On the Degeneracy of $\mathbb{N}$ and the Mutability of Primes

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October 9, 2018

Abstract

This paper sets forth a representation of the hyperbolic substratum that defines order on $\mathbb{N}$. Degeneracy of $\mathbb{N}$ at points of intersection with the substratum is observed as violations of the fundamental theorem of arithmetic in the form of mutable prime factorization. At a point of maximum symmetry on the representation manifold, an exact expression of π is available as a combination of three integers.

1 Synopsis

This paper sets forth a representation ($\mathfrak{R}$) of the hyperbolic substratum that provides order to $\mathbb{N}$. Representation is provided by third (κ) and fourth (ρ) roots of unity, where κ functions as the Absolute, facilitating absolute involutions of ρ.

With each involution incongruity betwen the linear system $\mathbb{N}$ and the hyperbolic system $\mathfrak{R}$ shifts, generating a set of primes representing linear traversal of the hyperbolic involution and providing a metric on $\mathbb{N}$. The first thirty terms are provided in appendix A, and the first sixteen primes are observed to emerge in sequence.

At intersections of $\mathbb{N}$ and $\mathfrak{R}$, $\mathbb{N}$ gains a hyperbolic degree of freedom permitting access to the ultraparallel lines of the representation manifold which results in direct violations of the fundamental theorem of arithmetic. At these points, symmetry may be manually enhanced and/or restricted depending on the symmetric patterns available on $\mathfrak{R}$, e.g.[1]

$$\frac{(7^2)(1231)(92567)}{(3^3)(5^7)} = \frac{14552123304620819}{2^{39}}. \tag{1}$$

Here, $(7^2)(1231)(92567)$ is a conventional set of prime factorization in the numerator. Division by 3^n, and 5^m is the restriction of high symmetry paths on $\mathfrak{R}$, with the maximal restriction shown. Restrictions on symmetry may be relaxed by reducing n and/or m, such as

$$\frac{(7^2)(1231)(92567)}{(3^2)(5^3)} = \frac{(59)(29027)(3111741449)}{2^{30}}. \tag{2}$$

Relaxing the symmetry restriction permits $\mathfrak{R}$ to assume a configuration that conforms better to the substratum, which is generally expressed by lower order primes. Symmetries available for

[1] All equallities are exact and confirmed with Matlab variable precision at 2000 digits. All numbers provided in equalities are prime except powers.

manipulation vary based on the state of the system, but the nature of the numbers and step count generally provide an indication of available symmetries.

At ρ_{30} the representation manifold expresses the maximal symmetry of $2 \times 3 \times 5$ permitting the exact[2] expression of π as,

$$\pi = \frac{24854370230898984}{(5290919)(1495277429)} \tag{3}$$

2 Defining the Representation

Construction of $\mathfrak{R}$ makes use of the third and fourth roots of unity and is straightforward. The third root is denoted by κ and defined as

$$\kappa_1 = \frac{1}{2} + i\frac{\sqrt{3}}{2}, \quad \kappa_2 = -\frac{1}{2} + i\frac{\sqrt{3}}{2}, \quad {}_i\kappa_1 = \frac{\sqrt{3}}{2} - i\frac{1}{2}, \quad {}_i\kappa_2 = \frac{\sqrt{3}}{2} + i\frac{1}{2}, \tag{4}$$

which cube to $-1, 1, i, -i$, respectively. This term was used by Dedekind and Hurwitz in their work on modular functions to tesselate the upper half-plane ($\mathbb{H}$), as provided and elucidated by Roy [2, 222, 458-459].

