DETERMINISTIC METHOD FOR SPECIAL EXPONETIAL EQUTAIONS

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ABSTRACT:
In this note, some mathematical equations where solved using a modified approach that introduces logarithm with its rules as well as presented as a certain determinant. While some ideas and theories presented in this note could generate issues of dispute, yet the progressive orderliness and agreement in the method cannot easily be set aside. Diverse equations were developed and conveniently solved by the proposed model. And this modification is called Determinant Method.

PHILOSOPHY OF THIS METHOD USED IN SOLVING THE EXPONETIAL EQUATIONS: INTRODUCTION

The approach used in solving the problems presented in this note is easy to learn, and interesting too. The method will be assimilated as you go through about four examples.

The following procedures provided clarity to the steps taken in solving the problems

METHOD

The method is applied where direct application of the laws of indices may be difficult.

SIMPLIFICATION AS USED

Simplify the equation as much as possible. This may show the possibility of solving the equation using the rules of indices or any formula else. It also enables proper application of the approach designed in this note.

REARRANGEMENT OF QUESTIONS

Rearrange the question such as the unknown variable may be on one side of the equation.

SET LIMITS

The approach employed by the note aims at defining the range or limits from which the value(s) of the unknown can be derived. Limits such as $2.1 \leq X \leq 4.4$ and $0.3 \leq y \leq 9.2$ may be taken as $2 \leq x \leq 5$ and $0 \leq y$ 10 respectively. Assumption always made is that the unknown variable is a whole number.

Note: the operator $\geq$ and $\leq$ are useful only in setting up the range or limits for finding the unknown.

USE OF LOGARITHM

Logarithm usage involves all the applications of rules of logarithm and at the same time the use of log as unknown factor or determinant.

SUMMARY OF METHOD/APPROACH

- Simplify the question.
- Set the unknown factor at one side of the equation.
- Use the denominator and the numerator at the left hand side separately with the figures on the right hand side alongside with the operator ≥ and ≤ to set up limits for the value of the unknown factor.
- Hence find the minimum and maximum limits from which the actual value of the unknown variable can be estimated.

QUESTIONS

Solve for x, y or n in the following equation

1. $4^{x-1} = 2x$
2. $4^{x} = 8x$
3. $2^{x} = 4x$
4. $3^{x} = 3x + 3$
5. $5^{x} = 5x + 3$
6. $3^{y} - y^{3} = 1$
7. $3^{n} - 2^{n} = 65$
8. $2^{x} = 6x + 2$
9. $3^{x} = 4x + 2$
10. $2^{\frac{x}{2}} = x^2$
11. $4^{\frac{x}{4}} = x$
12. $3^{\frac{x}{4}} = \frac{3x}{4}$
13. $169^{\frac{1}{x}} = \frac{26}{x}$
14. $5^{\frac{1}{x}} x + 1 = x^2 + 5^{\frac{1}{x}}$

15. $9^{\frac{1}{x}} = x+1$

16. $9^{\frac{1}{x}} - 1 = x^{y+1}$

$xy = 0$

17. $-xy^2 - 2x^{\frac{3}{2}}y + xy = 8$

$xy^2 + 3xy = -8$

18. $(y-2)^2 + xy + 2 = 0$

$2y(y-2)^x + 4xy + 6 = 0$

19. $3^{xy} - xy^3 = x^2$

$x^{2x} = 1$

20. $\log_{10} x = \log_5 2x$, find X

21. Find the relationship between x and a for which

$\log_a x = a^x$ ………. X > 1

Hence find the value(s) of a and x

22. Three airplanes X, Y and Z took off at one moment from two different airports, X and Y started from a point T: X moved North, and Y moved South. X covered the product of the distance covered by Y in any given instance (time), Z which took off from the South side of T, covered the sum of the distance covered by X and Y in any given instance (time). Z came across Y after covering 5Km. Z and X reached the same terminal point simultaneously, at the same time Y arrived the starting point of Z.

