Spiral Galaxy Rotation Curves and Arm Formation Without Dark Matter or MOND*

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Usual explanations of spiral galaxy rotation curves assume circular orbits of stars. The consequences of giving up this assumption were investigated through a couple of models in an earlier communication. Here, further investigations of one of these models (the spinner model) shows that it can explain the formation of the spiral arms as well. It is also shown that the behavior of the tail of the rotation curve is related to the age of the galaxy. The spinner model conjectures the existence of a spinning hot disk around a spherical galactic core. The disk is held together by local gravity and electromagnetic scattering forces. However, it disintegrates at the edge producing fragments that form stars. Once separated from the disk, the stars experience only the centrally directed gravitational force due to the massive core and remaining disk. A numerical simulation shows that a high enough angular velocity of the disk produces hyperbolic stellar trajectories that agree with the observed rotation curves. Besides the rotation curves, the simulation generates two other observable features of spiral galaxies. First, it shows the formation of spiral arms and their nearly equal angular separations. Second, it determines that, for large radial distances, younger galaxies have rotation curves that dip downwards and older galaxies have a rising trend. The strength of this model lies in the fact that it does not require the postulation of dark matter or MOND. This model also revisits the method of estimation of star age. As the stars are formed from an already hot disk, they do not start off as cold collections of dust and gas. Hence, their ages are expected to be significantly less than what current models estimate. This explains why they have not escaped the galaxy in spite of their hyperbolic trajectories.

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I. INTRODUCTION

Measured tangential velocities[17] of stars in the spiral arms of spiral galaxies have presented a challenge for theoretical modeling for some time[1–4]. In one class of models dark matter is postulated to explain these velocities[5–11]. Another class of models, called MOND (Modified Newtonian Dynamics) postulates a modification of Newtonian gravity for the same purpose[12–14]. Neither dark matter nor MOND has been observed directly yet. So here, a new kind of model (the spinner model) is revisited. This model has already been found to explain observed rotation curves without the postulation of any new kind of matter or laws of gravity[15]. In this communication it is shown that the spinner model can produce the observed spiral arm shapes as well as estimate the age of galaxies.

Anyone who has played with firework spinners as a child (or an adult) must have noticed their resemblance to spiral galaxies. Anyone who has not can always search "firework spinner" on YouTube to see it. The trajectories of the glowing embers in a spinner look like the stars of the spiral arms. Quite obviously, the actual motion of the stars cannot be observed directly. But the still picture that we see suggests characteristically hyperbolic trajectories (like the firework spinner) rather than the circular trajectories as assumed by most analyses. The assumption of hyperbolic star trajectories opens up the following questions.

- Do the stars of the spiral arms eventually escape the galaxy?
- If stars eventually escape the galaxy, the observed stars must be young enough to be seen as part of the galaxy. Then, why are significantly older stars still observed in the Milky Way?
- Why do the stars conspire to have almost the same tangential speed beyond a certain distance from the core?

The answer to the first question is "Yes". According to this model the "spiral" state of a galaxy is just one transient state in its time evolution. Due to their hyperbolic trajectories, the stars of the spiral arms are expected to escape the galaxy eventually. The answers to the other two questions are to be found in the following detailed discussion that includes a numerical simulation.

II. THE SPINNER MODEL

For this model, a spiral galaxy is considered to start as a compact spherical core surrounded by a functionally rigid spinning disk held together by gravity as well as electromagnetic scattering forces (see figure 1). Fragments of the disk break off at the edge in the form of stars. Here, in the simplest version of this model, we will assume that the stars separate from the disk edge with

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initial velocities equal to that of the edge. Then we can assume that the disk angular velocity remains constant while its radius decreases due to loss of material in the form of stars. Hence, we conclude that stars separating earlier have greater initial velocities than stars that separate later. Once a star separates, it experiences no local forces. Then, the only force on it is the much weaker long range gravitational force due to the core and the remaining disk. Hence, stars start off with significant tangential speeds due to the spinning disk and no radial speeds. But, once separated, they develop nonzero radial speeds and their tangential speeds decrease. The stars that separate later start with smaller tangential speeds due to the shrinking of the disk. Hence, if the disk shrinks at a certain rate, it could make the early-separated stars move at roughly the same speed as the later-separated ones. Due to outward radial speeds developed after separation from the disk, the stars are expected to have hyperbolic trajectories.

