ELEMENTARY SET THEORY USED TO PROVE FLT

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Abstract. An open problem is proving FLT *simply* (as Fermat might have) for each $n \in \mathbb{N}, n > 2$. Our *direct proof* (not BWOC) of FLT is based on our algebraic identity $((r^n + 2q^n)^{\frac{1}{n}})^n - ((r^n - 2q^n)^{\frac{1}{n}})^n = (2^{\frac{2}{n}}q)^n$ for which n is any given positive natural number, r is unrestricted positive real and q is all positive rationals such that the set of triples $\{((r^n+2q^n)^{\frac{1}{n}}, (r^n-2q^n)^{\frac{1}{n}}, 2^{\frac{2}{n}}q\}$ is not empty with $(r^n+2q^n)^{\frac{1}{n}}, (r^n-2q^n)^{\frac{1}{n}}, (2^{\frac{2}{n}}q) \in \mathbb{N}$. We relate this identity to the transposed *Fermat equation* $z^n - y^n = x^n$ for which z, y, x are natural numbers. We demonstrate, for any given value of n, that $2^{\frac{2}{n}}q = x$. Clearly, for $n > 2$, the term $2^{\frac{2}{n}}q$ with $q \in \mathbb{Q}$ is not rational. Consequently, for values of $n \in \mathbb{N}, n > 2$, it is true that $\{(x, y, z)|x, y, z \in \mathbb{N}, x^n + y^n = z^n\} = \varnothing$.

1. Introduction

FLT states, for $n \in \mathbb{N}, n > 2$, $x, y, z \in \mathbb{N}, x, y, z > 0$ that $x^n + y^n = z^n$ does not hold. A *simple* (using Fermat's tools) proof of FLT for each $n \in \mathbb{N}, n > 2$ is lacking.

For $n \in \mathbb{N}, n > 2$: We propose a simple *direct proof* (not the expected BWOC).

We want an algebraic identity *to relate with* the traditional Fermat equation $x^n + y^n = z^n$ $(x, y, z \in \mathbb{N})$, which, for convenience, we transpose as $z^n - y^n = x^n$. The simplest algebraic identity we have considered that contains $2^{\frac{2}{n}}q$, a term that is irrational for $n > 2$, is $((r^n + q^n)^{\frac{1}{n}})^n - ((r^n - q^n)^{\frac{1}{n}})^n = ((2^{\frac{1}{n}}q)^n$, with r being unrestricted positive real and q being all positive rationals such that the equation $((r^n + q^n)^{\frac{1}{n}})^n - ((r^n - q^n)^{\frac{1}{n}})^n = (2^{\frac{1}{n}}q)^n$ holds for $(r^n + q^n)^{\frac{1}{n}}, (r^n - q^n)^{\frac{1}{n}}, 2^{\frac{1}{n}}q \in \mathbb{N}$.

For $n = 2$: Eqn. $((r^n + q^n)^{\frac{1}{n}})^n - ((r^n - q^n)^{\frac{1}{n}})^n = (2^{\frac{1}{n}}q)^n$ does not hold for $(r^n+q^n)^{\frac{1}{n}}, (r^n-q^n)^{\frac{1}{n}}, 2^{\frac{1}{n}}q \in \mathbb{N}$. So, $((r^n+q^n)^{\frac{1}{n}})^n - ((r^n-q^n)^{\frac{1}{n}})^n = (2^{\frac{1}{n}}q)^n$ would be a false premise from which nothing would follow logically in our argument, below.

We decided to use $((r^n + 2q^n)^{\frac{1}{n}})^n - ((r^n - 2q^n)^{\frac{1}{n}})^n = (2^{\frac{2}{n}}q)^n$ such that n is any given positive natural number, r is unrestricted positive real numbers, and q is all positive rationals, such that $((r^n + 2q^n)^{\frac{1}{n}})^n - ((r^n - 2q^n)^{\frac{1}{n}})^n = (2^{\frac{2}{n}}q)^n$ holds for $(r^n + 2q^n)^{\frac{1}{n}}, (r^n - 2q^n)^{\frac{1}{n}}, 2^{\frac{2}{n}}q \in \mathbb{N}$.

Identity $((r^n + 2q^n)^{\frac{1}{n}})^n - ((r^n - 2q^n)^{\frac{1}{n}})^n = ((2^{\frac{2}{n}}q)^n$ clearly holds for $n = 1, 2$.

We have considered identities with the following general form :

For any given $n > 0$: $((r^n + 2^p q^n)^{\frac{1}{n}})^n - ((r^n - 2^p q^n)^{\frac{1}{n}})^n = (2^{\frac{p+1}{n}}q)^n$ such that $p \in \mathbb{I}, p \geq 0, r \in \mathbb{R}, q \in \mathbb{Q}$, with $r, q > 0$ for which the respective triples hold.

We reject identities with even $p \geq 0, q \in \mathbb{Q}$ since these identities *exclude* (which we define as "fails to hold for") $n = 2$. We reject identities with odd $p > 1, q \in \mathbb{Q}$ since these equally valid identities yield, with each value of odd $p > 1, q \in \mathbb{Q}$, a *different set of excluded* n. Our chosen identity with $p = 1, q \in \mathbb{Q}$ yields the composite set of all elements contained in these different sets of excluded n.