Find: i. The distance covered by X

ii. The distance between the point where Z met Y and where Z met X

23. No. 25 solve for x in $2^{x^{2^x}} = 9$.

SOLUTION

1. If $4^{x-1} = 2x$, Solve for x

Therefore:

$$4^{x-1} = 2x$$

$$\frac{4^x}{4^1} = 8$$

$$4^x = 8x$$

Setting the unknown variable at one side of the equation, we see that

$$\frac{4^x}{x} = 8$$

Setting the limits from the equation above, we will see that … $4^x \geq 8$, hence

$$x \log 4 = \log 8$$

$$x = \frac{\log 8}{\log 4}$$

$$= 1.5$$

We see also from equation 1 that If the above statement is true, then

$$x \leq 8$$

These two premises provided the limits for determining the value of x, hence

$$1.5 \leq x \leq 8$$

That is $1 \leq x \leq 8$

Putting the limits into the equation to determine the value of x

$$4^x = 8x$$

X = 2

1B: Again we attempt to solve the equation using logarithm function.

$4^x = 8x$ taking the log of both sides:

$$\log 4^x = \log(8x)$$
$$x\log 4 = \log 8 + \log x$$
$$2x\log 2 = 3\log 2 + \log x$$
$$2x\log 2 - 3\log 2 = \log x$$
$$\log 2(2x-3) = \log x$$

$\underline{\log}2(2x - 3) = \underline{\log}\ x$

$$2(2x-3) = x$$
$$4x - 6 = x$$
$$4x - x = 6$$
$$3x = 6$$
$$\underline{x = 2}$$

Remember that log is also used as a constant.

2. $2^x = 4x$, Solve for x.

Solution:

$$2^x = 4x$$
$$\frac{2^x}{x} = 4$$

And do we have

$$2^x \geq 4$$
$$2^x \geq 2^2$$
$$x \geq 2$$

Equally, the other extreme is

$$x \leq 4$$

Therefore we have: $2 \leq x \leq 4$

Putting the range into the equation shows that

$x = 4$

Using logarithm function on Question 2.

$$2^x = 4x$$
$$\log 2^x = \log(4x)$$
$$x\log 2 = \log 4 + \log x$$
$$x\log 2 - \log 2^2 = \log x$$
$$x\log 2 - 2\log 2 = \log x$$
$$(x-2)\log 2 = \log x$$
$$\log[2(x-2)] = \log x$$
$$2(x-2) = x$$
$$2x - 4 = x$$
$$2x - x = 4$$
$$x = 4$$

As before.

3. $3^x = 3x + 3$, Solve for x.

Solution:

$$3^x = 3x + 3$$
$$\frac{3^x}{3x+3} = 1$$
$$\frac{3^x}{(x+1)} = 3$$
$$3^x \geq 3$$
$$3^x \geq 3^1$$
$$x \geq 1$$
$$also$$
$$x + 1 \leq 3$$
$$x \leq 3 - 1$$
$$x \leq 2$$
$$Therefore$$
$$1 \leq x \leq 2$$

Putting this range into the equation gives: $x = 2$.

Using Logarithm on Q3.

$$3^x = 3x + 3$$
$$3^x = 3(x+1)$$
$$\log 3^x = \log[3(x+1)]$$
$$x\log 3 = \log 3 + \log(x+1)$$
$$x\log 3 - \log 3 = \log(x+1)$$
$$\log[3(x-1) = \log(x+1)$$
$$3(x-1) = (x+1)$$
$$3x - 3 = x + 1$$
$$3x - x = 1 + 3$$
$$2x = 4$$
$$x = 2.$$

4. $5^x = 25x + 50$, solve for x

Solution:

$$5^x = 25(x+2)$$
$$\frac{5^x}{x+2} = 25$$
$$therefore: 5^x \geq 25$$
$$5^x \geq 5^2$$
$$x \geq 2$$

Also

$$x + 2 \leq 25$$
$$x \leq 25 - 2$$
$$x \leq 23$$
$$hence: 2 \leq x \leq 23$$

Putting the limits into the equation

We have:

