

## AN OPERATIONAL DEFINITION OF LIFE, EVOLUTION AND THEIR PRIMEVAL OCCURRENCE

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We will examine one of the traits more frequently suggested in order to define life: living beings are able to produce new individual organisms (offspring), either asexually from a single parent organism, or sexually from two parent organisms. We will treat life's occurrence and reproduction in terms of algebraic topology, making clear that two of its more powerful theorems, i.e., the Borsuk-Ulam theorem and the ham sandwich theorem, are able to provide us with a mathematical definition of life, or at least one of its foremost traits. We discuss the advantages of describing life and evolution in topological terms and conclude with a novel "teleological", but physically-framed hypothesis concerning the role of the Universe.

KEYWORDS: symmetry; Topology; Borsuk-Ulam theorem; reproduction; mitosis; meiosis

After the Singularity/Big Bang, a gradual increase in thermodynamic entropy occurred in the Universe (Ellwanger, 2012). Because of the relationship between entropy and symmetry (Roldán et al., 2014), the number of cosmic symmetries, the highest possible occurring at the very start (Tozzi and Peters, 2016), have declined as time has passed. Here the evolution of living beings comes into play. As sentient organisms, we have conventionally determined our evolutionary history from the adult phenotype looking backwards in the retrograde direction, whereas the reality of the development and phylogeny of biota are recognized to have occurred from unicellular organisms to multicellular organisms in a prograde direction. This means that the mechanism of evolution can be formulated using a "bottom-up" approach in the context of cell biology (Torday and Rehan, 2012; Tozzi et al., 2018a). For example, the analogy between such a way of thinking about evolution and a Mobius strip, i.e. as a continuum, has been assessed (Torday and Rehan, 2017). Indeed, the life cycle can be seen from the perspective of the unicell as the primary level of selection pressure, given the centrality of the germ cells (egg and sperm) in sorting out the accumulated epigenetic marks as relevant or irrelevant to the organism's "history" by an as-yet-to-be-determined, but well recognized, "black box" mechanism. Starting from this central evolutionary role of the unicellular system, further steps are allowed towards seemingly higher complexity (Tozzi et al., 2017), yet in reality the unicellular state of the organism is the primary level of selection, the multicellular state acting as a phenotypic "agent" for the collection of epigenetic marks (Torday, 2016). Indeed, life only appears to be a space-limited increase in energy and complexity, and therefore of symmetries, evolution proceeding towards evermore complex systems (Chaisson, 2010), when in reality it resolves symmetries (Torday, 2018).

What is life? Despite different denotations having been proposed, there still is no unequivocal definition. Apart from the intrinsic difficulty in describing such an elusive phenomenon, the current limitation is that most of the proposed features are just descriptive, from the concept of homeostasis to organization, from growth to adaptation, from response to stimuli to reproduction (McKay, 2004; Trifonov, 2012). This lack of objectivity is a big restraint, because the sole language able

to describe scientific issues in quantitative terms is mathematics. Without mathematics, you do not have observables, whereas life is observable. Therefore, our aim is to try to give a mathematical, operational, quantifiable definition of what life is. One of the main features of life, a feature also able to define it, is the capacity for cellular duplication. Every cell gives rise to two identical “progeny”. In this paper, we show how, based on topological arguments, monocellular life and its primeval birth can be defined and operationalized in terms of a novel variant of BUT (termed split-BUT), and to the ham sandwich theorem, thanks to continuous mapping from one manifold to another. We show how, starting from an original assembly of macromolecules, it is possible to build a living cell. Furthermore, we tackle the issue to show how, from an ancestral cell, two progenitors are produced. In operational terms, the antipodal regions on a hypersphere synthesized by a variant of the Borsuk-Ulam Theorem (BUT) are viewed as tiny balls sliced, with a hyperplane, thanks to a ham sandwich view of the hypersphere. This leads to a novel view of cell division, which is the result of splitting a sphere? into two identical hyperspheric offspring cells. In such a novel context, life becomes a mathematical, operational, quantifiable phenomenon which stands for the continuous function required by the BUT.