This family of terms produces an exceptional multiplication table permitting its function as the Absolute. As a third root of unity, κ provides the minimal points necessary for conception of cyclic order [1, 19, 23].

$$\kappa_1 = \frac{1}{2} + i\frac{\sqrt{3}}{2}, \quad \kappa_2 = -\frac{1}{2} + i\frac{\sqrt{3}}{2}, \quad {}_i\kappa_1 = \frac{\sqrt{3}}{2} - i\frac{1}{2}, \quad {}_i\kappa_2 = \frac{\sqrt{3}}{2} + i\frac{1}{2} \tag{5}$$

$${\kappa_1}^2 = \kappa_2, \quad {\kappa_2}^2 = -\kappa_1, \quad {\kappa_1}^3 = -1, \quad {\kappa_2}^3 = 1, \quad \kappa_1 - \kappa_2 = 1 \tag{6}$$

$$\kappa_1\kappa_2 = -1, \quad \sqrt{\kappa_1} = {}_i\kappa_2, \quad \sqrt{-\kappa_2} = {}_i\kappa_1, \quad \sqrt{\tfrac{\kappa_1}{\kappa_2}} - \sqrt{\tfrac{\kappa_2}{\kappa_1}} = i, \quad {{}_i\kappa_1}^2 = \kappa_2, \quad {{}_i\kappa_2}^2 = \kappa_1 \tag{7}$$

$${}_i\kappa_1\kappa_1 = \kappa_1, \quad {}_i\kappa_2\kappa_2 = -{}_i\kappa_1, \quad {}_i\kappa_2\kappa_1 = {}_i\kappa_1\kappa_2 = {}_i\kappa_2 - {}_i\kappa_2 = i, \quad {}_i\kappa_1\,{}_i\kappa_2 = 1 \tag{8}$$

Although κ is not used explicitly in $\mathfrak{R}$, its cyclic permutations order the substratum on which ρ operates permitting conception of serial order. Definition of ρ, is given by

$$\rho_1 = \frac{3}{5} + i\,\frac{2^2}{5}, \quad \rho_2 = -\frac{3}{5} + i\,\frac{2^2}{5}, \quad {}_i\rho_1 = \frac{\rho_1}{i} = \frac{2^2}{5} - i\,\frac{3}{5}, \quad {}_i\rho_2 = \frac{\rho_2}{i} = \frac{2^2}{5} + i\,\frac{3}{5} \tag{9}$$

$$\sqrt{\rho_1} = \frac{2}{\sqrt{5}} + i\,\frac{1}{\sqrt{5}}, \quad \sqrt{\rho_2} = \frac{1}{\sqrt{5}} + i\frac{2}{\sqrt{5}}. \tag{10}$$

Thus ρ provides a representation of $\mathfrak{u}_1'$ as a primitive generator of serial order.[3]

[2] Confirmed with Matlab variable precision at ten-million digits.

[3] $\mathfrak{u}_1'$ denotes the augmented unitary group, which modifies the unitary group to include odd permutations. This was referenced by Weyl in analogy to augmenting $\mathfrak{o}_3$ to include reflections in the origin [3, 146, 164]. In native hyperbolic space, this analogy is expressed in the form of an absolute involution.

3 Use of the Representation

The representation is trivial to deploy. Access to each point may be considered an iterative process of reflections, begining with

$$\rho_1 : \quad r_1 = \frac{1}{2} + i\frac{\sqrt{3}}{2} \tag{11}$$

$$\rho_0 \to \rho_1 \to \rho_2 : \quad r_2 = {r_1}^2, \tag{12}$$

with r_n a variable storing the resulting value. The base of the exponential determines the axis of the reflection, so the result is twice the distance from the absolute relative to the base. The simplest next steps are reflections to sequential powers of 2 by

$$\rho_0 \to \rho_2 \to \rho_4 : \quad r_4 = {r_2}^2 \tag{13}$$

$$\rho_0 \to \rho_4 \to \rho_8 : \quad r_8 = {r_4}^2 \tag{14}$$

Access to any non-power of 2 region requires composite reflections about an ultra-infinite axis, which is represented by division. Sense on the ultra-infinite manifold is reversed, so division by r_1 reverses position by one step, and division by the complex conjugate of r_1 advances position by one step. One could reach points 3, 5, and 7, respectively, by

$$\rho_0 \to \rho_1 \to \rho_2 \to \rho_3 : \quad ({r_1}^2)({\bar{r}_1}^{-1}) \tag{15}$$

$$\rho_0 \to \rho_2 \to \rho_4 \to \rho_5 : \quad ({r_2}^2)({\bar{r}_1}^{-1}) \tag{16}$$