X = 3

Using logarithm as a function Q4.

$$5^x = 25(X + 2)$$
$$\log 5^x = \log[25(x+2)]$$
$$x\log 5 = \log 25 + \log(x+2)$$
$$x\log 5 = 2\log 5 + \log(x+2)$$
$$x\log 5 - 2\log 5 = \log(x+2)$$
$$\log[5(x-2) = \log(x+2)$$
$$5(x-2) = x+2$$
$$5x - 10 = x + 2$$
$$5x - x = 2 + 10$$
$$4x = 12$$
$$x = \frac{12}{4}$$
$$x = 3$$

5. Let us assume an equation of these form:

   A = 5B + 31 ……….(1)

   $2^A = 4B + 3$ ………….(2)

   By the simple examination in these two equations, it is evident that

   A ≥ 5B…….. for (1) and $2^A \geq 4B$ …..for (2), for every given value of A.

   In each case the constants (31 and 3) are ignored

Now let us use this assumption to solve the question 3 and 4.

$$3^x = 3x + 3$$

$$\frac{3^x}{3x+3} = 1$$

therefore

$$3^x \geq 3x$$

$$\log 3^x \geq \log 3 + \log x$$

$$x \log 3 \geq \log 3 + \log x$$

$$x \log 3 - \log 3 \geq \log x$$

$$\log[3(x-1)] \geq \log x$$

$$3(x-1) \geq x$$

$$3x - 3 \geq x$$

$$3x - x \leq 3$$

$$x \leq \frac{3}{2}$$

$$x \leq 1.5$$

$$x \approx\leq 2$$

Also

$$3^x \geq 1$$

$$3^x \geq 3^0$$

$$x \geq 0$$

Hence we have succeeded to set the limits once again.

That $0 \leq x \leq 2$, applying this limit to the equation gives **X=2** as before.

Now let consider the question 4.

$$5^x = 25(x+2)$$

$$\frac{5^x}{25x+50} = 1$$

$$5^x \geq 1$$

$$5^x \geq 5^0$$

$$x \geq 0$$

Also:

$$5^x \geq 25x$$
$$\log 5^x \geq \log(25x)$$
$$x \log 5 \geq \log 25 + \log x$$
$$x \log 5 - \log 25 \geq \log x$$
$$x \log 5 - 2\log 5 \geq \log x$$
$$\log[5(x-2)] \geq \log x$$
$$5(x-2) \geq x$$
$$5x - 10 \geq x$$
$$5x - x \leq 10$$
$$4x \leq 10$$
$$x \leq 2.5$$
$$x \approx\leq 3$$

Hence we have succeeded to set the limits once again.

That $0 \leq x \leq 3$, substituting for the value(s) of X gives

X = 3 as before.

Hence for equations of the form

$\frac{\beta}{\alpha + k_i} = k_j$ where k_i and k_j are constants, and β and α quantities with unknown variable(s). The method used in solving the above questions applies. See next questions.

6. Solve for y, $3^y = y^3 + 1$

   Solution:

$$3^y - y^3 = 1$$
$$3^y = 1 + y^3$$
$$\frac{3^y}{1+y^3} = 1$$
$$3^y \geq 1$$
$$3^y \geq 3^o$$
$$y \geq 0$$

Also

$$y^3 \le 3^y$$
$$y \le 3$$

Hene

$$0 \le y \le 3$$

There for the equation

$3^y = 1 + y^3$ ……. We see if the left hand side agrees with the right hand side of the equation.

Table 7.1

Range (y)	0	1	2	3
3^y	**1**	3	**9**	27
$1+y^3$	**1**	1	**9**	28

Hene we see that y = 2 or 0.

<u>NOTE</u>: In this example, if we attempt the question by

$$3^y \ge 1$$
$$3^y \ge 3^0$$
$$y \ge 0$$

And then

$$1 + y^3 \le 1$$
$$y^3 \le 1 - 1$$
$$y^3 \le 0$$

$$y \le 0$$

Then **y** will be identically zero, giving no room to set out the range which the formula intends to establish in other to solve for the unknown.

