

zeta odd-numbers

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Abstract

I tried to find a new expression for zeta odd-numbers, but it seems to have remained a general expression.

However, it may be a new expression and will be published here.

key words

zeta odd-numbers, new expression

1 introduction

$$\zeta(3) = \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3} + \sum_{n=1}^{\infty} \frac{1}{(2n)^3} = \frac{1}{8} \sum_{n=1}^{\infty} \frac{1}{n^3} + \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3} = \frac{1}{8} \zeta(3) + \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3} \quad (1)$$

$$\zeta(3) = \frac{8}{7} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3} \quad (2)$$

from Eq.(2)

$$\zeta(3) = \frac{2^3}{2^3-1} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3} \quad (3)$$

$$\zeta(3) \approx \frac{2^3}{2^3-1} \sum_{n=1}^{10000} \frac{1}{(2n-1)^3} \quad (4)$$

$$\approx 1.1428571428571428571428571428[1.0517997896396450013....] \quad (5)$$

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$$= 1.20205690244530857291428571422561144059202028564... \quad (6)$$

$$\zeta(3) = 1.2020569031596... \quad (7)$$

Do the same

$$\zeta(5) = \sum_{n=1}^{\infty} \frac{1}{(2n-1)^5} + \sum_{n=1}^{\infty} \frac{1}{(2n)^5} = \frac{1}{32} \sum_{n=1}^{\infty} \frac{1}{n^5} + \sum_{n=1}^{\infty} \frac{1}{(2n-1)^5} = \frac{1}{32} \zeta(5) + \sum_{n=1}^{\infty} \frac{1}{(2n-1)^5} \quad (8)$$

$$\zeta(5) = \frac{2^5}{2^5-1} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^5} = \frac{32}{31} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^5} \quad (9)$$

$$\approx \frac{32}{31} \sum_{n=1}^{10000} \frac{1}{(2n-1)^5} \approx 1.032258064516129032258064516129032258[1.00452376279513961535226...] \quad (10)$$

$$= 1.036927755143369925524913595084003046537673070891103... \quad (11)$$

$$\zeta(5) = 1.036927755143369926331... \quad (12)$$

do the same

$$\zeta(7) = \sum_{n=1}^{\infty} \frac{1}{(2n-1)^7} + \sum_{n=1}^{\infty} \frac{1}{(2n)^7} = \frac{1}{128} \sum_{n=1}^{\infty} \frac{1}{n^7} + \sum_{n=1}^{\infty} \frac{1}{(2n-1)^7} = \frac{1}{128} \zeta(7) + \sum_{n=1}^{\infty} \frac{1}{(2n-1)^7} \quad (13)$$

$$\zeta(7) = \frac{2^7}{2^7-1} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^7} = \frac{128}{127} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^7} \quad (14)$$

$$\approx \frac{128}{127} \sum_{n=1}^{10000} \frac{1}{(2n-1)^7} \approx 1.00787401574803149606299212598425[1.00047154865237655475511163...] \quad (15)$$

$$= 1.008349277381922826839797548351207339820116747971686595497... \quad (16)$$

$$\zeta(7) = 1.008349277381922826839797549849796759599863560565238706417... \quad (17)$$

do the same

$$\zeta(9) = \sum_{n=1}^{\infty} \frac{1}{(2n-1)^9} + \sum_{n=1}^{\infty} \frac{1}{(2n)^9} = \frac{1}{512} \sum_{n=1}^{\infty} \frac{1}{n^9} + \sum_{n=1}^{\infty} \frac{1}{(2n-1)^9} = \frac{1}{512} \zeta(9) + \sum_{n=1}^{\infty} \frac{1}{(2n-1)^9} \quad (18)$$

$$\zeta(9) = \frac{2^9}{2^9-1} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^9} = \frac{512}{511} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^9} \quad (19)$$

$$\approx \frac{512}{511} \sum_{n=1}^{10000} \frac{1}{(2n-1)^9} \approx 1.00195694716242661448140900195694[1.000051345183843772592817900542505\dots] \quad (20)$$

$$= 1.002008392826082214417852769232404892051595618903725788276\dots \quad (21)$$

$$\zeta(9) = 1.002008392826082214417852769232412060485605851394888756548\dots \quad (22)$$

do the same

$$\zeta(11) = \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{11}} + \sum_{n=1}^{\infty} \frac{1}{(2n)^{11}} = \frac{1}{2048} \sum_{n=1}^{\infty} \frac{1}{n^{11}} + \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{11}} = \frac{1}{2048} \zeta(11) + \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{11}} \quad (23)$$

$$\zeta(11) = \frac{2^{11}}{2^{11}-1} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{11}} = \frac{2048}{2047} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{11}} \quad (24)$$