## THE ROLE OF THE BORSUK-ULAM IN LIFE OCCURRENCE FROM INANIMATE MATTER

The Borsuk-Ulam Theorem (Borsuk 1933) is given in the following form:

Let  $f : S^n \rightarrow R^n$  be a continuous map. Then there exists  $x \in S^n \subseteq R^{n+1}$  such that  $f(x) = f(-x)$ .

This means that antipodal points on an  $n$ -sphere  $S^n$  map to  $R^n$ , which is then-dimensional Euclidean space (Matoušek 2003). Points on an  $n$ -sphere  $S^n$  are antipodal, provided the points are diametrically opposite. The assumptions in the original formulation of BUT display versatile ingredients that can be modified, resulting in useful extensions of BUT with different guises, *e.g.*, continuous mappings are replaced by piecewise continuous mappings, antipodal points are replaced by antipodal regions  $A, A'$  with matching descriptions (Peters 2016) and mappings are from  $S^n$  to  $R^k$ ,  $1 \leq k \leq n$  or  $k \geq n$ , which is a  $k$ -dimensional Euclidean feature space, where  $f(A)$  and  $f(A')$  describe regions  $A, A'$  respectively, and  $f(A), f(A')$  are matching descriptions, provided appropriate conditions are satisfied. See Tozzi and Peters (2016b) for further details.

In other words, the sphere  $S^n$  maps to the Euclidean space  $R^n$ , which stands for an  $n$ -dimensional Euclidean space. Note that the function  $f$  needs to be continuous and that  $n$  must be a natural number (although we will see that it is not completely true) (Matoušek 2003; Tozzi 2016a; Tozzi 2016b).

The notation  $S^n$  denotes an  $n$ -sphere, which is a generalization of the circle. An  $n$ -sphere is an  $n$ -dimensional structure embedded in an  $n+1$  space. For example, a 2-sphere ( $S^2$ ) is the 2-dimensional surface of a 3-dimensional ball (a beach ball is a good example). An  $n$ -sphere is formed by points which are at a constant distance from the origin in  $(n+1)$ -dimensions (Marsaglia 1972). For example, a 3-sphere (also called *glome* or *hypersphere*) of radius  $r$  (where  $r$  may be any positive real number) is defined as the set of points in 4D Euclidean space at distance  $r$  from some fixed center point  $c$  (which may be any point in 4D space) (Henderson 1996). A 3-sphere is a simply connected 3-dimensional manifold of constant, positive curvature, which is enclosed in a 4-dimensional Euclidean space called a 4-ball. A 3-sphere is thus the surface or boundary of a 4-dimensional ball, while a 4-dimensional ball is the interior of a 3-sphere. From a geometer’s perspective, we have different  $n$ -spheres, starting with the perimeter of a circle ( $S^1$ ), advancing to  $S^3$ , which is the smallest hypersphere, embedded in a 4-ball. Points on  $S^n$  are *antipodal*, provided they are diametrically opposite. Examples of antipodal points are the poles of a sphere. Further, every continuous function from an  $n$ -sphere  $S^n$  into Euclidean  $n$ -space  $R^n$  maps some pair of antipodal points of  $S^n$  to the same point of  $R^n$ . To make an example, if we use the mapping  $f: S^3 \rightarrow R^3$ , then  $f(x)$  in  $R^3$  is just a signal value (a real number associated with  $x$  in  $S^3$ ) and  $f(x) = f(-x)$  in  $R^3$ . Furthermore, when  $g: S^2 \rightarrow R^2$ , the  $g(x)$  in  $R^2$  is a vector in  $R^2$  that describes the  $x$  embedded in  $S^2$ . In other words, a point embedded in an  $R^n$  manifold is projected to two opposite points on an  $S^{n+1}$ -sphere, and vice versa.

