(pk mk qk) or an Unexpected Inconsistency

Ralf Wüsthofen [1]

Abstract. This note proves the inconsistency of the Peano arithmetic (PA) by deriving both the strong Goldbach conjecture and its negation.

Notations. Let $\mathbb{N}$ denote the natural numbers starting from 1 and let $\mathbb{P}_3$ denote the prime numbers starting from 3.

Theorem 1 (Strong Goldbach conjecture (SGB)). *Every even integer greater than 2 can be expressed as the sum of two primes*.

Proof. We define the set $S_g := \{ (pk, mk, qk) \mid k, m \in \mathbb{N}; p, q \in \mathbb{P}_3, p < q; m = (p + q)/2 \}$. Then, SGB is equivalent to saying that for any fixed k ≥ 1 all multiples xk, where x ≥ 4 is composite, are given by the triple components mk. Let us assume ¬SGB now. The difference between SGB and the negation ¬SGB is that in the former case for each k ≥ 1 there is no nk, n ≥ 4 composite, different from all the mk in S_g and that in the latter case there is at least one such nk.

Because of 4 = (3 + 5)/2 we consider n > 5. Then, for each k ≥ 1 the assumed nk is part of a sum $(s_1 + s_3)$, $(s_1, s_2, s_3) \in S_g$, by being written as some pk' when n is composite and not a power of 2, or as (3 + 5)k' when n is a power of 2; $p \in \mathbb{P}_3$, k, k' $\in \mathbb{N}$.

This implies that for all S_g triples (pk, mk, qk) the sums (pk + qk) are the same, regardless of whether one or more than one of the above nk exists or not. This means, all the (pk + qk) are the same in both cases, SGB and ¬SGB. So, all the mk = (pk + qk)/2 are the same in both cases, SGB and ¬SGB, which is a contradiction to the assumption ¬SGB. Therefore, ¬SGB is false.

□

Theorem 2 (¬SGB). *There is an even integer greater than 2 that cannot be expressed as the sum of two primes*.

Proof. Let us assume SGB. In the proof of Theorem 1 we have seen that for all S_g triples (pk, mk, qk) the arithmetic means mk = (pk + qk)/2 are the same in both cases, SGB and ¬SGB. This is a contradiction to the assumption SGB. Therefore, SGB is false.

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[1] rwuesthofen@gmail.com