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2. Our Direct Proof

Our argument, below, is a *direct proof*, one that does not rely on the deriving of a contradiction as is generally expected. Instead, we attempt to infer a series of true statements (conclusions) from justified statements (premises).

Per Sect. 1, the *identity* that, below, we relate to $z^n - y^n = x^n$ is :

$$\left((r+2q^n)^{\frac{1}{n}}\right)^n - \left((r-2q^n)^{\frac{1}{n}}\right)^n = (2^{\frac{2}{n}}q)^n. \tag{1}$$

For any given value of $n \in \mathbb{N}, n > 0 : r \in \mathbb{R}, q \in \mathbb{Q}, n, q, r > 0$ such that $r > 2q^n$.

Variable q must be be rational for our proof to work since we want term $2^{\frac{2}{n}}q$ of (1) to be irrational for $n > 2$. Also, we must exclude $q \in \mathbb{R} - \mathbb{Q}$ from our argument (based upon (1)) since, for $n = 2$, if $q \in \mathbb{R} - \mathbb{Q}$, then, term $2^{\frac{2}{n}}q$ is not rational. Luckily, *our use of solely rational q is sufficient for our argument*, as shown, below.

Note, for $n = 2$, with $q \in \mathbb{R} - \mathbb{Q}$, identity $((r^n+q^n)^{\frac{1}{n}})^n - ((r^n-q^n)^{\frac{1}{n}})^n = (2^{\frac{1}{n}}q)^n$, which we have rejected, above, does hold for $(r^n+q^n)^{\frac{1}{n}}, (r^n-q^n)^{\frac{1}{n}}, 2^{\frac{1}{n}}q \in \mathbb{N}$. However, for $n > 2$, with $q \in \mathbb{R} - \mathbb{Q}$, term $2^{\frac{1}{n}}q$ gives us no useful new information.

Temporarily, we *generalize* equation (1) so that this equation (also an algebraic identity) holds for $(r^n + 2q^n)^{\frac{1}{n}}, (r^n - 2q^n)^{\frac{1}{n}}, 2^{\frac{2}{n}}q \in \mathbb{R}$, with $r \in \mathbb{R}, q \in \mathbb{Q}$, $r, q > 0$.

So, for $n > 0$, such $((r^n+2q^n)^{\frac{1}{n}})^n - ((r^n+2q^n)^{\frac{1}{n}})^n = (2^{\frac{2}{n}}q)^n$ is a *true statement.*

Temporarily, *generalize* $z^n - y^n = x^n$ so that this equation holds for $z, y, x \in \mathbb{R}$.

Hence, for any given $n > 0$, such $z^n - y^n = x^n$ is a *true statement.*

For *any given* $n \in \mathbb{N}, n > 0$: With any given $q \in \mathbb{Q}, q > 0$, unrestricted $r \in \mathbb{R}, r > 0$ *varies such that* positive real $((r^n + 2q^n)^{\frac{1}{n}})^n - ((r^n - 2q^n)^{\frac{1}{n}})^n$ of (1) takes every positive real value of $z^n - y^n$ of $z^n - y^n = x^n$. By definition, positive real $z^n - y^n$ takes every value of positive real $((r^n + 2q^n)^{\frac{1}{n}})^n - ((r^n - 2q^n)^{\frac{1}{n}})^n$.

Thus, for any given value of $n > 0$: $((r^n + 2q^n)^{\frac{1}{n}})^n - ((r^n - 2q^n)^{\frac{1}{n}})^n = z^n - y^n$.

So, for any given $n > 0$, it is uniquely determined that $(2^{\frac{2}{n}}q)^n \in \mathbb{R} = x^n \in \mathbb{R}$.

Consequently, for any given value of n, it is true that $2^{\frac{2}{n}}q \in \mathbb{R} = x \in \mathbb{R}$.

3. Results and Conclusion

Hence, for $n \in \mathbb{N}, n > 2$: $\{2^{\frac{2}{n}}q \in \mathbb{R} | q \in \mathbb{Q}$, (1) holds $\} = \{x \in \mathbb{R} | z^n - y^n = x^n\}$.

So, the respective subsets are also equal, with both sides of the equation being empty sets, or with both sides of the equation being non-empty sets, as follows :

For $n \in \mathbb{N}, n > 2$: $\{2^{\frac{2}{n}}q \in \mathbb{N} | q \in \mathbb{Q}$, (1) holds $\} = \{x \in \mathbb{N} | z^n - y^n = x^n\}$.

Per above, for $n \in \mathbb{N}, n > 2$: $\{2^{\frac{2}{n}}q \in \mathbb{N} | q \in \mathbb{Q}$, (1) holds $\} = \varnothing$.

Consequently, for any given value of $n \in \mathbb{N}, n > 2$: $\{x \in \mathbb{N} | z^n - y^n = x^n\} = \varnothing$.

It logically follows, for $n \in \mathbb{N}, n > 2$, that the following statement is true :

Equation $x^n + y^n = z^n$ does not hold for (x, y, z) with $x, y, z \in \mathbb{N}, x, y, z > 0$.

QED.