Spiral arm formation is explained by having stars ejected only from some localized regions on the disk edge. This is discussed in some detail in a later section.

As an individual star moves away from the core, its outward radial speed increases and could eventually become measurable. However, several mitigating factors are expected to make such measurement difficult. First, the density of stars decreases with increasing radial distance. Second, the stars at greater radial distances are expected to be colder and dimmer. Third, the radial velocity component must have a component along the line of sight of the observer from Earth to allow measurement using the Doppler effect. This would require the star to have the bright galactic core region in its background as seen from Earth. Such a bright background might wash out the light of the star.

III. A NUMERICAL SIMULATION OF THE SPINNER MODEL

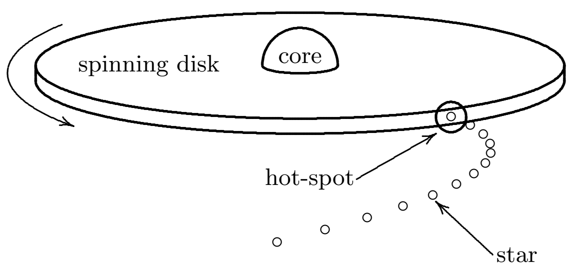

FIG. 1: The spinner model for spiral galaxies.

The spinner model (see figure 1) in conjunction with standard Newtonian gravity can now be used to simulate a spiral galaxy. Let us assume that the disk has constant areal density and it disintegrates at a constant rate to produce stars of equal mass. So, the area of the disk will reduce at some constant rate q. Hence,

$$A = A_0 - qt, \tag{1}$$

where A is the area of the disk at time t and A_0 the initial area. If R is the radius of the disk at time t and R_0 the initial radius, then

$$R^2 = R_0^2 - at, \text{ where } a = q/\pi. \tag{2}$$

Hence, the radius of the disk at time t is given by

$$R = \sqrt{R_0^2 - at}. \tag{3}$$

It is to be expected that, below a certain minimum radius R_m, there will not be enough centrifugal force to produce more stars.

Now, let there be a total of N stars created[18] by the disk at equal time intervals of T. Let the i^{th} star have a radial coordinate r_i after it is created. At the time of creation, each star has an initial radial coordinate equal to the current radius R of the disk given by equation 3. The initial radial velocity is zero. The initial angular momentum is important to record as it is expected to be conserved under the radially directed gravitational force from the galactic core and disk. If Ω is the constant angular velocity of the disk, then the initial angular momentum is $mR^2\Omega$ where m is the mass of the star. This will be different for different stars as they are created at different times with different values of R. However, for each star this angular momentum will be conserved. As the trajectory of a star is independent of its mass, the relevant conserved quantity related to angular momentum is

$$l_i = r_i^2\dot{\phi}_i = R^2\Omega, \tag{4}$$

where ϕ_i is the angular coordinate of the i^{th} star, $\dot{\phi}_i = d\phi_i/dt$ and R is the initial radial coordinate. So, the nonrelativistic tangential velocity of the i^{th} star at any time is

$$v'_{ti} = r_i\dot{\phi}_i = \frac{l_i}{r_i}. \tag{5}$$

If v'_{ti} is comparable or larger than c, the speed of light, it needs a relativistic correction. The relativistic tangential velocity is (see section VII),

$$v_{ti} = \frac{v'_{ti}}{\sqrt{1 + v'^2_{ti}/c^2}}. \tag{6}$$

The next section discusses the formation of spiral arms with the assumption that stars are ejected only from some localized regions called hot-spots on the disk edge. Then, the initial angle ϕ_{i0} of a star just ejected would be given by,

$$\phi_{i0} = \Omega t. \tag{7}$$

Here we are assuming there is only one hot-spot on the disk and it emits stars at times t which are multiples of the interval T. In reality, there could be multiple hot-spots. With the above initial condition, the following standard differential equation is used to find the angular position of each star.

$$\dot{\phi}_i = l_i/r_i^2. \quad (8)$$

After creation, the i^{th} star trajectory can be tracked using the above equation 8 for the angular position and the following equation 9 for the radial position. Equation 9 arises from Newtonian gravity (see section VII for relativistic considerations).

$$\ddot{r}_i - \frac{l_i^2}{r_i^3} + \frac{GM}{r_i^2} + f_c(r_i, R) = 0, \quad (9)$$

where G is the gravitational constant, M is the mass of the galactic core and $f_c(r, R)$ is the gravitational acceleration produced at a distance r from the center by the disk when its radius is R. It can be seen that

$$f_c(r, R) = G\sigma \int_0^{2\pi} \int_0^R \frac{\rho(r - \rho\cos\theta)d\rho\, d\theta}{(\rho^2 + r^2 - 2\rho r\cos\theta)^{3/2}}, \quad (10)$$

where σ is the areal density of the disk. The above integral needs to be computed numerically at each stage of the computation.