$$\rho_0 \to \rho_3 \to \rho_6 \to \rho_7 : \quad ({r_3}^2)({\bar{r}_1}^{-1}) \tag{17}$$

$$\tag{18}$$

For general use, symbolic math is preferable to avoid precision loss, but double precision floats should provide exact solutions to ρ_{12}. Simple pseudo-code takes the form

```
//traverse to r11
r1=sym(3/5+i*4/5);
r2=r1^2;
r5=r2^2/r1';
r11=r5^2/r1'
r11 =- 34867797/48828125 - 34182196i/48828125

//evaluate real portion with sufficient precision
vpa(factor(sym(34867797/48828125)))
ans = [ 6431973276730577.0, 0.5, 0.5, 0.5, 0.5, 0.5, 0.5 .... ]

//test available symmetries at this point (5,6,7,...) for new prime factors
>> vpa(factor(sym(5*34867797/48828125)))
ans = [ 3.0, 31.0, 86451253719497.0, 0.5, 0.5, 0.5, 0.5, .... ]

>> vpa(factor(sym(6*34867797/48828125)))
ans = [ 7.0, 13.0, 19.0, 1061.0, 13187.0, 199411.0, 0.5, 0.5, .... ]

>> vpa(factor(sym(7*34867797/48828125)))
 ans = [ 3.0, 3.0, 3.0, 5.0, 17.0, 101.0, 167.0, 145389067.0, 0.5, 0.5, ....]
```

Appendix A Initial Terms of $\boldsymbol{\rho}$

$$\rho_0 = \frac{2}{\sqrt{5}} + i\,\frac{1}{\sqrt{5}}$$

$$\rho_0{}^2 = \rho_1 = \frac{3}{5} + i\,\frac{2^2}{5}$$

$$\rho_1{}^2 = \rho_2 = -\frac{7}{5^2} + i\,\frac{(2^3)\,(3)}{5^2}$$

$$\frac{\rho_1{}^2}{\bar{\rho}_1} = \rho_3 = -\frac{(3^2)\,(13)}{5^3} + i\,\frac{(2^2)\,(11)}{5^3}$$

$$\rho_2{}^2 = \rho_4 = -\frac{(17)\,(31)}{5^4} - i\,\frac{(2^4)\,(3)\,(7)}{5^4}$$

$$\frac{\rho_2{}^2}{\bar{\rho}_1} = \rho_5 = -\frac{(3)(79)}{5^5} - i\,\frac{(2^2)(19)(41)}{5^5}$$

$$\rho_3{}^2 = \rho_6 = \frac{(7)(23)(73)}{5^6} - i\,\frac{(2^3)(3^2)(11)(13)}{5^6}$$

$$\frac{\rho_3{}^2}{\bar{\rho}_1} = \rho_7 = \frac{(3)(83)(307)}{5^7} + i\,\frac{(2^2)(29)(139)}{5^7}$$

$$\rho_4{}^2 = \rho_8 = \frac{(191)(863)}{5^8} + i\,\frac{(2^5)(3)(7)(17)(31)}{5^8}$$

$$\frac{\rho_4{}^2}{\bar{\rho}_1} = \rho_9 = -\frac{(3^3)(13)(37)(71)}{5^9} + i\,\frac{(2^2)(11)(109)(359)}{5^9}$$

$$\rho_5{}^2 = \rho_{10} = -\frac{(7)(479)(2897)}{5^{10}} + i\,\frac{(2^3)(3)(19)(41)(79)}{5^{10}}$$

$$\frac{\rho_5{}^2}{\bar{\rho}_1} = \rho_{11} = -\frac{(3)(43)(89)(3037)}{5^{11}} - i\,\frac{(2^2)(1321)(6469)}{5^{11}}$$

$$\rho_6{}^2 = \rho_{12} = \frac{(17)(31)(47)(1297)}{5^{12}} - i\,\frac{(2^4)(3^2)(7)(11)(13)(23)(73)}{5^{12}}$$

$$\frac{\rho_6{}^2}{\bar{\rho}_1} = \rho_{13} = \frac{(3)(53)(157)(42641)}{5^{13}} - i\,\frac{(2^2)(8839)(16901)}{5^{13}}$$

