The formula of zeta odd number ver.10

Toshiro Takami*
Mmm82889@yahoo.co.jp

【Abstract】

I calculated ζ(3), ζ(5). ζ(7), ζ(9)……… ζ(23).
And the formula indicated.
For example, in ζ(3)

$$\zeta(3)=\frac{8}{7}(1+\sum_{n=1}^{\infty}\frac{1}{(2n+1)^3})$$

For example, in ζ(5)

$$\zeta(5)=\frac{2^5}{2^5-1}(1+\sum_{n=1}^{\infty}\frac{1}{(2n+1)^5})$$

And ultimately the following formula is required

$$\zeta(2m+1)=\frac{2^{2m+1}}{2^{2m+1}-1}(1+\sum_{n=1}^{\infty}\frac{1}{(2n+1)^{2m+1}})$$

n and m are positive integer.

【Discussion】

$$\zeta(3)=\sum_{n=1}^{\infty}\frac{1}{n^3}=1+\sum_{n=1}^{\infty}\frac{1}{(2n)^3}+\sum_{n=1}^{\infty}\frac{1}{(2n+1)^3}$$

calculated

$$\sum_{n=1}^{\infty}\frac{1}{n^3}=1+\frac{1}{8}\sum_{n=1}^{\infty}\frac{1}{n^3}+\sum_{n=1}^{\infty}\frac{1}{(2n+1)^3} \tag{1}$$

$$\frac{7}{8}\sum_{n=1}^{\infty}\frac{1}{n^3}=1+\sum_{n=1}^{\infty}\frac{1}{(2n+1)^3} \tag{2}$$

$$\zeta(3)=\frac{8}{7}(1+\sum_{n=1}^{\infty}\frac{1}{(2n+1)^3})$$

n=1,　ζ (3)≒ 8/7*(1+1/27)= 1.185185185185185185185185185185185185185185……
n=1,2,　ζ (3)≒ 8/7*(1+1/27+1/125)= 1.194328042328042328042328042……
n=1,2,3　ζ (3)≒ 8/7*(1+1/27+1/125+1/343)= 1.19765998735094945007481……
n=1,2,3,4　ζ (3)≒ 8/7*(1+1/27+1/125+1/343+1/729)=
1.1992276926223584251……
n=1,2,3,4,5　ζ (3)≒ 8/7*(1+1/27+1/125+1/343+1/729+1/1391)=
1.20004930092060…..
n=1,2,3,4,5,6　ζ (3)≒ 8/7*(1+1/27+1/125+1/343+1/729+1/1391+1/2197)=
1.2005694907899045327877…..
n=1,2,3,4,5,6,7　ζ (3)≒ 8/7*(1+1/27+1/125+1/343+1/729+1/1391+1/2197+1/3375)=
1.2009081151285288714121…..
n=1,2,3,4,5,6,7,8　ζ(3)=
8/7*(1+1/27+1/125+1/343+1/729+1/1391+1/2197+1/3375+1/4913)=
1.20114073412768562754019085……..
…….
ζ (3) = 1.202056903159594285399738161511449……..

and

$$\zeta(5)=\sum_{n=1}^{\infty}\frac{1}{n^5}=1+\sum_{n=1}^{\infty}\frac{1}{(2n)^5}+\sum_{n=1}^{\infty}\frac{1}{(2n+1)^5}$$

calculated

$$\sum_{n=1}^{\infty}\frac{1}{n^5}=1+\frac{1}{32}\sum_{n=1}^{\infty}\frac{1}{n^5}+\sum_{n=1}^{\infty}\frac{1}{(2n+1)^5} \tag{1}$$

$$\frac{31}{32}\sum_{n=1}^{\infty}\frac{1}{n^5}=1+\sum_{n=1}^{\infty}\frac{1}{(2n+1)^5} \tag{2}$$

$$\zeta(5)=\frac{2^5}{2^5-1}(1+\sum_{n=1}^{\infty}\frac{1}{(2n+1)^5})$$

$$\zeta(5)=\frac{32}{31}(1+\sum_{n=1}^{\infty}\frac{1}{(2n+1)^5})$$

n=1,　ζ (5)≒ 32/31*(1+1/243)= 1.03650604009026948095048……

n=1,2,　ζ (5)≒ 32/31*(1+1/243+1/3125)=1.03683636267091464224080

…….

n=1,2,3,　ζ (5)≒ 32/31*(1+1/243+1/3125+1/16807)=
1.036897781012350718222972……

n=1,2,3,4,　ζ (5)≒ 32/31*(1+1/243+1/3125+1/16807+1/59049)=
1.0369152623933142591640938344…….

…….

