Notes on Perfect Numbers

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Abstract

A set of relations between perfect numbers is presented. Then some properties of this relations and how they behave, next, a geometric interpretation, a function and finally, the way this function works.

Introduction.

The intention of this introductory paper is to show a few notes of properties and relations on Perfect Numbers. As the elements displayed here are material for many theorems and proofs, we will let them for another paper. Our purpose here is discovery.

A Perfect Number is a natural number such that it's value is equal to the sum of it's proper divisors[1]. The first seven Perfect Numbers are:
6, 28, 496, 8128, 33550336, 8589869056, 137438691328. In this paper we use the terms Perfect Number=Pf, Superperfect Number=Sp, Mersenne Prime=Mp, Mersenne Exponent=Me. The first Pf will be the 28 and we will call it Pf_1, 496 will be Pf_2, etc.

Relation between two consecutive Perfect Numbers.

Assuming that all Perfect Numbers have the form: $[(2 \cdot n^2) - 1]n^2$.
Then:
$[(2 \cdot (n_1)^2) - 1](n_1)^2 = Pf_1$.
$[(2 \cdot (n_2)^2) - 1](n_2)^2 = Pf_2$.

$[(2 \cdot (n_k)^2) - 1](n_k)^2 = Pf_k$.

Exists a relation (r) of the form:
$\frac{n_k}{n_{k-1}}$.
for every Pf_k and Pf_{k-1}.

This is:

$\sqrt{\frac{Sp_k}{Sp_{k-1}}}$

For example, the relation between 28 and 496 is equal to: $\sqrt{\frac{16}{4}} = 2$

Table 1: Relation between consecutive Perfect Numbers.

Pf 1	Pf 2	Relation (r)
28	496	2
496	8128	2
8128	33550336	8
33550336	8589869056	4
8589869056	137438/691328	2
137438/691328	2305843008139952128	64

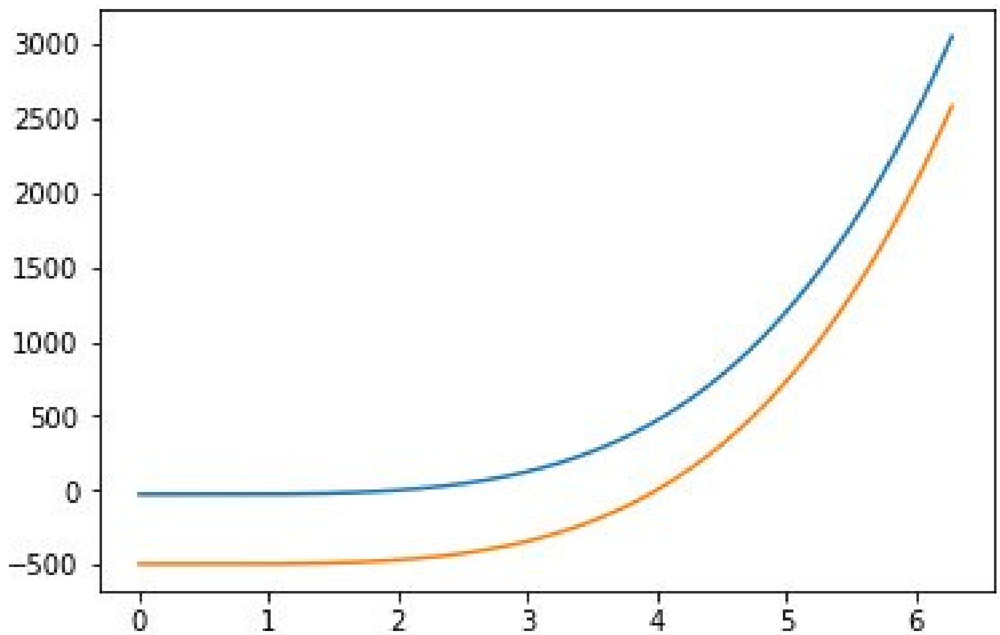

Figure 1: (r) of 28 and 496

Relation between two non-consecutive Perfect Numbers.