7. $3^n - 2^n = 65$, solve for n.

Solution:

$$3^n - 2^n = 65$$
$$\frac{3^n}{65 + 2^n} = 1$$
$$3^n \geq 1$$
$$3^n \geq 3^0$$
$$n \geq 0$$

And, lets onsider a scenario first of finding the other extreme of the range by

$$2^n \leq 3^n \ ,$$
$$\log 2^n \leq \log 3^n$$
$$n \log 2 - n \log 3 \geq 0$$
$$n[\log 2 - \log 3] \geq 0$$
$$n \geq \frac{0}{\log 2 - \log 3}$$
$$n \geq 0$$

Mathematically this still states same statement as before whih does not set the limits as required by the method.

$$3^n \leq 65$$
$$\log 3^n \leq \log 65$$
$$n \log 3 \leq \log 65$$
$$n \leq \frac{\log 65}{\log 3}$$
$$n \leq 3.8$$
$$n \approx\leq 4$$

Hene the range $0 \leq n \leq 4$,

When these values are applied it gives n = 4.

8. Solve for x in $2^x = 6x + 2$

Solution:

$$\frac{2^x}{3x+1} = 2$$
$$2^x \geq 2$$
$$x \geq 1$$

Also simplifying the original equation in this arrangement below

$$2^x = 6x + 2$$
$$\frac{2^x}{x} = 6 + \frac{2}{x}$$

We consider the maximum and minimum value of the right hand of the equation in other to determine the left hand side.

The right hand of the equation is greatest when x = 1, and least when x = -1 giving the value of 8 and 4 respectively for the right hand of the equation.

Therefore $6 + \frac{2}{x} = 8$ …….when x = 1

$6 + \frac{2}{x} = 4$ ……..when x = -1

Now we have

$$\frac{2^x}{x} = 4 \ldots\ldots\ldots\ldots\ldots \min imum$$
$$2^x = 4$$
$$x = 2$$

And

$$\frac{2^x}{x} = 8 \ldots\ldots\ldots\ldots \mathit{for} \max imum$$
$$x = 8$$

we can now interpolate the values to determine the actual range, and this gives

$$1 \leq x \leq 6$$

When this is applied as shown in Table 7.1

We find that **<u>x = 5</u>**

9. Solve for x in $3^x = 4x+1$.

   Solution:

$$\frac{3^x}{4x+1} = 1$$

   Therefore

$$3^x = 1 = 3^0$$
$$x = 0$$

   To get the other extreme:

$$3^x = 4x+1$$
$$\frac{3^x}{x} = \frac{4x+1}{x}$$
$$\frac{3^x}{x} = 4 + \frac{1}{x}$$

   The right hand of the equation is minimum when x = -1, and maximum when x = 1.

   For minimum

$$\frac{3^x}{x} = 4-1$$
$$3^x = 3^1$$
$$x = 1$$

For maximum

$$\frac{3^x}{x} = 4+1$$
$$x = 5$$

The constraints therefore are $0 \le x \le 5$, hence

<u>X=0 or 2</u>

No 11. $2^{\frac{x}{2}} = x^2$, Solve for x.

Solution:

$$2^{(\frac{x}{2})2} = x^{(2)2}$$
$$2^x = x^4$$

Further simplifying

$$2^{(x)4} = x^{(4)4}$$
$$16^x = x^{16}$$
$$x = 16$$

No.12. $4^{\frac{x}{4}} = x$, solve for x

Solution:

$$4^x = x^4$$
$$\frac{4^x}{x^4} = 1$$

$$4^x \geq 1$$
$$4^x \geq 4^0$$
$$x = 0$$

For maximum

$$x^4 \leq 4^x$$
$$x \leq 4$$

Setting the range that $0 \leq x \leq 4$

Hence we see that **x = 2 or 4**

No. 13. Solve for x, $3^{\frac{x}{4}} = \frac{3x}{4}$

Solution:

$$3^{(\frac{x}{4})4} = (\frac{3x}{4})^4$$

$$3^x = \frac{3^4 x^4}{4^4}$$

$$\frac{3^x}{x^4} = \frac{3^4}{4^4}$$

By nature of the identity of the left hand and the right hand sides,

X = 4

No. 15, Solve for x

$x\sqrt{169} = \frac{26}{x}$.

