February 2021

Estimation and Prediction of Neutrino Mass Based on the Kinetic Theory of Gases

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Abstract

Neutrinos and molecules co-exist in space. Because both have physical properties of mass and speed, it is a logical assumption that neutrinos can interact with molecules according to the kinetic theory of gases. If neutrinos interact with gases such as nitrogen and oxygen, the mass of neutrino can be estimated using the kinetic theory of gases. According to the kinetic theory of gases, the estimated mass range of neutrino is 0.022-0.037 $[eV/c^2]$. This estimated mass range of neutrino not only agrees with the KamLAND-Zen's lowest upper limit of neutrino mass of 0.06-0.161 $[eV/c^2]$, but also provides a smaller range of neutrino mass than any other previously reported values, to the authors' best knowledge.

1. Introduction

Since 1998, when the Super-Kamiokande neutrino detector discovered the non-zero mass of neutrinos [1], there have been great efforts to measure the exact mass of neutrinos by various groups, including but not limited to, KamLAND [2,3], MINOS [4], and OPERA [5]. Instead of measuring the exact mass of neutrinos, we can estimate the mass of neutrinos based on the kinetic theory of gases.

Neutrinos and molecules co-exist in space. Because both have physical properties of mass and speed, it is a logical assumption that neutrinos can interact with molecules according to the kinetic theory of gases. If neutrinos interact with gases such as nitrogen and oxygen, the mass of a neutrino can be estimated using the kinetic theory of gases.

For this calculation, oxygen molecules and neutrino particles are assumed to be interacting in a well-defined closed environment. Based on the kinetic theory of gases, the RMS (root mean square) collision speeds of oxygen molecules and neutrion particles are used.

Using the given approximate information,

$u_{\_oxygen}$ = 477 m/s (RMS collision speed of oxygen molecules at 20 °C)

$m_{\_oxygen}$ = 5.31×10^{-26} kg (mass of one oxygen molecule)

$u_{\_neutrino\_LowerLimit}$ = 4.02×10^{8} m/s (Lower limit of the RMS collision speed of neutrinos)

$u_{\_neutrino\_UpperLimit}$ = 5.18×10^{8} m/s (Upper limit of the RMS collision speed of neutrinos)

the mass of a neutrino can be estimated using the following mass-speed relationship.

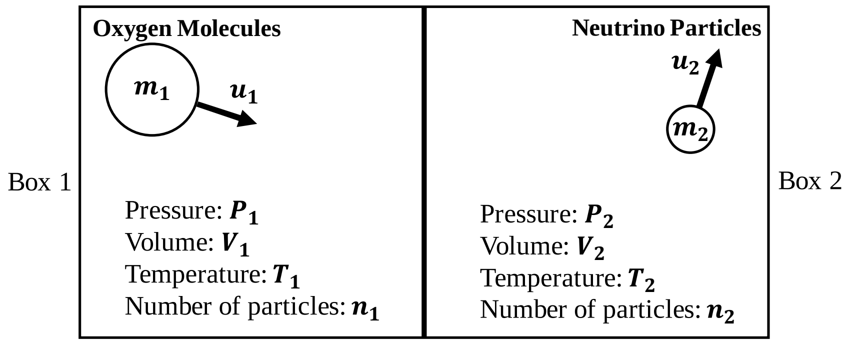
$$m_{\_oxygen}\left(u_{\_oxygen}\right)^2 = m_{\_neutrino}\left(u_{\_neutrino}\right)^2$$

Based on the above equation, the estimated range of neutrino mass of 0.022-0.037 $[eV/c^2]$. The goal of this estimation is to predict a smaller range of neutrino mass than any other previously reported values, to the authors' best knowledge.

The mass-speed relationship is derived, in the following three sections, from the kinetic theory of gases [6-10] and Avogadro's law [11] using a hypothesized setup of two boxes containing separated oxygen molecules and neutrino particles.