The numerical implementation of this simulation is done by looping through the following steps at small intervals of time $\Delta t = h$ for a total time duration of $NT + T_a$. As stated earlier, N is the number of stars created, T the time interval at which they are created and T_a is the time elapsed after the last star is created.

- If current time $t = iT$ for $i = 0, 1, 2, \ldots$, create a new star as long as the disk radius R is greater than the minimum radius R_m. Find initial r_i using equation 3. Set initial $\dot{r}_i$ to be zero. Find the constant of motion l_i using equation 4. Set initial angle using equation 7.
- In a nested loop, loop through all stars created so far computing their next values for ϕ_i, r_i and $\dot{r}_i$ after each time interval h using equations 8 and 9. Use a fourth order Runge-Kutta algorithm for this purpose.

This numerical simulation has already been shown to produce the observed rotation curves of spiral galaxies[15]. In the following, it will be seen that the model also demonstrates how spiral arms are formed and how the shapes of rotation curves are related to galaxy age.

IV. SPIRAL ARM FORMATION

Besides the rotation curves, a working model of spiral galaxies should explain the shape of the spiral arms and their near equal angular separations. To do this, here it is assumed that stars are ejected only from certain localized spots on the edge of the disk. They may be called "hot-spots". The dynamics of hot-spots can, by itself, be a subject of significant study. At present, we will think of them as localized regions of thermonuclear activity. In principle, there can be multiple hot-spots on a disk. Each hot-spot could be at a fixed point on the

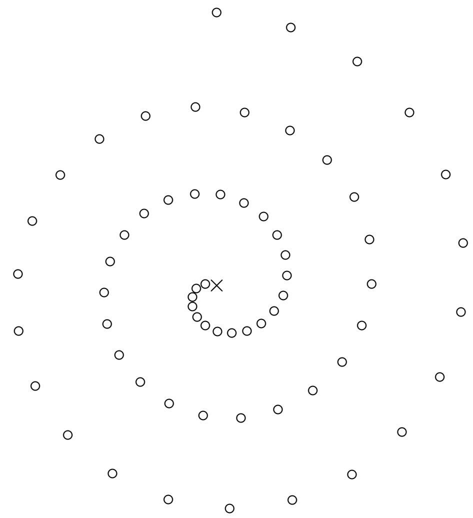

FIG. 2: Spiral arm with one hot-spot: Hot-spot rotation time period $T_h = 2\pi/\Omega = 2.09 \times 10^{11}$s and time between star ejection bursts $T = 2.00 \times 10^{11}$s.

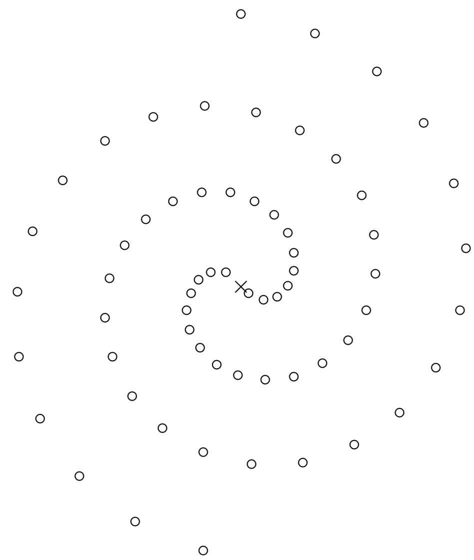

FIG. 3: Spiral arms with one hot-spot: Hot-spot rotation time period $T_h = 2\pi/\Omega = 4.19 \times 10^{11}$s and time between star ejection bursts $T = 2.00 \times 10^{11}$s.