A new form of shape theory (called homotopy) discovered by K. Borsuk makes it possible to assess the properties that are preserved through deformation, stretching and twisting of objects (Beyer and Zardecki 2004; Manetti 2015). Homotopy is a theory of shape deformation (Borsuk, 1971; Borsuk and Dydak, 1980), *e.g.*, how some shapes can be deformed into other shapes. In this context, the term “shape” means a finite, bounded region with a nonempty interior (a nonempty interior can have holes (Peters, 2017, 2018a, 2018b) and a “deformation” is a mapping from one shape into another one. A classic example is the deformation retract (a shrinking) of a coffee cup into a torus (Bredon, 1993). A more recent example is the deformation retract of a question mark “?” to the letter “J” (Peters, 2016). The combination of various forms of BUT and homotopy theory provides a methodological approach with countless possible applications. The theory of shape, in simple terms, focuses on the global properties of geometric objects such as polyhedra and tori, neglecting the complications of the local structures of the objects (Borsuk and Dydak, 1980). What shape theory and BUT tell us is that processes relating to storage, retrieval and reorganization interact with memory banks comparable to barcodes in the sense that complex process structures with similarities and bi-symmetries can be remembered and recalled in terms of simpler, fundamental representations.

In our case, the primeval, inanimate two-dimensional template, equipped with the proper macromolecular assemblies, gives rise to a three-dimensional sphere, i.e., a primeval living cell with its limiting membrane. Such living cells necessarily display antipodal features, thanks to the BUT dictates. Indeed, as the BUT states that on the Earth there are two antipodal points with the same temperature and pressure, in the same way we achieve, from the original lower-dimensional inanimate manifold, a higher-dimensional living cell equipped with opposite features with matching description. See the left part of the **Figure**.

We are now required to provide a biological name for such topological matching features. Or in other terms, what are such antipodal features in the first cell? We need to focus our attention on symmetrical features in the first progenitors, e.g., a protein conformation, a racemic structure, a peculiar polarization, a recognizable symmetry break, or other features which come in two or more copies. For example, we might hypothesize that, when the protocell formed due to the generation of micelles from lipids immersed in water it constituted the antipodes we seek, i.e., the internal and external environments described by Bernard and Cannon. Which then went on to distinguish themselves from one another through negative and positive entropy within and without of the cell, respectively. In sum, it might be speculated that chemiosmosis and homeostasis evolved in service to the entropic antipodes. This topological approach would also make it possible to define life via a continuous function that maps chunks (pieces in cellular division) or facets (particular cellular pieces with distinguishing features) from one manifold to another.

### **THE ROLE OF THE HAM SANDWICH THEOREM IN CELLULAR REPRODUCTION**

Define a projection  $\pi$  of an  $S^n$  hypersphere into  $S^n \times S^n$  (a pair of hyperspheres with duplicated antipodal regions, i.e., the progeny of parent cell  $S^n$  inherit the characteristics of regions  $S^n$ ). This can be done by replacing each pair of antipodes with tiny balls and inserting a hyperplane between the balls, effectively slicing the antipodal portions of  $S^n$  into two parts. This is analogous to the result for a special case of Ulam's ham sandwich theorem.

Ham Sandwich Theorem (Matoušek, 2003).