2. Separated Oxygen Molecules and Neutrinos

A hypothesized setup is designed to demonstrate the interactions between oxygen molecules and neutrino particles. Let each oxygen molecule have a mass m_1 and RMS collision speed u_1, and each neutrino have a mass m_2 and RMS collision speed u_2. Additionally, let the number of oxygen molecules be n_1 and the number of neutrinos be n_2. Assume that the oxygen molecules and neutrino particles are initially separated in two adjacent boxes, as illustrated below.



u_1 and u_2 indicate RMS Speeds of m_1 and m_2 respectively

Assume that the two boxes have the same pressure, volume, and temperature:

$$P_1 = P_2 = P$$
$$V_1 = V_2 = V$$
$$T_1 = T_2 = T$$

3. Kinetic Theory of Gases

Based on the kinetic theory of gases, the amounts, masses, and RMS collision speeds of the particles in Box 1 and Box 2 have the following relationship.

$$n_1 m_1 u_1^2 = n_2 m_2 u_2^2$$

The above relationship can be derived from the fundamental relationship between pressure, density, and RMS collision speed of particles:

$$P = \frac{1}{3}\rho u^2$$

Since density is defined by the ratio of mass m to volume V, the above equation can be modified into

$$P = \frac{1}{3}\frac{n\,m}{V}u^2$$

or, equivalently.

$$n\,m\,u^2 = 3PV$$

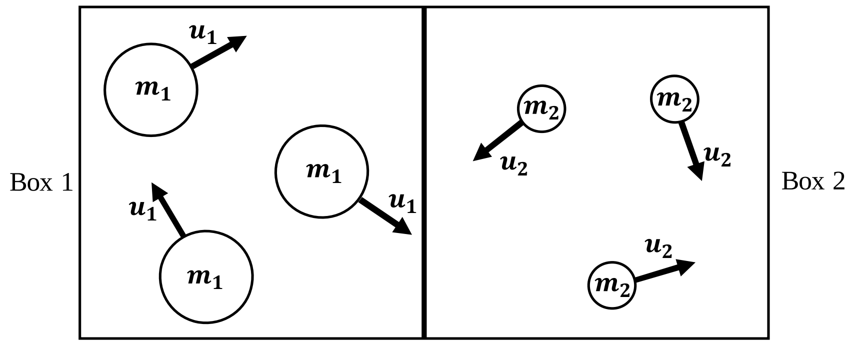
Because Box 1 and Box 2 have the same pressure and volume, the relationship between the amounts, masses, and RMS collision speeds of the particles in Box 1 and Box 2 can be formulated as

$$n_1 m_1 u_1^2 = n_2 m_2 u_2^2$$

4. Avogadro's Law

Avogadro's law states that "equal volumes of all gases, at the same temperature and pressure, have the same number of molecules". Because Box 1 and Box 2 have the same temperatures and pressures, it can be concluded from Avogadro's law that the number of oxygen molecules, n_1, and neutrino particles, n_2, are the same:

$$n_1 = n_2 = n$$



u_1 and u_2 indicate RMS Speeds of m_1 and m_2 respectively

By removing the common factor n from the last equation in the last section, the relationship between the masses and RMS collision speeds of the particles in Box 1 and Box 2 can be formulated as

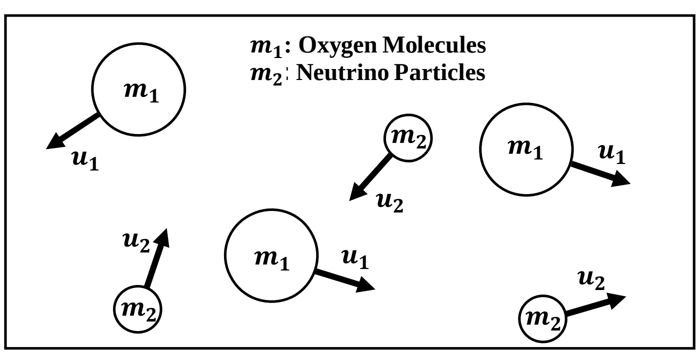
$$m_1 u_1^2 = m_2 u_2^2$$

or rearranged as

$$m_2 = m_1 \frac{u_1^2}{u_2^2}$$

5. Mixed Oxygen Molecules and Neutrinos

Because Box 1 and Box 2 have the same pressures, removing the divider between two boxes will not change RMS collision speeds (u_1, u_2) of the particles.



u_1 and u_2 indicate RMS Speeds of m_1 and m_2 respectively

6. Neutrino Mass

Let m_1 and u_1 be the mass and RMS collision speed of oxygen molecules respectively; m_2 and u_2 be the mass and RMS collision speed of neutrinos respectively, the above equation can be represented as

$$m_{\_neutrino} = m_{\_oxygen} \left(\frac{u_{\_oxygen}}{u_{\_neutrino}} \right)^2$$