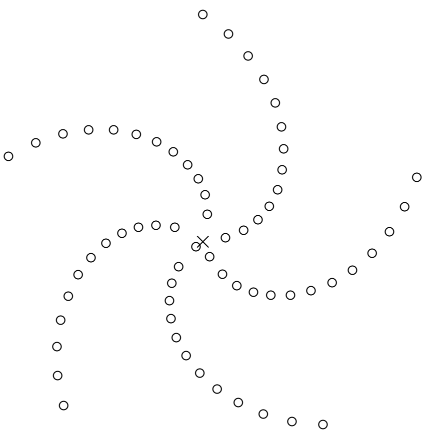

FIG. 4: Spiral arms with one hot-spot: Hot-spot rotation time period $T_h = 2\pi/\Omega = 2.51 \times 10^{11}$s and time between star ejection bursts $T = 2.00 \times 10^{11}$s.

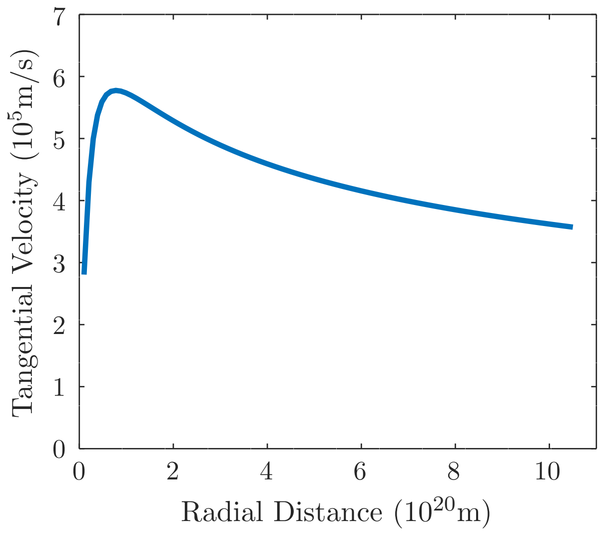

FIG. 5: Rotation curve for $T_a = 0$, $M = 1.0 \times 10^{40}$kg, $R_0 = 5.0 \times 10^{18}$m, $\Omega = 1.5 \times 10^{-11}\text{s}^{-1}$, $\sigma = 0$ and $a = 2.0 \times 10^{24}\text{m}^2/\text{s}$.

edge of the disk or it could be moving along the edge. The movement could occur due to the relative availability of local thermonuclear fuel. This would be similar to what happens in certain kinds of firework spinners. Hence, a hot-spot could be spinning at the same speed as the disk or not. For now, we will assume a hot-spot to be spinning at the same speed as the disk. So, its time period will be,

$$T_h = 2\pi/\Omega, \tag{11}$$

where Ω is the angular velocity of the disk as defined earlier.

Multiple spiral arms could be created due to multiple hot-spots. However, there is no reason for multiple hot-spots to be equally spaced in angle. But actual spiral galaxies seem to have close to equally spaced arms. So, the present model uses a single hot-spot and demonstrates how it could produce multiple equally spaced arms. The single hot-spot may be assumed to become active only in short bursts. These bursts are separated by the time period T defined earlier in the description of the model. Each burst can produce a bunch of stars even though the simulation uses only one representative star. If the time period of rotation of the hot-spot T_h is close to some multiple of T, that is, for some integer n,

$$T_h \simeq nT, \tag{12}$$

then n is the number of equally spaced spiral arms. Note that the ratio of T_h and T does not have to be exactly n to produce the n arms. Figure 2 shows a simulation for $n = 1$ and figure 3 a simulation for $n = 2$. However, equation 12 is not the only condition under which a small number of spiral arms are formed. A more general condition is as follows.

$$n'T_h \simeq nT, \tag{13}$$

where n' and n are both integers and n is the number of arms. Figure 4 shows a simulation illustrating such a case where $n' = 4$ and $n = 5$. Note that equation 13 supports the observed equal angular separation of spiral arms.

V. AGE OF GALAXIES

The simulation shows that in early stages of the formation of a spiral galaxy, the rotation curve dips downwards at large distances after rising to a peak value at some critical distance (see figure 5). In this figure, the time elapsed after last star creation $T_a = 0$. After some time elapsed ($T_a = 1.0 \times 10^{13}$s), the curve becomes flat at large distances (see figure 6). After some more time elapsed ($T_a = 4.0 \times 10^{13}$s), the curve has a rising trend even for the farthest stars (see figure 7). This time progression can be used to estimate the age of galaxies.