ζ (5) = 1.0369277551433699263313654864570

and

$$\zeta(7)=\sum_{n=1}^{\infty}\frac{1}{n^7}=1+\sum_{n=1}^{\infty}\frac{1}{(2n)^7}+\sum_{n=1}^{\infty}\frac{1}{(2n+1)^7}$$

calculated

$$\zeta(7) = \frac{2^7}{2^7 - 1}(1 + \sum_{n=1}^{\infty} \frac{1}{(2n+1)^7})$$

$$\zeta(7) = \frac{128}{127}(1 + \sum_{n=1}^{\infty} \frac{1}{(2n+1)^7})$$

n=1,　ζ (7) ≒ 128/127*(1+1/2187)= 1.0083348634918577…..

n=1,2,　ζ (7) ≒128/127*(1+1/2187+1/78125)= 1.00834776427925933126667……

n=1,2,3,　ζ (7) ≒128/127*(1+1/2187+1/78125+1/823543)=

1.0083489881060853112471…

…….

ζ (7)= 1.0083492773819228268839…….

and

$$\zeta(9) = \sum_{n=1}^{\infty} \frac{1}{n^9} = 1 + \sum_{n=1}^{\infty} \frac{1}{(2n)^9} + \sum_{n=1}^{\infty} \frac{1}{(2n+1)^9}$$

calculated

$$\zeta(9) = \frac{2^9}{2^9 - 1}(1 + \sum_{n=1}^{\infty} \frac{1}{(2n+1)^9})$$

$$\zeta(9) = \frac{512}{511}(1 + \sum_{n=1}^{\infty} \frac{1}{(2n+1)^9})$$

n=1, ζ (9)≒512/511*(1+1/3^9)= 512/511*(1+1/19683)= 1.002007851849068001......
n=1,2, ζ (9)≒512/511*(1+1/3^9+1/5^9)= 512/511*(1+1/19683+1/1953125)=
1.002008364851024948963577597284.........
……...
ζ (9) = 1.002008392826082214417852769232412......

and

$$\zeta(11) = \sum_{n=1}^{\infty} \frac{1}{n^{11}} = 1 + \sum_{n=1}^{\infty} \frac{1}{(2n)^{11}} + \sum_{n=1}^{\infty} \frac{1}{(2n+1)^{11}}$$

calculated

$$\zeta(11) = \frac{2^{11}}{2^{11}-1}(1 + \sum_{n=1}^{\infty} \frac{1}{(2n+1)^{11}})$$

$$\zeta(11) = \frac{2048}{2047}(1 + \sum_{n=1}^{\infty} \frac{1}{(2n+1)^{11}})$$

n=1,
ζ (11)≒ 2048/2047*(1+1/3^11)= 2048/2047*(1+1/177147)=
1.00049416757202925667272........
n=1,2,
ζ(11)≒2048/2047*(1+1/3^11+1/5^11)=2048/2047*(1+1/177147+1/48828125)=
1.00049418806203414187057225..........
……...
ζ (11) =1.000494188604119464558702282526......

and

$$\zeta(13) = \sum_{n=1}^{\infty} \frac{1}{n^{13}} = 1 + \sum_{n=1}^{\infty} \frac{1}{(2n)^{13}} + \sum_{n=1}^{\infty} \frac{1}{(2n+1)^{13}}$$

calculated

$$\zeta(13) = \frac{2^{13}}{2^{13}-1}(1+\sum_{n=1}^{\infty}\frac{1}{(2n+1)^{13}})$$

$$\zeta(13) = \frac{8192}{8191}(1+\sum_{n=1}^{\infty}\frac{1}{(2n+1)^{13}})$$

n=1,　ζ(13)≒ 8192/8191*(1+1/3^13)= 8192/8191*(1+1/1594323)=
1.00012271251752974 88……

…….

ζ(13) = 1.000122713347578489146751 83……

and,

$$\zeta(15) = \sum_{n=1}^{\infty}\frac{1}{n^{15}} = 1+\sum_{n=1}^{\infty}\frac{1}{(2n)^{15}}+\sum_{n=1}^{\infty}\frac{1}{(2n+1)^{15}}$$

calculated

$$\zeta(15) = \frac{2^{15}}{2^{15}-1}(1+\sum_{n=1}^{\infty}\frac{1}{(2n+1)^{15}})$$

$$\zeta(15) = \frac{32768}{32767}(1+\sum_{n=1}^{\infty}\frac{1}{(2n+1)^{15}})$$

n=1,　ζ(15)≒ 32768/32767*(1+1/3^15)= 32768/32767*(1+1/14348907)=
1.000030588203322260846 8038……..

…….