$$\approx \frac{2048}{2047} \sum_{n=1}^{10000} \frac{1}{(2n-1)^{11}} \approx 1.00048851978505129457743038593063019[1.0000056660510901093513982\dots] \quad (25)$$

$$= 1.000494188604119464558702253835447194036760299214\dots \quad (26)$$

$$\zeta(11) = 1.00049418860411946455870228\dots \quad (27)$$

do the same

$$\zeta(13) = \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{13}} + \sum_{n=1}^{\infty} \frac{1}{(2n)^{13}} = \frac{1}{8192} \sum_{n=1}^{\infty} \frac{1}{n^{13}} + \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{13}} = \frac{1}{8192} \zeta(13) + \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{13}} \quad (28)$$

$$\zeta(13) = \frac{2^{13}}{2^{13}-1} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{13}} = \frac{8192}{8191} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{13}} \quad (29)$$

$$\approx \frac{8192}{8191} \sum_{n=1}^{10000} \frac{1}{(2n-1)^{13}} \approx 1.000122085215480405322915394945672[1.00000006280554218023194634\dots]$$

$$= 1.000122713347578489146751821853316305\dots \quad (31)$$

$$\zeta(13) = 1.0001227133475784891467518\dots \quad (32)$$

do the same

$$\zeta(15) = \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{15}} + \sum_{n=1}^{\infty} \frac{1}{(2n)^{15}} = \frac{1}{32768} \sum_{n=1}^{\infty} \frac{1}{n^{15}} + \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{15}} = \frac{1}{32768} \zeta(15) + \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{15}} \quad (33)$$

$$\zeta(13) = \frac{2^{15}}{2^{15}-1} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{15}} = \frac{32768}{32767} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{15}} \quad (34)$$

$$\approx \frac{32768}{32767} \sum_{n=1}^{10000} \frac{1}{(2n-1)^{15}} \approx 1.0000305185094759971922971282082583[1.00000006972470312928809233\dots]$$

$$= 1.000030588236307020493551727940916854\dots \quad (36)$$

$$\zeta(15) = 1.0000305882363070204935517285\dots \quad (37)$$

do the same

$$\zeta(17) = \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{17}} + \sum_{n=1}^{\infty} \frac{1}{(2n)^{17}} = \frac{1}{131072} \sum_{n=1}^{\infty} \frac{1}{n^{17}} + \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{17}} = \frac{1}{131072} \zeta(17) + \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{17}} \quad (38)$$

$$\zeta(17) = \frac{2^{17}}{2^{17}-1} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{17}} = \frac{131072}{131071} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{17}} \quad (39)$$

$$\approx \frac{131072}{131071} \sum_{n=1}^{10000} \frac{1}{(2n-1)^{17}} \approx 1.0000076294527393550060654149[1.000000007744839455869605736267\dots]$$

$$= 1.0000076371976378997622736002650939\dots \quad (41)$$

$$\zeta(17) = 1.00000763719763789976227360029356302921308\dots \quad (42)$$

do the same

$$\zeta(19) = \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{19}} + \sum_{n=1}^{\infty} \frac{1}{(2n)^{19}} = \frac{1}{524288} \sum_{n=1}^{\infty} \frac{1}{n^{19}} + \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{19}} = \frac{1}{524288} \zeta(19) + \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{19}} \quad (43)$$

$$\zeta(19) = \frac{2^{19}}{2^{19}-1} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{19}} = \frac{524288}{524287} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{19}} \quad (44)$$

$$\approx \frac{524288}{524287} \sum_{n=1}^{1000} \frac{1}{(2n-1)^{19}} \approx 1.00000190735227079824599885177[1.0000000008604441145228910749615316...] \quad (45)$$

$$= 1.000001908212716553938925656953862044069423427900110139813... \quad (46)$$

$$\zeta(19) = 1.000001908212716553938925656957795101353258571144838630235... \quad (47)$$

make official

$$\zeta(2m-1) = \frac{2^{2m-1}}{2^{2m-1}-1} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{2m-1}} \quad (48)$$

m is a positive integer greater than or equal to 2

2 Postscript

These calculations were performed with WolframAlpha.

References

- [1] B.Riemann.: Uber die Anzahl der Primzahlen unter einer gegebenen Grosse, Mon. Not. Berlin Akad pp.671-680, 1859
- [2] John Derbyshire.: Prime Obsession: Bernhard Riemann and The Greatest Unsolved Problem in Mathematics, Joseph Henry Press, 2003
- [3] S.Kurokawa.: Riemann hypothesis, Japan Hyoron Press, 2009
- [4] Marcus du Sautoy.: The Music of The Primes, Zahar Press, 2007
- [5] T.Takami.: Consideration of the Riemann hypothesis, viXra:1905.0546

I was finally crazy because of the curse of Riemann.

Please raise the prize money to my little son and daughter who are still young.