Lorentz Force in Special Relativity theory

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ABSTRACT

In the Special Relativity theory, we tell undergraduate how Lorentz 4-force is invariant in Special Relativity theory.

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1. Introduction

Our article's aim is that we tell an undergraduate Lorentz 4-force is invariant by electro-magnetic field transformations in Special Relativity theory.

At first, the coordinate transformation is in Special relativity theory,

$$ct = \gamma(ct' + \frac{v}{c}x'), x = \gamma(x' + \frac{v}{c}ct'), \quad y = y', z = z' \qquad (1)$$

Therefore, Minkowski 4-force is in Special Relativity theory[5]

$$f^0 = m_0 c \frac{d^2t}{d\tau^2} = m_0\gamma(c\frac{d^2t'}{d\tau^2} + \frac{v}{c}\frac{d^2x'}{d\tau^2}) = \gamma(f'^0 + \frac{v}{c}f'^1),$$

$$f^1 = m_0 \frac{d^2x}{d\tau^2} = m_0\gamma(\frac{d^2x'}{d\tau^2} + \frac{v}{c}\frac{cd^2t'}{d\tau^2}) = \gamma(f'^1 + \frac{v}{c}f'^0)$$

$$f'^0 = m_0 c\frac{d^2t'}{d\tau^2}, f'^1 = m\frac{d^2x'}{d\tau^2}$$

$$f^2 = m_0\frac{d^2y}{d\tau^2} = m_0\frac{d^2y'}{d\tau^2} = f'^2, f^3 = m_0\frac{d^2z}{d\tau^2} = m_0\frac{d^2z'}{d\tau^2} = f'^3 \qquad (2)$$

Hence, in inertial frame, Lorentz 4-force is

$$F^0 = m_0\frac{d}{dt}(\frac{cdt}{d\tau}) = q\frac{\vec{u}}{c}\cdot\vec{E} \qquad (2)$$

$$\vec{F} = m_0\frac{d}{dt}(\frac{d\vec{x}}{d\tau}) = q[\vec{E} + \frac{\vec{u}}{c}\times\vec{B}], \quad \vec{u} = \frac{d\vec{x}}{dt} \qquad (3)$$

$$F'^0 = m_0\frac{d}{dt'}(\frac{cdt'}{d\tau}) = q\frac{\vec{u}'}{c}\cdot\vec{E}' \qquad (4)$$

$$\vec{F}' = m_0\frac{d}{dt'}(\frac{d\vec{x}'}{d\tau}) = q[\vec{E}' + \frac{\vec{u}'}{c}\times\vec{B}'], \quad \vec{u}' = \frac{d\vec{x}'}{dt'} \qquad (5)$$

2. Invariant Lorentz 4-force

In this time, Minkowski 4-force is in inertial frame.

$$f^0 = m_0 c\frac{d^2t}{d\tau^2} = q\frac{\vec{u}}{c}\cdot\vec{E}\frac{dt}{d\tau} \qquad ,\vec{u} = \frac{d\vec{x}}{dt}, \ \vec{u}' = \frac{d\vec{x}'}{dt'}$$

$$= m_0\gamma(c\frac{d^2t'}{d\tau^2} + \frac{v}{c}\frac{d^2x'}{d\tau^2}) = \gamma(f'^0 + \frac{v}{c}f'^1)$$

$$= \gamma q\frac{\vec{u}'}{c}\cdot\vec{E}'\frac{dt'}{d\tau} + \gamma\frac{v}{c}[qE_x' + q\frac{1}{c}(u_y'B_z' - u_z'B_y')]\frac{dt'}{d\tau} \qquad (6)$$

$$f^1 = m_0\frac{d^2x}{d\tau^2} = m_0\gamma(\frac{d^2x'}{d\tau^2} + \frac{v}{c}\frac{cd^2t'}{d\tau^2}) = \gamma(f'^1 + \frac{v}{c}f'^0)$$

$$= \gamma[qE_x' + q\frac{1}{c}(u_y'B_z' - u_z'B_y')]\frac{dt'}{d\tau} + \gamma\frac{v}{c}(q\frac{\vec{u}'}{c}\cdot\vec{E}')\frac{dt'}{d\tau} \qquad (7)$$

$$f'^0 = m_0 c \frac{d^2 t'}{d\tau^2} = q \frac{\vec{u}'}{c} \cdot \vec{E}' \frac{dt'}{d\tau}, f'^1 = m_0 \frac{d^2 x'}{d\tau^2} = q[E_x' + \frac{1}{c}(u_y' B_z' - u_z' B_y')] \frac{dt'}{d\tau} \quad (8)$$

In this time, the transformation of electromagnetic field is in Special Relativity theory.