VI. AGE OF STARS

One criticism of the spinner model is due to the hyperbolic nature of star trajectories. Clearly, such stars are not going to remain in the galaxy for long. So, the stars that are still a part of the galaxy are expected to be relatively young. The ages of some observed stars in the Milky Way (a spiral galaxy) are estimated to be too large

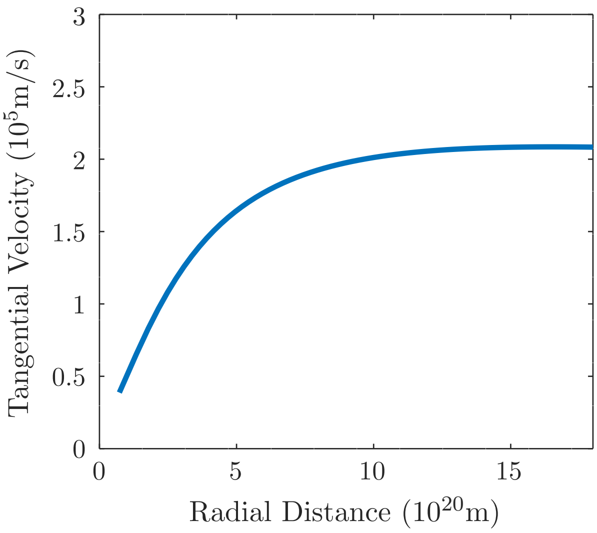

FIG. 6: Rotation curve for $T_a = 1.0\times10^{13}$s, $M = 1.0\times10^{40}$kg, $R_0 = 5.0 \times 10^{18}$m, $\Omega = 1.5 \times 10^{-11}\text{s}^{-1}$, $\sigma = 0$ and $a = 2.0 \times 10^{24}\text{m}^2/\text{s}$.

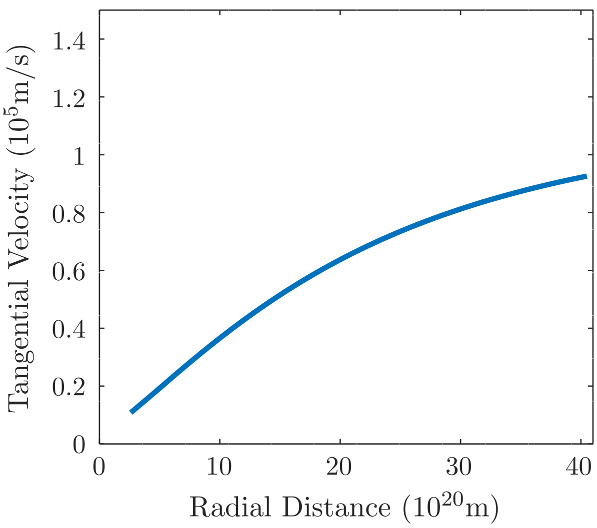

FIG. 7: Rotation curve for $T_a = 4.0\times10^{13}$s, $M = 1.0\times10^{40}$kg, $R_0 = 5.0 \times 10^{18}$m, $\Omega = 1.5 \times 10^{-11}\text{s}^{-1}$, $\sigma = 0$ and $a = 2.0 \times 10^{24}\text{m}^2/\text{s}$.

to satisfy this condition[16]. The problem is resolved by reexamining the method of star age estimation. Standard methods assume that a star begins formation as a cold collection of dust and gas and then, over time, collapses to a hot light emitting object. In the spinner model a star is born already as a hot light emitting object from the hot-spot of the disk. So, the standard method will significantly overestimate the age of a star for the purpose of the spinner model.

VII. RELATIVISTIC CONSIDERATIONS

The simulation uses the two differential equations 8 and 9 that arise from Newtonian gravity. Hence, it is important to consider if they are adequate or relativistic effects need to be included. Relativistic effects can be due to high space-time curvature or high velocities. High space-time curvature is of consequence if the distance of a star from the center is close to or less than the Schwarzschild radius of the spherical core which contains most of the mass. Then there are additional terms in equation 9. For the core mass of M, the Schwarzschild radius is

$$r_s = 2GM/c^2, \tag{14}$$

where c is the speed of light. For all situations considered here, the distances of stars from the center are much larger than this. Hence, relativistic effects due to high space-time curvature can be ignored. The relativistic version of equation 9 also has an additional radial velocity ($\dot{r}$) term. This can also be ignored as $\dot{r} \ll c$.