The mass of an oxygen molecule is

$$m_{\_oxygen} \cong 5.31 \times 10^{-26} \ [kg]$$

The speed of sound in oxygen molecules at 20 °C is

$$c_{\_oxygen} \cong 326 \left[\frac{m}{s}\right]$$

The speed of light in neutrinos is

$$c_{\_neutrino} \cong 3.00 \times 10^8 \left[\frac{m}{s}\right]$$

Based on the kinetic theory of gases, the RMS collision speed (u) is related to the wave propagation speed (c) by the specific heat ratio (γ) as

$$u = c\sqrt{\frac{3}{\gamma}}$$

Since the specific heat ratio γ of oxygen is 1.4, the RMS collision speed of oxygen molecules at 20 °C is

$$u_{\_oxygen} = c_{\_oxygen}\sqrt{\frac{3}{\gamma}} \cong 326 \left[\frac{m}{s}\right] \sqrt{\frac{3}{1.4}} = 477 \left[\frac{m}{s}\right]$$

Since the specific heat ratio of neutrinos is still unknown, the lower limit and upper limit of the specific heat ratios γ (1 and 5/3 respectively) are used for calculating the lower limit and the upper limit of the RMS collision speeds of neutrino as

$$u_{\_neutrino\_LowerLimit} = c_{\_neutrino}\sqrt{\frac{3}{\gamma}} \cong 3.00 \times 10^8 \left[\frac{m}{s}\right] \sqrt{\frac{3}{\left(\frac{5}{3}\right)}} = 4.02 \times 10^8 \left[\frac{m}{s}\right]$$

$$u_{\_neutrino\_UpperLimit} = c_{\_neutrino}\sqrt{\frac{3}{\gamma}} \cong 3.00 \times 10^8 \left[\frac{m}{s}\right] \sqrt{\frac{3}{1}} = 5.18 \times 10^8 \left[\frac{m}{s}\right]$$

It is surprising that even the lower limit of the collision speed of neutrino ($u = 4.02 \times 10^8 \ m/s$) is faster than the propagation speed of neutrino, which is close to the speed of light. However, this is an inevitable result of kinetic theory and the authors believe that this can be validated in the future.

Based on these values, the lower limit of a neutrino mass can be calculated as

$$m_{\_neutrino\_LowerLimit} = m_{\_oxygen} \left(\frac{u_{\_oxygen}}{u_{\_neutrino\_UpperLimit}} \right)^2$$

$$\cong 5.31 \times 10^{-26}\ [kg] \left(\frac{477 \left[\frac{m}{s}\right]}{5.18 \times 10^8 \left[\frac{m}{s}\right]} \right)^2 \cong 3.96 \times 10^{-38}\ [kg]$$

$$\cong 3.96 \times 10^{-38}\ [kg] \left(\frac{3.00 \times 10^8 \left[\frac{m}{s}\right]}{c_{\_neutrino}} \right)^2 \frac{1[eV]}{1.60 \times 10^{-19}\ [J]} \cong 0.022 \left[\frac{eV}{c^2}\right]$$

And the upper limit of a neutrino mass can be calculated as

$$m_{\_neutrino\_UpperLimit} = m_{\_oxygen} \left(\frac{u_{\_oxygen}}{u_{\_neutrino\_LowerLimit}} \right)^2$$

$$\cong 5.31 \times 10^{-26}\ [kg] \left(\frac{477 \left[\frac{m}{s}\right]}{4.02 \times 10^8 \left[\frac{m}{s}\right]} \right)^2 \cong 6.55 \times 10^{-38}\ [kg]$$

$$\cong 6.55 \times 10^{-38}\ [kg] \left(\frac{3.00 \times 10^8 \left[\frac{m}{s}\right]}{c_{\_neutrino}} \right)^2 \frac{1[eV]}{1.60 \times 10^{-19}\ [J]} \cong 0.037 \left[\frac{eV}{c^2}\right]$$

Therefore, the estimated mass range of neutrino is 0.022-0.037 $[eV/c^2]$ based on the kinetic theory of gases.

7. Comparison to Measurements of Neutrino Mass

In June of 2008, KamLAND-Zen reported a lowest upper limit of neutrino mass of 0.06 – 0.161 $[eV/c^2]$, based on their measured data from their neutrino mass detector [12]. The estimated mass range of neutrino is 0.022-0.037 $[eV/c^2]$ is within the KamLAND-Zen's lowest upper limit of neutrino mass.

8. Conclusions

By comparing neutrinos to molecules, the range of a neutrino mass is predicted to be in a range of 0.022 – 0.037 $[eV/c^2]$ according to Avogadro's law and the kinetic theory of gases. This estimated range of neutrino mass not only agrees with the KamLAND-Zen's lowest upper limit of neutrino mass but also provides a smaller range of neutrino mass than any previous values.

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