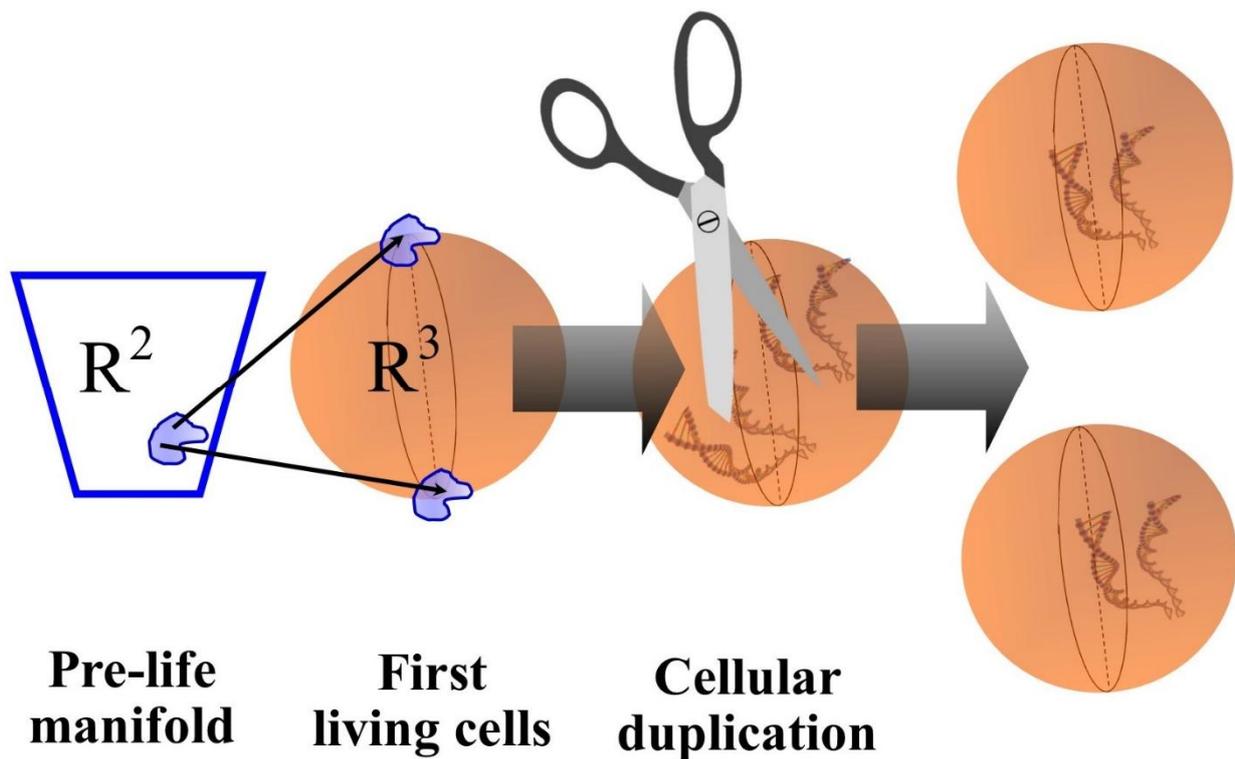
Let  $A_1, A_2 \subset \mathbb{R}^n$ . Then there exists a hyperplane that simultaneously bisects  $A_1$  and  $A_2$ .

This is analogous to what occurs during cell division: a single cell gives rise to two cells identical to itself.

In topological terms, the advantage is that we achieve antipodal points not due to an increase in dimensions of  $S$ , but due to a SPLIT of  $S^n$  into two parts, with no change in dimensions of the bifurcated  $S^n$ . We want to identify the features of antipodal regions on a physical  $n$ -dimensional hypersphere. A region of a physical chunk of space-time such as a cell has very interesting features such as mass, weight, area, diameter.

Each physical manifold maps piecewise continuously to a  $k$ -dimensional feature space  $\mathbb{R}^k$ , where  $k$  is the number of features of an antipodal region.

For further details, see the right side of the **Figure**.



**Figure 1.** Topological framework for biological events. The **left side** of the Figure displays a two-dimensional manifold (standing for pre-biological assemblies of macromolecules) and a three-dimensional sphere (standing for the first organization of membrane-delimited living cells). This case can be described in terms of the Borsuk-Ulam theorem for different values of  $S^n$ : indeed, a single region on  $S^n$  maps to two matching regions on  $S^{n+1}$ , and vice versa. The regions stand for particularly important biological features, needed as more than one copy in order to generate life. The **right side** of the figure illustrates how, for the ham-sandwich theorem, the three-dimensional cell with two antipodal regions can be split in two three-dimensional spheres, each one equipped with one of the antipodal regions. This case occurs during cellular division: we only need a manifold of a given dimension that splits into two manifolds with the same given dimension.