Equation 8 remains unchanged by relativity. However, measured tangential velocity of a star is no longer given by the simple relation

$$v_t' = l/r = r\dot{\phi}, \tag{15}$$

where l is the angular momentum per unit mass, r the radial distance and $\dot{\phi}$ the angular velocity. Note that for large enough r and $\dot{\phi}$ this can be greater than the speed of light! The measured tangential velocity must be corrected for length contraction[19]. It is given by

$$v_t = v_t'\sqrt{1 - v_t^2/c^2}. \tag{16}$$

Hence,

$$v_t = \frac{v_t'}{\sqrt{1 + v_t'^2/c^2}}. \tag{17}$$

This can be seen to be always less than c. This correction is used in the simulation. However, in most cases, even this is not necessary.

VIII. CONCLUSION

The spinner model for spiral galaxies has been previously used to explain observed rotation curves without introducing new kinds of matter like dark matter or forces like in MOND. Here, the same model is used to explain spiral arm shapes and to relate rotation curve shapes to ages of galaxies.

Using a single hot-spot on the spinning disk, the model explains the equal angular separation of multiple arms. The number of arms produced depends on the ratio of the time period of the disk and the time period of star ejection.

The slope of the tail end of the rotation curve is found to be related to the age of the galaxy. For younger galaxies, the slope is negative and as galaxies age, the slope increases and becomes positive for the oldest galaxies.

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[1] V. C. Rubin and W. K. Ford, Astrophysical Journal, **159**, 379-403, (1970).
[2] V. C. Rubin, W. K. Ford and N. Thonnard, Astrophysical Journal, **225**, L107-L111, (1978).
[3] V. C. Rubin, W. K. Ford and N. Thonnard, Astrophysical Journal, **238**, 471-487, (1980).
[4] A. Bosma, Astronomical Journal, **86**, 1791-1824, (1981).
[5] M. Persic and P. Salucci, Mon. Not. R. Astron. Soc. **245**, 577-581, (1990).
[6] M. Persic, P. Salucci and F. Stel, Mon. Not. R. Astron. Soc. **281**, 27-47, (1996).
[7] E. Corbelli and P. Salucci, Mon. Not. R. Astron. Soc. **311**, 441-447, (2000).
[8] G. Gentile, P. Salucci, U. Klein, D. Vergani and P. Kalberla, Mon. Not. R. Astron. Soc. **351**, 903-922, (2004).
[9] D. Merritt, J. F. Navarro, A. Ludlow, and A. Jenkins, Astrophysical Journal, **624**, L85-L88, (2005).
[10] I. A. Yegorova and P. Salucci, Mon. Not. R. Astron. Soc. **377**, 507-515, (2007).
[11] A. R. Duffy, J. Schaye, S. T. Kay, C. D. Vecchia, R. A. Battye and C. M. Booth, Mon. Not. R. Astron. Soc. **405**, 2161-2178, (2010).
[12] S. S. McGaugh, and W. J. G. de Blok, Astrophysical Journal, **499**, 66-81, (1998).
[13] S. S. McGaugh, Astrophysical Journal Letters, **832**, L8, (2016).
[14] S. S. McGaugh, F. Lelli and J. M. Schombert, Phys. Rev. Lett. **117**, 201101, (2016).
[15] T. Biswas, arXiv:1801.09304 [astro-ph.GA] (2018).
[16] S. S. McGaugh, private communications, (2018).
[17] The terms “tangential” and “radial” are used here in reference to the galactic center.
[18] In reality, stars are expected to be created in bunches. The number N is the number of bunches. But, for the purpose of numerical simulation, it is sufficient to track just one star out of a bunch.
[19] A relativistic definition of rigidity requires this correction. The non-relativistic definition of rigidity implies instantaneous signal transmission. Hence, rigidity needs to be redefined for relativistic purposes. Unfortunately, there are multiple possibilities for such redefinition. Here, one such definition is used. Fortunately, the final results of the present simulation does not seem to be affected by this relativistic correction.