Solution:

$$13^{2(\frac{1}{x})} = \frac{26}{x}$$

$$x(13)^{\frac{2}{x}} = 26 = 2(13)^1$$

The two sides of the equation are identical, hence

$$x = 2$$

$$or$$

$$13^{\frac{2}{x}} = 13^1$$

$$\frac{2}{x} = 1$$

$$x = 2.$$

No. 16. Solve the equation: $5^{\frac{1}{x}} x + 1 = x^2 + 5^{\frac{1}{x}}$

Solution:

$$5^{\frac{1}{x}}x - x^2 = 5^{\frac{1}{x}} - 1$$

$$x(5^{\frac{1}{x}} - x) = 1(5^{\frac{1}{x}} - 1)$$

The equation is identical. Hene

X = 1.

No. 17, Solve for X : $9^{\frac{1}{x}} = x + 1$

Solution:

$$9^{\frac{1}{x}} = x + 1$$

$$9 = (x+1)^x$$

$$3^2 = (x+1)^x$$

$$2\log 3 = \log(x+1)^x$$

$$2\log(2+1) = x\log(x+1)$$

The two sides of the equation are identical

Therefore X = 2

No. 18.

Solve the simultaneous equation: $9^{\frac{1}{x}} - 1 = x^{y+1}$.............(1)

$xy = 0$......................(2)

Solution:

From (2), X = 0

Then there will be no solution for the equation. Hence for

xy = 0.

y=0.

Therefore putting the value of y in (1).

$$9^{\frac{1}{x}} - 1 = x^{y+1} \ldots\ldots\ldots\ldots\ldots\ldots(1)$$

$$9^{\frac{1}{x}} - 1 = x^{0+1}$$

$$9^{\frac{1}{x}} = x + 1$$

$$3^2 = (x+1)^x$$

$$2\log 3 = x\log(x+1)$$

$$2\log(2+1) = x\log(x+1)$$

This is identical equation, hence X = 2 .

Therefore we have (2, 0).

19. Solve for x and y.

$$-xy^2 - 2x^{\frac{3}{2}}y + xy = 8 \ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

$$xy^2 + 3xy = -8 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

solution :

$$-xy^2 - 2x^{\frac{3}{2}}y + xy = 8 \ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

$$-xy(-y - 2x^{\frac{1}{2}} + 1) = 8 \ldots\ldots\ldots\ldots\ldots\ldots(1)$$

$$xy(y+3) = -8 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

The ratio of equation to (1) to (2).

$$\frac{-xy(-y - 2x^{\frac{1}{2}} + 1}{xy(y+3)} = \frac{8}{-8}$$

$$\frac{-y - 2x^{\frac{1}{2}} + 1}{y+3} = -1$$

$$-y - 2x^{\frac{1}{2}} + 1 = -y - 3$$

$$-y + y - 2x^{\frac{1}{2}} = -3 - 1$$

$$-2x^{\frac{1}{2}} = -4$$

$$x^{\frac{1}{2}} = 2$$

$$x = 4$$

Substituting for x = 4 in equation (2).

$$xy(y+3)=-8\text{..........................}(2)$$
$$4y(y+3)=-8$$
$$y(y+3)=-2$$
$$y^2+3y+2=0$$
$$(y+1)(y+2)=0$$
$$y=-1, y=-2$$

20. Solve for x :

$$(y-2)^x+xy+2=0$$
$$2y(y-2)^x+4xy+6=0$$

Solution:

$$(y-2)^x+xy+2=0\text{................................}(1)$$
$$2y(y-2)^x+4xy+6=0\text{..........................}(2)$$