#### A TOPOLOGICAL ACCOUNT OF LIVING BEINGS: WHAT FOR?

In the theoretical biological context of living beings, the Borsuk-Ulam theorem stands for a general methodological and ontological approach able also to assess and operationalize its very own definition (Tozzi and Peters, 2016a). This allows us to topologically define life as the required continuous function that maps from one manifold to another. An important by-product of a topology of cells is the identification of cellular facets (components in a cell at the instant of cellular division) with matching feature vectors and the presence of dynamically changing piecewise continuous mappings from collections (clumps) of cellular facets in spacetime to their descriptions. The effect of a sequences of these facet mappings from a parent cell to its progeny over time is the detection of Efremovich trails (Boltyanskii and Efremovich, 2001) that lead from a feature vector ( $n$ -dimensional vertex) in a parent cell to progeny cells with matching feature vectors. An *Efremovich trail* is a sequence of connected edges between a succession of vertices whose union is homeomorphic to a single segment with only the endpoints in the original sequence. The end result is the unfolding of trails between vertices (spread over time) that make it possible to trace the lineage of a cellular division tree. Another important result is the approximation of geodesic lines by moving a very small 2-wheeled buggy along the trails between the feature vectors in a succession of manifolds during cell division. The discovery of such geodesic lines as the straightest lines between distant vertices depends on the way a succession of cell divisions is embedded in spacetime (Hilbert and Cohn-Vossen, 1952). At each juncture, a cellular geodesic line has the smallest curvature among the curves through a vertex on the surface, and has the same tangent at the vertex of the geodesic.

A topological approach also helps to elucidate another frequently unnoticed feature of living beings, in particular of the intentional ones such as humans. Every living being does not want to just maintain homeostasis, rather he wants to increase his adaptability in an ever-changing environment. In human terms, we are never satisfied with the things we have achieved: we always want more. In turn, inanimate systems do not tend to increase their power and influence over their own environment or niche (Torday and Miller, 2016). This is an evolutionary necessity, not an intrinsic feature of life: put simply, living beings not equipped with will power are negatively selected. One of the tenets of Friston's "variational free energy" account of life and nervous systems is that living organisms are equipped a priori with (possibly due to evolution) intrinsic generative models (modes?) which, when coping with external inputs, try to reduce the "surprise". However, in our own experience, human individuals are curious and look for novelties and surprises. We humans do not like routine, we like novelty. Therefore, Friston does not take into account that our mind WANTS to increase the surprise: the human brain seems to PREFER an increase of variational free energy, and therefore a DECREASE in thermodynamic/informational entropy. This is a natural, biological, intentional feature of our brain/mind and of living cells. Every human individual wants novelties, quantum jumps, breaks. The "continuous" state is despised, rejected: everyone wants to be unique, different from others, as if life required a break from the original, "boring" symmetries, rather favouring coping with local transformations instead of general symmetries. Therefore, because change is required by living beings in order to survive, we might state that homeostasis and preservation of homeostasis could be a wrong account. In order to improve their possibilities, living beings want change, not merely to preserve themselves in the same state. Biological codes could be built in order to modify individuals, not to keep them constant. Here topology comes into play: indeed, a recent variant of the BUT, termed bio-BUT suggests that, with increases in complexity, the occurring matching features do not display exactly the same description, rather very slight changes that allow short-term transformations (epigenetic changes) and long-term evolutionary changes (DNA changes). For further details concerning bio-BUT, see: (Tozzi and Peters, 2018). From a dynamical systems perspective of self-organization, it's all about (topological) symmetry breaking. It is suspected that this is at the heart of how we are compelled to "break the original, boring" symmetries.

In the next, final chapter, we will speculate how a topological approach to life and evolution might also elucidate general features of cosmic evolution.

