

# (pk mk qk) or an Unexpected Inconsistency

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**Abstract.** This note proves the inconsistency of the Peano arithmetic (PA) by deriving both a strengthened form of the strong Goldbach conjecture and its negation.

**Notations.** Let  $\mathbb{N}$  denote the natural numbers starting from 1 and let  $\mathbb{P}_3$  denote the prime numbers starting from 3.

**Theorem.** *The Peano arithmetic (PA) is inconsistent.*

*Proof.* We define the set  $S_g := \{ (pk, mk, qk) \mid k, m \in \mathbb{N}; p, q \in \mathbb{P}_3, p < q; m = (p + q) / 2 \}$  and we consider the following two cases:

- (G) The numbers  $m$  in the components  $mk$  take all integer values  $x \geq 4$ .
- $\neg$ (G) The numbers  $m$  in the components  $mk$  do not take all integer values  $x \geq 4$ .

The case  $\neg$ (G) means that for each  $k \geq 1$  there is an  $n_k$ ,  $n \geq 4$ , different from all the  $mk$ , where all pairs  $(p, q)$  of odd primes, which determine the numbers  $m$ , are used in  $S_g$ . For each  $k \geq 1$ , such an  $n_k$  can be written as some  $pk$  when  $n$  is prime, as some  $pk'$  when  $n$  is composite and not a power of 2, or as  $4k'$  when  $n$  is a power of 2;  $p \in \mathbb{P}_3$ ;  $k, k' \in \mathbb{N}$ .

The expression  $pk'$  for  $n_k$  with  $k' = k$  or  $k' \neq k$  is a first component of  $S_g$  triples and the expression  $4k'$  for  $n_k$  is component of the triple  $(3k', 4k', 5k')$ . Thus, the  $S_g$  triples are always the same, regardless of whether  $n_k$  as a component of them exists or not.

Therefore, we obtain that the triples of  $S_g$  are the same in both cases, (G) and  $\neg$ (G). So, the components  $mk$  are the same in both cases, (G) and  $\neg$ (G), which is a contradiction. □

Actually, the above argument uses a strengthened form of the strong Goldbach conjecture and its negation:

**Strengthened strong Goldbach conjecture (SSGB):** *Every even integer greater than 6 can be expressed as the sum of two different primes.*

$\neg$ **SSGB:** *There is an even integer greater than 6 that cannot be expressed as the sum of two different primes.*

SSGB is equivalent to saying that all integers  $x \geq 4$  appear as  $m$  in a component  $mk$  of  $S_g$ . Therefore, SSGB is equivalent to the case (G) and the negation  $\neg$ SSGB is equivalent to the case  $\neg$ (G). We have seen above that the  $S_g$  triples are the same in these two cases. This means that both SSGB and  $\neg$ SSGB hold.