From (1)

$$xy=-2-(y-2)^x \quad \text{................}(3)$$

Putting (3) into (2)

$$2y(y-2)^x+4xy+6=0 \quad \text{.............}(2)$$

$$2y(y-2)^x+4(-2-(y-2)^x)+6=0$$
$$2y(y-2)^x+[-8-4(y-2)^x]+6=0$$
$$2y(y-2)^x-8-4(y-2)^x+6=0$$
$$2y(y-2)^x-4(y-2)^x=2$$
$$(y-2)^x(2y-4)=2$$
$$(y-2)^x=\frac{1}{(y-2)}$$
$$(y-2)^x=(y-2)^{-1}$$
$$x=-1$$

From (1)

$$(y-2)^x + xy + 2 = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$
$$(y-2)^{-1} - y + 2 = 0$$
$$\frac{1}{y-2} - y = -2$$
$$1 - y(y-2) = -2(y-2)$$
$$1 - y^2 + 2y = -2y + 4$$
$$-y^2 + 2y + 2y + 1 - 4 = 0$$
$$-y^2 + 4y - 3 = 0$$
$$y^2 - 4y + 3 = 0$$
$$(y-3)(y-1) = 0$$
$$y = 3, y = 1$$

Hene

X = -1, y = 3 or 1.

No. 21.

$$3^{xy} - xy^3 = x^2$$
$$x^{2x} = 1$$

Solution:

If $x^{2x} = 1$ ....(2)

Then $x^{2x} = 1^{2(1)}$

$x = 1$

Or taking the log of both sides

$$\log x^{2x} = \log 1$$
$$2x \log x = 0$$
$$\log x = 0$$

Now lets consider that this is log to base p

$$\log_p x = 0$$
$$x = p^0$$
$$x = 1$$

As before.

Putting the value of x in (1)

$$3^{xy} - xy^3 = x^2 ....(1)$$

$$3^y - y^3 = 1 ....(3)$$

$$3^y \geq 1$$

$$3^y \geq 3^0$$

$$y \geq 0$$

Also

$$y^3 \leq 3^y$$

$$y \leq 3$$

Hence we establish the range as $0 \leq y \leq 3$, Putting the range into (3)

$$3^y - y^3 = 1$$

We have y = 0 or y = 2.

Hence x = 1, y =0 or 2

No. 22

If $\log_{10} x = \log_5 2x$, find x

Solution:

Let

$$\log_{10} x = y$$

$$10^y = x ....(1)$$

Hence

$$\log_5 2x = y$$

$$5^y = 2x ....(2)$$

The ratio of (1) to (2)

$$\frac{10^y}{5^y} = \frac{x}{2x}$$

$$\frac{(2\times 5)^y}{5^y} = \frac{1}{2}$$

$$\frac{2^y \times 5^y}{5^y} = \frac{1}{2}$$

$$2^y = 2^{-1}$$

$$y = -1$$

From (1)

$$x = 10^y$$

$$x = 10^{-1} = \frac{1}{10}$$

No. 23, Find the relationship between a and x for which

$$\log_a x = a^x, \qquad x > 1.$$

Hence find the value(s) of a and x.

Solution:

$$\log_a x = a^x$$

Let

$$\log_a x = p \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

$$a^x = p \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

(1) x (2)

$$a^x \log_a x = p^2$$

$$\log_a x^{a^x} = p^2$$

$$p^{a^x} = p^2$$

$$a^x = 2$$

Therefore from (2)

P = 2

Hence from (1)

$$\log_a x = p = 2$$

$$a^2 = x$$

$$a = \sqrt{x}$$

This is the relationship between a and x.