## **CONCLUSION: A SYMMETRY-BASED ACCOUNT OF LIFE AND EVOLUTION.**

After the Big Bang, a gradual increase in thermodynamic entropy occurred in our Universe (Ellwanger, 2012). Because of the relationships between entropy and symmetry (Roldán et al., 2014), the number of cosmic symmetries, the highest possible occurring at the very start, declined as time passed. It is noteworthy that the very tenets of our current scientific account of the Universe, i.e., symmetries, symmetry breaks, gauge theories, thermodynamic entropy, can all be described in term of the Borsuk-Ulam theorem (for gauge theories and topology, see Tozzi et al., 2017a and 2017b; for thermodynamics and topology, see: Tozzi et al., 2018b).

Here the evolution of living beings comes into play: indeed, more advanced forms of life were able to artificially increase the symmetries of the world. The human brain's cognitive abilities are not just able to imagine objects and events more complex than the physical ones existing in Nature, but are also able to build highly symmetric crafts constructs?. For example, human beings can watch a rough stone, imagine an amygdala, and fashion it from the same stone. Humankind is able, through its ability to manipulate tools and technology, to produce objects (and ideas, i.e., equations) with complexity levels higher than the objects and systems encompassed in the pre-existing physical world. Therefore, human beings are naturally built by evolution in order to increase the number of environmental symmetries. This is in keeping with recent claims suggesting that the brain is equipped with a number of functional and anatomical dimensions higher than the 3D environment (Peters et al., 2017). Intentionality, typical of living beings, particularly that of the human mind, may be seen as a mechanism able to increase symmetries.

In touch with Spencer's (1860) and Tyler's (1881) claims, it looks like evolutionary mechanisms tend to achieve increases in environmental complexity, and therefore symmetries (Tozzi and Peters, 2017b). Life is produced in our Universe in response to the initial loss of symmetries as a result of its ambiguous origins, emanating from the difference in entropy within and outside of the cell (Torday and Miller, 2017). Like its highly ambiguous origins, life recognizes asymmetries and resolves them through endosymbiosis and compartmentation (Torday and Rehan, 2012). At the beginning of life, increases in symmetries were just local, e.g., they were related to the environmental niches where living beings existed. However, on long timescales, they extended to the whole Universe. For example, Homo sapiens, in just 250.000 years, has been able to build the Large Hadron Collider, where artificial physical processes try to approximate the initial symmetric state of the Universe. Therefore, life is a sort of gauge field (Sengupta et al., 2016), e.g., a combination of forces and fields that try to counterbalance and restore, in very long timescales, the original cosmic symmetries, lost after the Big Bang. Due to physical issues, the "homeostatic" cosmic gauge field must be continuous, e.g., life must stand, proliferate and increase in complexity over very long timescales. This is the reason why every living being has an innate tendency towards self-preservation and proliferation. With death, continuity is broken. This speaks in favor of intelligent life scattered everywhere in the Universe: if a few species become extinct, others might continue to proliferate and evolve on remote planets, consistent with the ultimate restitution of the symmetry emulating the Singularity of the Big Bang

(Torday 2018). In touch with long timescale requirements, it must be kept in mind that life formed after a long gestation: a childbearing which encompasses the cosmic birth of fermions, then atoms, then stars able to produce the more sophisticated matter (metals) required for molecular life.

With a symmetry-based framework, we achieve an answer to long-standing questions: “*why are we here?*”, “*Why is evolution the way it is?*”, an answer that relieves our most important concerns and anxieties. On the other hand, this framework does not give us any hope. We are immortalized by passing our genetic and epigenetic knowledge of our environment to our offspring. And our microbiome may be immortal since it re-enters the soil and aquifer, eventually recycling by being ingested by plants and animals, and ultimately by humans. However, we are just micro-systems programmed in order to contribute to restoring a partially “broken” macro-system. And, in case we succeed in restoring the initial symmetries through our mathematical abstract thoughts and craftsmanship, we are nevertheless doomed to die: indeed, the environment equipped with the starting symmetries does not allow for the presence of life, though life is destined to reform symmetries as the ‘equal and opposite reaction to the Big/Bang formulated by Newton’s Third Law of Motion (Torday, 2018).

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