Acta Mathematica Sinica, English Series
Http://www.ActaMath.com

A third note on Bell's theorem

Han GEURDES
Address
Geurdes Datascience, KvK 64522202, C vd Lijnstraat 164, 2593 NN, Den Haag Netherlands
E-mail:*han.geurdes@gmail.com*

Abstract In the paper it is demonstrated that Bell's formula for ± 1 measurement functions, is inconsistent.

Keywords Inconsistency, Bell's theorem.

MR(2010) Subject Classification 60A99, 26E99, 32B05

1 Introduction

In 1964, John Bell wrote a paper [1] on the possibility of hidden variables [2] causing the entanglement correlation $E(a, b)$ between two particles. In the present paper, an inconsistency in the starting formula will be demonstrated.

Bell, based his hidden variable description on particle pairs with entangled spin, originally formulated by Bohm [3]. Bell used hidden variables λ that are elements of a universal set Λ and are distributed with a density $\rho(\lambda) \geq 0$. Suppose, $E(a, b)$ is the correlation between measurements with distant A and B that have unit-length, i.e. $||a|| = ||b|| = 1$, real 3 dim parameter vectors a and b.

Then with the use of the λ we can write down the classical probability correlation between the two simultaneously measured spins of the particles. This is what we will call Bell's formula.

$$E(a, b) = \int_{\lambda \in \Lambda} \rho(\lambda) A(a, \lambda) B(b, \lambda) d\lambda \tag{1.1}$$

The spin measurement functions are, $A(a, \lambda) \in \{-1, 1\}$ and $B(b, \lambda) \in \{-1, 1\}$. The probability density is normalized, $\int \rho(\lambda) d\lambda = 1$. The equation (1.1) will be specified a little bit more in the section below.

2 Bell formula

Suppose, $\lambda = (\lambda_1, \lambda_2, \dots, \lambda_n)$. Equation (1.1) is then rewritten as

$$E(a, b) = \int_{\lambda \in \Lambda} \rho(\lambda) A(a, \lambda) B(b, \lambda) d\lambda_1 d\lambda_2 \dots d\lambda_n \tag{2.1}$$

Nothing much changed. We have, $\Lambda = \Lambda_1 \times \Lambda_2 \times \dots \Lambda_n$. If the ρ is defined on a restricted sub "interval", outside this "interval" the values do not add to the integral. Therefore, we may

Received 7, 10, 2018, accepted x x, 201x
Supported by . . . (Grant No. . . .)

write: $\Lambda = \mathbb{R}^n$. Suppose for continuous hidden variables λ.

$$\rho(\lambda) = \rho(\lambda_1, \lambda_2, \dots, \lambda_n) = \frac{\partial}{\partial \lambda_m} \mathcal{R}_m(\lambda_1, \lambda_2, \dots, \lambda_n) \tag{2.2}$$

For $m = 1, 2 \dots n$.As an example we may look at

$$\rho_{Gauss}(x) = \frac{1}{\sqrt{2\pi}} \frac{\partial}{\partial x} \int_{-\infty}^{x} e^{-y^2/2} dy$$

Returning to the main line of argument, this implies e.g.

$$E_1(a, b) = \int_{\mathbb{R}^n} \left(\frac{\partial}{\partial \lambda_1} \mathcal{R}_1(\lambda) \right) A(a, \lambda) B(b, \lambda) d\lambda_1 d\lambda_2 \dots d\lambda_n \tag{2.3}$$

Suppose, we abbreviate the $2, 3, \dots n$ integration with

$$\int_{-\infty}^{\infty} d\lambda_2 \dots \int_{-\infty}^{\infty} d\lambda_n f(\lambda) = \langle f(\lambda) \rangle_{2,3,\dots n} \tag{2.4}$$

When we use continuous hidden variables λ. we are allowed to employ partial integration. Partial integration on E_1 in (2.3), introducing the angular notation for $\int_{-\infty}^{\infty} d\lambda_2 \dots \int_{-\infty}^{\infty} d\lambda_n$ gives

$$\begin{aligned} E_1(a, b) &= \left\langle \mathcal{R}_1(\lambda) A(a, \lambda) B(b, \lambda) |_{\lambda_1 = -\infty}^{\lambda_1 = +\infty} \right\rangle_{2,3 \dots n} \\ &- \left\langle \int_{\mathbb{R}} \mathcal{R}_1(\lambda) \left(\frac{\partial}{\partial \lambda_1} A(a, \lambda) B(b, \lambda) \right) d\lambda_1 \right\rangle_{2,3 \dots n} \\ &= \left\langle \mathcal{R}_1(\lambda) A(a, \lambda) B(b, \lambda) |_{\lambda_1 = -\infty}^{\lambda_1 = +\infty} \right\rangle_{2,3 \dots n} \\ &- \left\langle \int_{\mathbb{R}} \mathcal{R}_1(\lambda) \left(B(b, \lambda) \frac{\partial}{\partial \lambda_1} A(a, \lambda) + A(a, \lambda) \frac{\partial}{\partial \lambda_1} B(b, \lambda) \right) d\lambda_1 \right\rangle_{2,3 \dots n} \end{aligned} \tag{2.5}$$

