_s H \, ds = d/dt \iint_S D \, dS$$

Maxwell-Faraday hypothesis

$$\int_s E \, ds = d/dt \iint_S B \, dS$$

4. The Electro Magnetic wave

Given the fact that both Maxwell hypotheses must, due to the operator d/dt , describe a dynamic situation, $\oint_s H \, ds$ can be written as $I(t)$ and $\iint_S D \, dS$ as $Q(t)$. So we just have the equation $I=dQ/dt$. Applying the definition of Φ_D , as proposed here: $I=d\Phi_D/dt$.

Because an electric charge is a quantity that can exist independent of other quantities, the start of the generation of an EM wave must be found in a source that initially produces a $I = d\Phi_D/dt$. This has already been considered shortly in the second example in section 2.4.

Jumping to the Maxwell-Faraday hypothesis results in $V(t) = d\Phi_B(t)/dt$, shortly: $V = d\Phi_B/dt$.

This voltage V closes the loop of the two Maxwell hypotheses, effectively causing: $V = \alpha d^2V/dt^2$

Because of this second order differential equation, V has to be a sinusoidal function.

Up to now EM waves are drawn as *in phase* E and H-fields. See for example figure 1, copied from [1] at page 318. Such a model is thus wrong. The real one is shown in figure 2.

Another, more direct, argumentation to reject the present model is that where dE/dt is maximum, H has to be maximum and where dH/dt is maximum, E has to be maximum.

In both cases the E- and H-fields are always generated in planes perpendicular to each other.

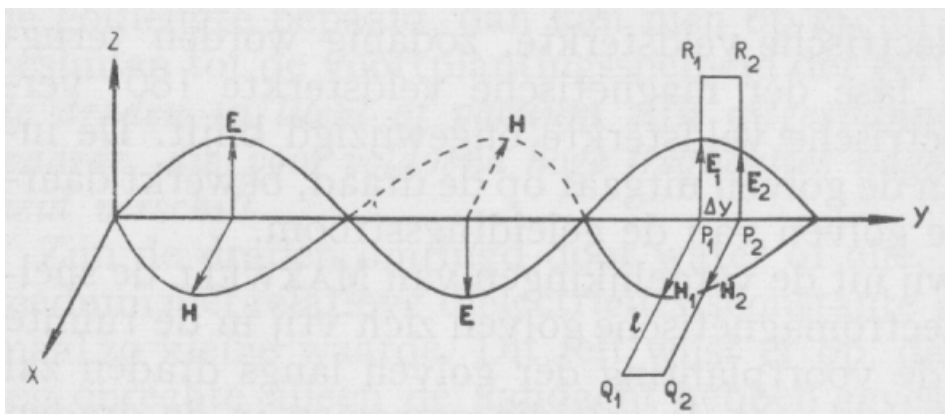


Figure 1 showing wrong EM wave

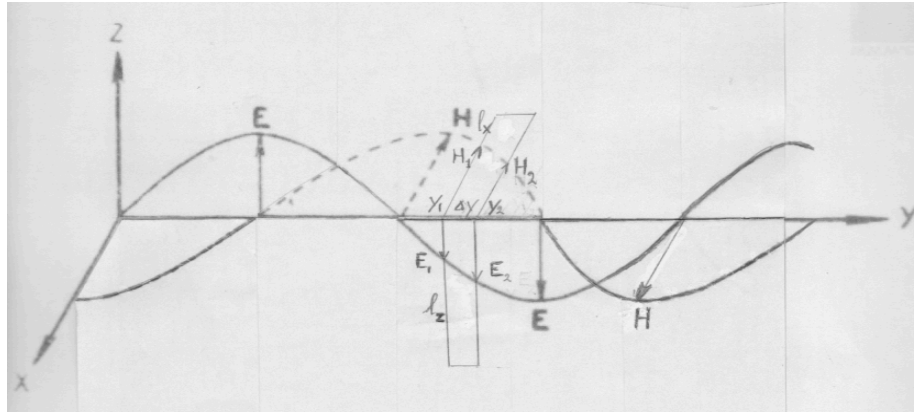


Figure 2 showing real EM wave

Reference [1] shows a simple derivation of the speed of light based on figure 1. The translation of that derivation, a bit adapted here and based on figure 2, is presented now.

The propagation direction of the EM wave is along the y-axis. The electric resp. magnetic fields at position y_1 resp. y_2 are: E_1 and H_1 resp. E_2 and H_2 . Between these moments the EM wave is supposed to travel the distance Δy during the time Δt with speed $v = \Delta y / \Delta t$.

The Maxwell-Ampère respectively Maxwell-Faraday hypothesis applied in figure 2, leads to*:
 $\oint_S H ds = (H_2 - H_1)l_x = d/dt \iint_S D dS = dD/dt \cdot l_z \Delta y = \epsilon dE/dt \cdot l_z \Delta y = \epsilon (E_2 - E_1) / \Delta t \cdot l_z \Delta y = \epsilon v (E_2 - E_1) l_z$
 resp.,
 $\oint_S E ds = (E_2 - E_1)l_z = d/dt \iint_S B dS = dB/dt \cdot l_x \Delta y = \mu dH/dt \cdot l_x \Delta y = \mu (H_2 - H_1) / \Delta t \cdot l_x \Delta y = \mu v (H_2 - H_1) l_x$

* The electric and magnetic fields in the y-direction are zero!

So $(H_2 - H_1)l_x = \epsilon v (E_2 - E_1)l_z$ respectively $(E_2 - E_1)l_z = \mu v (H_2 - H_1)l_x$

Applying the first equation in the second equation shows:

$(E_2 - E_1)l_z = \mu v \{ \epsilon v (E_2 - E_1)l_z \} = \epsilon \mu v^2 (E_2 - E_1)l_z$, so $\epsilon \mu v^2 = 1$, resulting in $v = 1 / \sqrt{\epsilon \mu}$.

Another important conclusion of this consideration is the remark made already at the beginning: "Because an electric charge is a quantity that can exist independent of other quantities, the start of the generation of an EM wave must be found in a source that eventually produces a dQ/dt ." That means that the very first dQ/dt is generated in the source of the EM wave, with the conclusion that thus the propagation speed of an EM wave is $1/\sqrt{\epsilon \mu}$ relative to its source only. With this conclusion the hypothesis in the Special Theory of Relativity: the speed of light is c relative to any reference, has to be rejected, and thus this theory! See also [3].

In[2] it has been argued that, given a certain ΔQ , the smaller Δt is, the higher the frequency of the emitted photon. Along such a way the source determines the frequency of the EM wave.

Conclusions

1. The article shows in the simplest way how the EM wave can mathematically be deduced from the Maxwell-Ampère and Maxwell-Faraday hypotheses, with, as spin of, the evidence that the reference for the propagation speed of an EM wave can only be its source. Such a conclusion forces to reject the Special Theory of Relativity.
2. It also turns out that the E and H field are mutually shifted 90° , thus not in phase, like generally accepted.

References

- [1] Leerboek der natuurkunde, Prof. Dr. R. Kronig, Delft, 1962 (in Dutch)
- [2] Why a photon is not a particle <http://vixra.org/abs/1505.0225>
- [3] Einstein's and Galilei's Principle of Relativity <http://vixra.org/abs/1611.0111>