$$\rho_7{}^2 = \rho_{14} = \frac{(7^2)(1231)(92567)}{5^{14}} + i\,\frac{(2^3)(3)(29)(83)(139)(307)}{5^{14}}$$

$$\frac{\rho_7{}^2}{\bar{\rho}_1} = \rho_{15} = \frac{(3^2)(13)(79)(239)(3119)}{5^{15}} + i\,\frac{(2^2)(11)(19)(41)(59)(61)(241)}{5^{15}}$$

$$\rho_{16} = -\frac{(193)(2689)(189311)}{5^{16}} + i\,\frac{(2^6)(3)(7)(17)(31)(191)(863)}{5^{16}}$$

$$\rho_{17} = -\frac{(3)(67)(13397)(282881)}{5^{17}} - i\,\frac{(2^2)(12239)(873121)}{5^{17}}$$

$$\rho_{18} = -\frac{(7)(23)(73)(4967)(36217)}{5^{18}} - i\,\frac{(2^3)(3^3)(11)(13)(37)(71)(109)(359)}{5^{18}}$$

$$\rho_{19} = \frac{(3)(151)(6917)(2029123)}{5^{19}} - i\,\frac{(2^2)(1782959)(2521451)}{5^{19}}$$

$$\rho_{20} = \frac{(17)(31)(359041)(480959)}{5^{20}} - i\,\frac{(2^4)(3)(7)(19)(41)(79)(479)(2879)}{5^{20}}$$

$$\rho_{21} = \frac{(1787)(2746191911977)}{(43)(151)(377171)(2111041)} - i\,\frac{(21539611)(75499609)}{(43)(151)(377171)(2111041)}$$

$$\rho_{22} = \frac{(7)(97943)(69049993)}{5^{22}} + i\,\frac{(2^3)(3)(43)(89)(1321)(3037)(6469)}{5^{22}}$$

$$\rho_{23} = -\frac{(2^2)(7)(181)(1853362418387)}{5^{23}} + i\,\frac{(2^2)(919)(3221)(5519)(112331)}{5^{23}}$$

$$\rho_{24} = -\frac{(2^5)(3^2)(7)(13)(17)(29)(311)(1409)(10163)}{5^{24}} - i\,\frac{(2^5)(3^2)(7)(11)(13)(17)(23)(31)(47)(73)(1297)}{5^{24}}$$

$$\rho_{25} = -\frac{(2^5)(3^2)(5)(7^2)(13)(17)(19)(4079)(91369)}{5^{25}} - i\,\frac{(2^5)(3^2)(5)(7)(13)(17)(44909)(2766931)}{5^{25}}$$

$$\rho_{26} = \frac{(2^7)(11)(441587)(1248048743)}{5^{26}} - i\,\frac{(2^{10})(173)(7180978404151)}{5^{26}}$$

$$\rho_{27} = \frac{(2^13)(1553)(582953615527)}{5^{27}} - i\,\frac{(2^8)(83257)(33426918589)}{5^{27}}$$

$$\rho_{28} = \frac{(2^3)(5)(7^2)(227)(6886314427)}{4547473508864641} + i\,\frac{(79)(17683)(4811027497)}{(2)(4547473508864641)}$$

$$\rho_{29} = \frac{(3^4)(5^2)(73)(587)(5099)(19211)}{(2^3)(41)(138642485026361)} + i\,\frac{(2)(14071)(225977)(878089)}{(41)(138642485026361)}$$

$$\rho_{30} = -\frac{(373)(4957)(33613)(102673)}{(2)(3552713678800501)} + i\,\frac{(19)(919)(358010863279)}{(2^2)(3552713678800501)}$$

References

[1] H. S. M. Coxeter. *Non-Euclidean Geometry: Fifth Edition.* Univ of Toronto Pr, 1965.

[2] Ranjan Roy. *Elliptic and Modular Functions from Gauss to Dedekind to Hecke.* Cambridge University Press, 2017.

[3] Hermann Weyl. *The Theory of Groups and Quantum Mechanics.* Martino Publishing, 2014.