Given two Perfect Numbers Pf_1 and Pf_2 and their respective Mersenne Primes Mp_1 and Mp_2:

$$\begin{bmatrix} Pf_1 & Mp_1 \\ Pf_2 & Mp_2 \end{bmatrix}$$

Exists a relation (r) of the form:

$r = \frac{\sqrt{Pf_1 * Mp_1 * Pf2 * Mp_2}}{Pf_1 * Mp_2}$

This means that the relation (r) between two non-consecutive Perfect Numbers is equal to: $r_n * r_{n+1} * r_{n+2} * ...r_k$

For example, the relation between 8589869056 and 28 is equal to:
$r = \frac{\sqrt{28 * 7 * 8589869056 * 131071}}{28 * 131071}$=128=2*2*8*4

Table 2: Relation between non-consecutive Perfect Numbers.

Perfect Number	**28**	**496**	**8128**	**33550336**	**8589869056**
28	1	2	4	32	128
496	2	1	2	16	64
8128	4	2	1	8	32
33550336	32	16	8	1	4
8589869056	128	64	32	4	1

Relation between number 28 and upper Perfect Numbers.

Given two Perfect Numbers Pf_1 and Pf_2 and their respective Mersenne Primes Mp_1 and Mp_2.
Exists a relation (r) such that: r= $2^{\frac{Me_{Pf2}-3}{2}}$, where Pf_1=28 and Me_{Pf2} is the Mersenne Exponent of the Mersenne Prime of the upper Perfect Number Pf_2.

For example: $Pf_1 = 28$ and $Pf_2 = 2305843008139952128$.
$Mp_1 = 7$ and $Mp_2 = 2147483647$ and $Me_2 = 31$.

We have:
$r = \frac{\sqrt{28*7*2305843008139952128*2147483647}}{28*2147483647} = 16384$

This is equal to the product of the relations between this two Perfect Numbers, this is:
r=2*2*8*4*2*64=16384.

And this is:
r= $2^{\frac{Me_{Pf2}-3}{2}} = 2^{\frac{31-3}{2}} = 2^{14} = 16384$.

Table 3: Relation between 28 and upper Perfect Numbers.

Pf 1	**Pf 2**	**Relation (r)**	$2^{\frac{Me-3}{2}}$
28	496	2	2^1
496	8128	2	2^2
8128	33550336	8	2^5
33550336	8589869056	4	2^7
8589869056	137438/691328	2	2^8
137438/691328	2305843008139952128	64	2^{14}

Geometrical interpretation of the relation between Perfect Numbers.

Assuming that all Perfect Numbers have the form $(2n^2-1)n^2$.
This is $2n^4-n^2$.
When solving for $2n^4-n^2=Pf_n$.
We obtain four roots, two Complex of the form:
$i\sqrt{\frac{Mp_n}{2}}$ and $-i\sqrt{\frac{Mp_n}{2}}$
And two Real roots running on the x axis such that:
Given two Perfect Numbers Pf_1 and Pf_2 equaled to the polynomial $2n^4-n^2$, the relation between their Real roots is equal to the relation (r).
$2n^4-n^2-Pf_1=x_1$
$2n^4-n^2-Pf_2=x_2$
Then:
$r=\frac{x_2}{x_1}$

***Also is possible to use the polynomial $2n^4+8n^3+11n^2+6n+1$. The only difference is that all the Complex roots will have Real part (-1).

Example:
Given two Perfect Numbers $Pf_1 = 28$ and $Pf_2 = 496$.
Solving for:
$2n^4 - n^2 = 28$ and $2n^4 - n^2 = 496$.
We get their roots on the xy axis.