ζ(15) =1.000030588236307020493 5517…….

and

$$\zeta(17)=\sum_{n=1}^{\infty}\frac{1}{n^{17}}=1+\sum_{n=1}^{\infty}\frac{1}{(2n)^{17}}+\sum_{n=1}^{\infty}\frac{1}{(2n+1)^{17}}$$

calculated

$$\zeta(17)=\frac{2^{17}}{2^{17}-1}(1+\sum_{n=1}^{\infty}\frac{1}{(2n+1)^{17}})$$

$$\zeta(17)=\frac{131072}{131071}(1+\sum_{n=1}^{\infty}\frac{1}{(2n+1)^{17}})$$

n=1,　ζ (17) ≒ 131072/131071*(1+1/3^17)=131072/131071*(1+1/129140163)=
=2^17/2^17-1*(1+1/3^17)=131072/131071*(1+1/129140163)=
1.0000076371963228089989……
…….
ζ (17) = 1.0000076371976378997622……

and

$$\zeta(19)=\sum_{n=1}^{\infty}\frac{1}{n^{19}}=1+\sum_{n=1}^{\infty}\frac{1}{(2n)^{19}}+\sum_{n=1}^{\infty}\frac{1}{(2n+1)^{19}}$$

calculated

$$\zeta(19)=\frac{524288}{524287}(1+\sum_{n=1}^{\infty}\frac{1}{(2n+1)^{19}}) \tag{1}$$

$$\zeta(19)=\frac{2^{19}}{2^{19}-1}(1+\sum_{n=1}^{\infty}\frac{1}{(2n+1)^{19}}) \tag{2}$$

n=1,　ζ (19) ≒ 524288/524287*(1+1/3^19)= 524288/524287*(1+1/1162261467)=
1.000001908212664036553598 01……..

…….

ζ (19) =1.0000019082127165539389 2…….

And

$$\zeta(21)=\sum_{n=1}^{\infty}\frac{1}{n^{21}}=1+\sum_{n=1}^{\infty}\frac{1}{(2n)^{21}}+\sum_{n=1}^{\infty}\frac{1}{(2n+1)^{21}}$$

calculated

$$\zeta(21)=\frac{2097152}{2097151}(1+\sum_{n=1}^{\infty}\frac{1}{(2n+1)^{21}})$$

$$\zeta(21)=\frac{2^{21}}{2^{21}-1}(1+\sum_{n=1}^{\infty}\frac{1}{(2n+1)^{21}})$$

n=1,　ζ (21) ≒ 2^21/(2^21-1)*(1+1/3^21)=
2097152/2097151*(1+1/10460353203)=
1.0000004769329846888538………

…….

ζ (21) =1.0000004769329867878064 6……

and

$$\zeta(23)=\sum_{n=1}^{\infty}\frac{1}{n^{23}}=1+\sum_{n=1}^{\infty}\frac{1}{(2n)^{23}}+\sum_{n=1}^{\infty}\frac{1}{(2n+1)^{23}}$$

calculated

$$\zeta(23)=\frac{8388608}{8388607}(1+\sum_{n=1}^{\infty}\frac{1}{(2n+1)^{23}})$$

$$\zeta(23)=\frac{2^{23}}{2^{23}-1}(1+\sum_{n=1}^{\infty}\frac{1}{(2n+1)^{23}})$$

n=1, ζ (23)≒ 2^23/(2^23-1)*(1+1/3^23)=
8388608/8388607*(1+1/ 94143178827)=
1.0000001192199258813883310б………

…….

ζ (23) =1.0000001192199259653110730…….

And the formula will be

$$\zeta(2m+1)=\sum_{n=1}^{\infty}\frac{1}{n^{2m+1}}=1+\sum_{n=1}^{\infty}\frac{1}{(2n)^{2m+1}}+\sum_{n=1}^{\infty}\frac{1}{(2n+1)^{2m+1}}$$

calculated

$$\sum_{n=1}^{\infty}\frac{1}{n^{2m+1}}=1+\frac{1}{2^{2m+1}}\sum_{n=1}^{\infty}\frac{1}{n^{2m+1}}+\sum_{n=1}^{\infty}\frac{1}{(2n+1)^{2m+1}} \qquad (1)$$

$$\frac{2^{2m+1}-1}{2^{2m+1}}\sum_{n=1}^{\infty}\frac{1}{n^{2m+1}}=1+\sum_{n=1}^{\infty}\frac{1}{(2n+1)^{2m+1}} \qquad (2)$$

$$\zeta(2m+1)=\frac{2^{2m+1}}{2^{2m+1}-1}(1+\sum_{n=1}^{\infty}\frac{1}{(2n+1)^{2m+1}})$$

n and m are positive integer.

【References】

1) https://en.wikipedia.org/wiki/Riemann_hypothesis

☃～☃～☃～☃～☃～

☃～☃～☃～☃～☃～

I am a psychiatrist now and also a doctor of brain surgery before.

(home)
〒854-0067
47-8 kuyamadai, Isahaya City, Nagasaki Prefecture, Japan

mmm82889@yahoo.co.jp
I would like to receive an email. I will not answer the phone.

Currently 56 years old
Born on November 26, 1961
(I am very poor of English. Almost all document are google-translation.)
When converted to English by Google translation, it becomes cryptic to me.
But, I read letter by google translation.