$$E_x = E'_x ,$$

$$E_y = \gamma E'_y + \gamma \frac{v}{c} B'_z ,$$

$$E_z = \gamma E'_z - \gamma \frac{v}{c} B'_y$$

$$B_x = B'_x ,$$

$$B_y = \gamma B'_y - \gamma \frac{v}{c} E'_z$$

$$B_z = \gamma B'_z + \gamma \frac{v}{c} E'_y \quad (9)$$

Hence,

$$f^0 = m_0 c \frac{d^2 t}{d\tau^2} = q \frac{\vec{u}}{c} \cdot \vec{E} \frac{dt}{d\tau}$$

$$= q \frac{1}{c} (u_x E_x + u_y E_y + u_z E_z) \frac{dt}{d\tau}$$

$$= q \frac{1}{c} [(\frac{u_x' + v}{1 + \frac{u_x' v}{c^2}}) E'_x + \frac{u_y'}{\gamma(1 + \frac{u_x' v}{c^2})} \gamma (E'_y + \frac{v}{c} B'_z) + \frac{u_z'}{\gamma(1 + \frac{u_x' v}{c^2})} \gamma (E'_z - \frac{v}{c} B'_y)]$$

$$\times \gamma \frac{dt'}{d\tau} (1 + \frac{u_x' v}{c^2})$$

$$= \gamma q \frac{1}{c} (u_x' E_x' + u_y' E_y' + u_z' E_z') \frac{dt'}{d\tau} + \gamma \frac{v}{c} [q E_x' + q \frac{1}{c} (u_y' B_z' - u_z' B_y')] \frac{dt'}{d\tau}$$

$$= \gamma (f'^0 + \frac{v}{c} f'^1) \quad (10)$$

$$f^1 = m_0 \frac{d^2 x}{d\tau^2} = [q E_x + q \frac{1}{c} (u_y B_z - u_z B_y)] \frac{dt}{d\tau}$$

$$= [q E'_x + q (\frac{1}{c} (\frac{u_y'}{\gamma(1 + \frac{u_x' v}{c^2})}) \gamma (B'_z + \frac{v}{c} E'_y) - \frac{u_z'}{\gamma(1 + \frac{u_x' v}{c^2})} \gamma (B'_y - \frac{v}{c} E'_z))]$$

$$\times \gamma \frac{dt'}{d\tau} (1 + \frac{u_x' v}{c^2})$$

$$= \gamma [q E_x' + q \frac{1}{c} (u_y' B_z' - u_z' B_y')] \frac{dt'}{d\tau} + \gamma \frac{v}{c} [q \frac{1}{c} (u_x' E_x' + u_y' E_y' + u_z' E_z')] \frac{dt'}{d\tau}$$

$$= \gamma(f'^{1} + \frac{v}{c} f'^{0}) \tag{11}$$

$$f^{2} = m_0 \frac{d^2 y}{d\tau^2} = [qE_y + q\frac{1}{c}(u_z B_x - u_x B_z)]\frac{dt}{d\tau}$$

$$= [q\gamma(E'_y + \frac{v}{c}B'_z) + q(\frac{1}{c}(\frac{u_z'}{\gamma(1+\frac{u_x'v}{c^2})})B'_x - \frac{u_x'+v}{1+\frac{u_x'v}{c^2}}\gamma(B'_z + \frac{v}{c}E'_y))]$$

$$\times \gamma \frac{dt'}{d\tau}(1+\frac{u_x'v}{c^2})$$

$$= [qE_y' + q\frac{1}{c}(u_z' B_x' - u_x' B_z')]\frac{dt'}{d\tau} = f'^{2} \tag{12}$$

$$f^{3} = m_0 \frac{d^2 z}{d\tau^2} = [qE_z + q\frac{1}{c}(u_x B_y - u_y B_x)]\frac{dt}{d\tau}$$

$$= [q\gamma(E'_z - \frac{v}{c}B'_y) + q(\frac{1}{c}(\frac{u_x'+v}{1+\frac{u_x'v}{c^2}})\gamma(B'_y - \frac{v}{c}E'_z) - \frac{u'_y}{\gamma(1+\frac{u_x'v}{c^2})}B'_x)]$$

$$\times \gamma \frac{dt'}{d\tau}(1+\frac{u_x'v}{c^2})$$

$$= [qE_z' + q\frac{1}{c}(u_x' B_y' - u_y' B_x')]\frac{dt'}{d\tau} = f'^{3} \tag{13}$$

3. Conclusion

We know Lorentz 4-force is invariant by the Lorentz transformation in Special relativity theory .

Hence, We want to know the form of Lorentz 4-force in accelerated frame(Lorentz force in Rindler Spacetime).

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