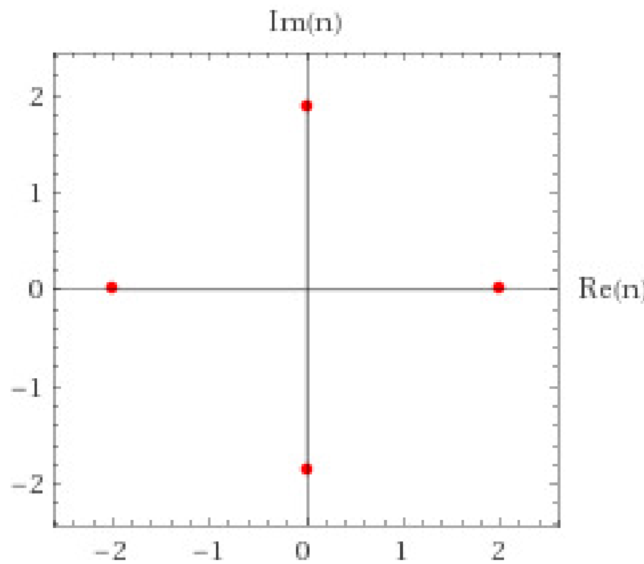

Figure 2: $2n^4 - n^2 = 28$

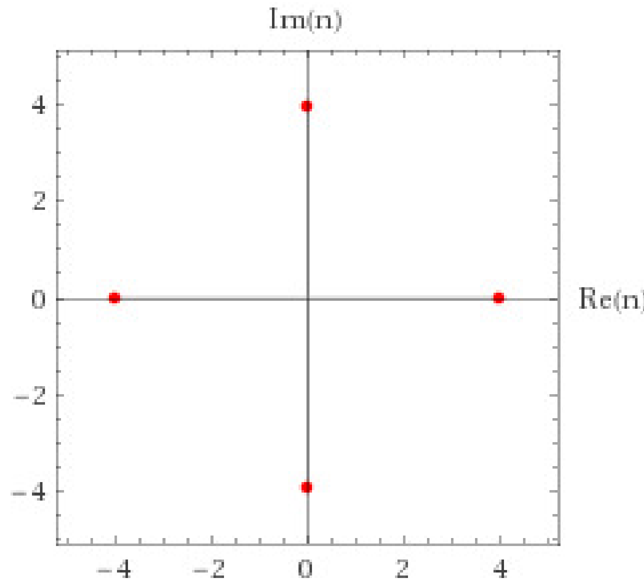

Figure 3: $2n^4 - n^2 = 496$

The Real roots of 496 are [4,-4] and the Real roots of 28 are [-2,2], or the lenght between two points are 8 and 4, in any case, the relation (r) is equal to 2.
The Complex roots are $\sqrt{\frac{7}{2}}$ and $-\sqrt{\frac{7}{2}}$ in the case of $2n^4 - n^2 = 28$ and $\sqrt{\frac{31}{2}}$ and $-\sqrt{\frac{31}{2}}$ in the case of $2n^4 - n^2 = 496$.
The figure they form apparently tend to be a perfect square (as we will see in the next section), but this never happens because the area of this figures is equal to $Mp_n + \frac{1}{2}$.

Graphic of Perfect Numbers.

$\log_{10}(2n^4 - n^2 = Pf_n) \approx \log_{10}\sqrt{\frac{Mp_n}{2}}$

Table 4: log-log.

$2n^4 - n^2 = Pf_n$	$\sqrt{\frac{Mp_n}{2}}$
.301029995664	.272034022175
.602059991328	.595165849085
.903089986992	.901386862646
1.80617997398	1.80615346513
2.40823996531	2.4082383086
2.70926996098	2.7092695468
4.51544993496	4.51544993486
9.03089986992	9.03089986992

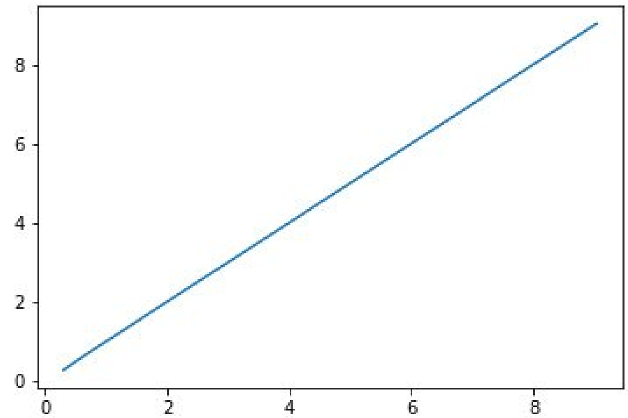

Figure 4: log-log

References

[1] George E. Andrews. *Number Theory. Combinatorial and Computational Number Theory.* Dover Publications, Inc. New York.1994.

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