Note that when $A \in \{-1, 1\}$ then $\frac{1}{A} \in \{-1, 1\}$ and, of course, $B \in \{-1, 1\}$ then $\frac{1}{B} \in \{-1, 1\}$. Hence, we can write

$$E_2(a, b) = \int_{\mathbb{R}^n} \left(\frac{\partial}{\partial \lambda_1} \mathcal{R}_1(\lambda) \right) \frac{1}{A(a, \lambda)} \frac{1}{B(b, \lambda)} d\lambda_1 d\lambda_2 \dots d\lambda_n \tag{2.6}$$

And by definition of the measurement functions A and B we *must* have $E_1(a, b) = E_2(a, b)$. If we are allowed to employ partial integration in (2.3), there is no reason why this operation is disallowed in (2.6). Hence,

$$\begin{aligned} E_2(a, b) &= \left\langle \mathcal{R}_1(\lambda) \frac{1}{A(a, \lambda)} \frac{1}{B(b, \lambda)} |_{\lambda_1 = -\infty}^{\lambda_1 = +\infty} \right\rangle_{2,3 \dots n} \\ &- \left\langle \int_{\mathbb{R}} \mathcal{R}_1(\lambda) \left(\frac{\partial}{\partial \lambda_1} \frac{1}{A(a, \lambda)} \frac{1}{B(b, \lambda)} \right) d\lambda_1 \right\rangle_{2,3 \dots n} \\ &= \left\langle \mathcal{R}_1(\lambda) \frac{1}{A(a, \lambda)} \frac{1}{B(b, \lambda)} |_{\lambda_1 = -\infty}^{\lambda_1 = +\infty} \right\rangle_{2,3 \dots n} \\ &- \left\langle \int_{\mathbb{R}} \mathcal{R}_1(\lambda) \left(\frac{1}{B(b, \lambda)} \frac{\partial}{\partial \lambda_1} \frac{1}{A(a, \lambda)} + \frac{1}{A(a, \lambda)} \frac{\partial}{\partial \lambda_1} \frac{1}{B(b, \lambda)} \right) d\lambda_1 \right\rangle_{2,3 \dots n} \end{aligned} \tag{2.7}$$

If we take the A and B as functions that can, in the series, be approximated with $A_k(a, \lambda)$ and $B_k(b, \lambda)$, with $k = 1, 2 \dots$, then we may write for e.g. differentiation of $\frac{1}{A}$

$$\frac{\partial}{\partial \lambda_1} \frac{1}{A(a, \lambda)} = -\frac{1}{\{A(a, \lambda)\}^2} \frac{\partial}{\partial \lambda_1} A(a, \lambda) \tag{2.8}$$

Because, $\{A(a,\lambda)\}^2 = 1$ when $A(a,\lambda) \in \{-1,1\}$, we have

$$\frac{\partial}{\partial \lambda_1} \frac{1}{A(a,\lambda)} = -\frac{\partial}{\partial \lambda_1} A(a,\lambda) \tag{2.9}$$

If we again note $A = \frac{1}{A}$ etc, then

$$E_2(a,b) = \left\langle \mathcal{R}_1(\lambda) A(a,\lambda) B(b,\lambda) |_{\lambda_1=-\infty}^{\lambda_1=+\infty} \right\rangle_{2,3\dots n} \tag{2.10}$$

$$+ \left\langle \int_{\mathbb{R}} \mathcal{R}_1(\lambda) \left(B(b,\lambda) \frac{\partial}{\partial \lambda_1} A(a,\lambda) + A(a,\lambda) \frac{\partial}{\partial \lambda_1} B(b,\lambda) \right) d\lambda_1 \right\rangle_{2,3\dots n}$$

Generally speaking we may assume that

$$\left\langle \int_{\mathbb{R}} \mathcal{R}_1(\lambda) \left(B(b,\lambda) \frac{\partial}{\partial \lambda_1} A(a,\lambda) + A(a,\lambda) \frac{\partial}{\partial \lambda_1} B(b,\lambda) \right) d\lambda_1 \right\rangle_{2,3\dots n} \tag{2.11}$$

$$= \int_{\mathbb{R}^n} \mathcal{R}_1(\lambda) \left(B(b,\lambda) \frac{\partial}{\partial \lambda_1} A(a,\lambda) + A(a,\lambda) \frac{\partial}{\partial \lambda_1} B(b,\lambda) \right) d\lambda_1 \dots d\lambda_n \neq 0$$

with $\lambda = (\lambda_1, \dots, \lambda_n)$. Therefore, $E_1(a,b) \neq E_2(a,b)$ whereas because of $A = \frac{1}{A}$ and $B = \frac{1}{B}$ the result should have been $E_1(a,b) = E_2(a,b)$.

3 Conclusion

From the previous considerations one can conclude that Bell's formula and the inequalities derived thereof are not so very general as is widely claimed. In fact the Bell formula allows inconsistencies in concrete mathematics. Note that in the derivation of the inconsistency, use is made of perfect measurement, i.e. leaving out the possibility that $A = 0$ or $B = 0$ representing "missed" measurements.

References

[1] J.S. Bell, "On the Einstein Podolsky Rosen paradox," Physics, 1, 195, (1964).

[2] A. Einstein, B., Podolsky, and N. Rosen, "Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?," Phys. Rev. 47, 777, (1935).

[3] D. Bohm, "Quantum Theory," pp 611-634, Prentice-Hall, Englewood Cliffs, 1951.