Therefore from (2)

$$a^x = p$$
$$x \log a = \log p \text{..........................}(3)$$

From (1)

$$\log_a x = p$$
$$a^p = x$$
$$p \log a = \log x \text{............}(4)$$

The ratio of (3) to (4)

$$\frac{x \log a}{p \log a} = \frac{\log p}{\log x}$$
$$\frac{x}{p} = \frac{\log p}{\log x}$$

$$\frac{x}{p} = \log_x p$$ ....By the laws of logarithm.

$$x = p \log_x p$$
$$x^x = p^p$$
$$x^x = 2^2 \text{.................................}(p = 2)$$
$$x = 2$$

Therefore the relationship

$$a = \sqrt{x} = \sqrt{2}$$

Therefore : $a = \sqrt{2}$
$x = 2$

No. 24

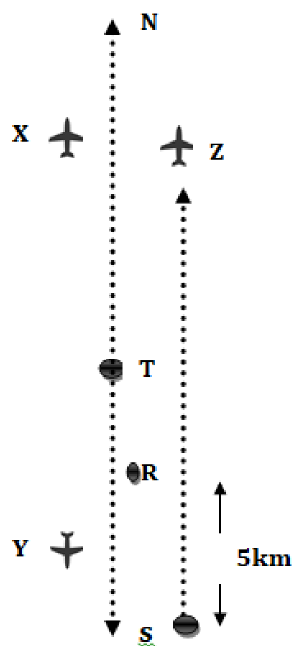
Three airplanes X, Y and Z took off at one moment from two different airports, X and Y started from a point T: X moved North, and Y moved South. X covered the product of the distance covered by Y in any given instance (time), Z which took off from the South side of T, covered the sum of the distance covered by X and Y in any given instance (time). Z came across Y after covering 5Km. Z and X reached the same terminal point simultaneously, at the same time Y arrived the starting point of Z.

Find: i. The distance covered by X

ii. The distance between the point where Z met Y and where Z met X

Solution:

Working with a sketch to gain clarity:



Let the terminal point of X and Z plane be N

Let the starting point of Z plane be S

Let the starting point of X and Y Plane be T

Let the terminal point of Y plane be S

Let the point where Z and Y plane crossed each other be R (whih is 5km from point S)

ke $p \times p$ (km) or p^2 (km)

Hence :

$$5 = p^2 + p \text{ ....(1)}$$

Solving for P

$$p^2 + p - 5 = 0$$

$$p = -1 \pm \sqrt{\frac{1^2 - 4 \times 1 \times (-5)}{2 \times 1}}$$

$$p = -2.79$$

or

$$p = 1.79$$

(i), the distance covered by X plane is the Distance from point T to point N say TN

When Z plane covered 5km, X plane covered $p^2 km$ or 3.2041km

Y plane made 5km to reach destination, plane X covered 25km to reach destination.

Hence the total distance covered by plane X: = 25km + 3.2041km

= 28.2041km

(ii) The distance between the meeting point of the planes YZ and the XZ.:

This is the distance from T to N say TN.

This is equal to the total distance covered by Z minus 5Km

When X covered RS ie 5km,

Z covered RN = 5 + 25 =30km.

No. 25 solve for x in $2^{x^{2^x}} = 9$.

Solution:

Our idea here about the use of log implies using all the attributes of log but at the same considering it as an constant operator.

Thus

$$2^{x^{2^x}} = 9$$

$$x^{2^x} \log 2 = 2 \log 3$$

$$2^x \log x \log 2 = \log 2 \log 3$$

$$x \log 2 \log x \log 2 = \log 1 \log 2 \log 3$$

$$\log x \log \cancel{2 \log} x \log 2 = \log 1 \log 1 \cancel{\log 2} \log 3$$

$$(\log x)^2 \log 2 = (\log 1)^2 \log 3$$

$$(\log x)^2 = \frac{(\log 1)^2 \log 3}{\log 2}$$

$$\log x = \sqrt{\frac{(\log 1)^2 \log 3}{\log 2}}$$

$$\log x = \log 1 \sqrt{\frac{\log 3}{\log 2}}$$

$$\cancel{\log} x = \cancel{\log} 1\sqrt{\frac{\log 3}{\log 2}}$$

$$x = \sqrt{\frac{\log 3}{\log 2}}$$

$$x = 1.2590$$

Reference:

K. A Stroud, Engineering mathematics, Fifth Edition, 2001