The distance between two inertial observers in relative motion in special relativity

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Abstract

When two inertial observers A and B in relative motion measure the distance between them, will they obtain the same value? Although there is a lack of detailed expositions on this issue, many relativity articles and books seem to suggest that the observers on the earth measure a longer distance than that measured by the observers moving relative to the earth. The present study has examined this issue in detail, using two fundamental conditions of special relativity: 1) the space time interval between two events in the Minkowski space is independent of the inertial reference frame chosen; and 2) there is no privileged reference frame and all inertial reference frames are equal. The results of the present study shows that the value of the distance between A and B measured by observer B in a frame where B is stationary is the same as that obtained by observer A in a frame where A is stationary. The idea that distance measured by observer A is longer than that measured by observer B contradicts special relativity, because it designates de facto more privileged reference frames, which cannot be correct within the framework of special relativity.

Keywords: Minkowski diagram; distance; space time interval; special relativity; Lorentz ether theory; privileged frame.

1. Introduction

The principle of relativity requires that physical laws have the same form in all inertial frames of reference, which is one of the most fundamental laws in physics. How to use the principle of relativity to predict the outcome of an observation especially in special relativity is sometimes a matter of debate because different commentators may interpret the implication of the principle differently. In Newtonian mechanics, physical quantities such as distance/length, time and mass have same values in all inertial reference frames, so the application of the principle of relativity is more straightforward. In special relativity, the implication of the principle is less straightforward because distance/length, time and mass might have different values in different frames. The lengths of rods, time and mass measured in different inertial reference frames have been extensively discussed in the relativity literature, but how a spatial distance is measured by different observers has not received as much attention.

What values will two inertial observers in relative motion obtain when they measure the distance between them? This question can be illustrated by Fig.1; two observers A and B move toward each other at a velocity of $0.99c$, where c is the speed of light in vacuum. If observer A measured the distance AB being 1907 m in her frame, what distance observer B would obtain when she measures the distance in her frame? A naïve application of the principle of relativity would give the same value of 1907 m, because according to special relativity there is no privileged observer or frame of reference. Since the two observers are identical in every aspect, there seems to be no reason for observer B to obtain a different value for the same distance. However, recently a reviewer of

Foundations of Physics disagrees with this naïve application of the principle of relativity and claims that observer B should obtain a different value, a distance shorter than 1907m.

Fig.1 The distance between two inertial observers A and B in relative motion. The velocity $v_{B,A}$ is that of observer B as measured by observer A, $v_{A,B}$ is that of observer A as measured by observer B.

That reviewer thinks that only observers which are at rest with respect to each other measure an identical distance between them; but observer B and observer A are not stationary with respect to each other, therefore, according to special relativity the distance they measure is not the same. The reason that the reviewer gave for observer B to measure a different value is that there is a velocity between the observers because they are not at rest with respect to each other. This velocity entails that the hyperplanes of simultaneity that correspond to the rest-frames of the observers are different. Now, since the distance between two observers is defined on a simultaneity hyperplane, the distances are measured by the observers are, *a fortiori*, different. The reviewer cited basic relativity textbooks such as the one by Sartori (1996) for supporting different values measured by observers A and B for the distance between A and B. This distance-between-A-and-B-being-different-for-A-and-B view seems to be shared by many relativity researchers.

Although that reviewer did not spell out the exact value that observer B should obtain according to this distance-between-A-and-B-being-different-for-A-and-B view, it is obvious from the context and the Minkowski diagram presented by the reviewer that in the reviewer's opinion the distance measured by observer B is

$$d_{AB,B} = d_{BA,A}\sqrt{1 - v^2/c^2}, \qquad (1)$$

because any value with $d_{AB,B} > d_{BA,A}\sqrt{1 - v^2/c^2}$ would support the conclusion which the reviewer dismissed. In Eq. (1), $d_{AB,B}$ is the distance between A and B measured by observer B in the frame where observer B is stationary, $d_{AB,A}$ the distance between A and B measured by observer A in the frame where observer A is stationary, and *v* the velocity between A and B.

The reviewer also claims that the view of $d_{AB,B} = d_{BA,A}\sqrt{1 - v^2/c^2}$ is common knowledge and the view of $d_{AB,B} = d_{BA,A}$ is a failure in understanding special relativity. The opinion from the aforementioned reviewer shows that there are fundamental differences in physicists' understanding of the frame-dependence of distance within the framework of special relativity. According to special relativity, there is no privileged frame. If two identical observers in relative motion must obtain different values for the same distance between them, then what determines who should obtain which value? Why should it be $d_{AB,B} = d_{BA,A}\sqrt{1 - v^2/c^2}$, i.e. observer A measures a longer distance between A and B? Why should it not be $d_{AB,A} = d_{BA,B}\sqrt{1 - v^2/c^2}$ instead, i.e. observer B measures a longer distance between A and B?

In the relativity literature, there is a lack of in-depth discussion of the distance issue. Many articles and books using the lifetime of unstable high speed elementary particles as experimental evidence of time dilation do implicitly share the view of the aforementioned reviewer. What values of the distance between two inertial observers in relative motion would be measured by the two observers in their own stationary frames? The answer to this question might have wide implications to our understanding of special relativity and it deserves an in-depth investigation and discussion.

The aim of the present study is to investigate which of the following two assertions is correct:

Assertion 1: the distance between observers A and B measured by observer B in a frame where B is stationary ($d_{AB,B}$) is shorter than the distance between observers A and B measured by observer A in a frame where A is stationary ($d_{BA,A}$), with a relationship

$$d_{AB,B} = d_{BA,A}\sqrt{1 - v^2/c^2}$$

Assertion 2: the distance between observers A and B measured by observer B in a frame where B is stationary ($d_{AB,B}$) is equal to the distance between observers A and B measured by observer A in a frame where A is stationary ($d_{BA,A}$),

$$d_{AB,B} = d_{BA,A} \tag{2}$$

The present study reveals that Assertion 2 is a logical consequence of special relativity and Assertion 1 seems to be inconsistent with non-existence of privileged reference frames. The rest of the paper is organized as follows: section 2 examines the arguments of Assertion 1 supporters for different values of the distance measured by two identical observers in relative motion; section 3 presents a truly relativistic analysis of the distance between two inertial observers in relative motion; and section 4 discusses and concludes.

2. How could the two observers obtain different values for the same distance?

The reviewer uses a Minkowski diagram similar to Fig. 2A to illustrate how the two observers obtain different values for the same distance AB, to support Assertion 1. The worldline of the observer A is represented by the line AD and its simultaneity hyperplane at A is represented by the line AB. The lines BD and BC represent the worldline of the observer B and her simultaneity hyperplane respectively.

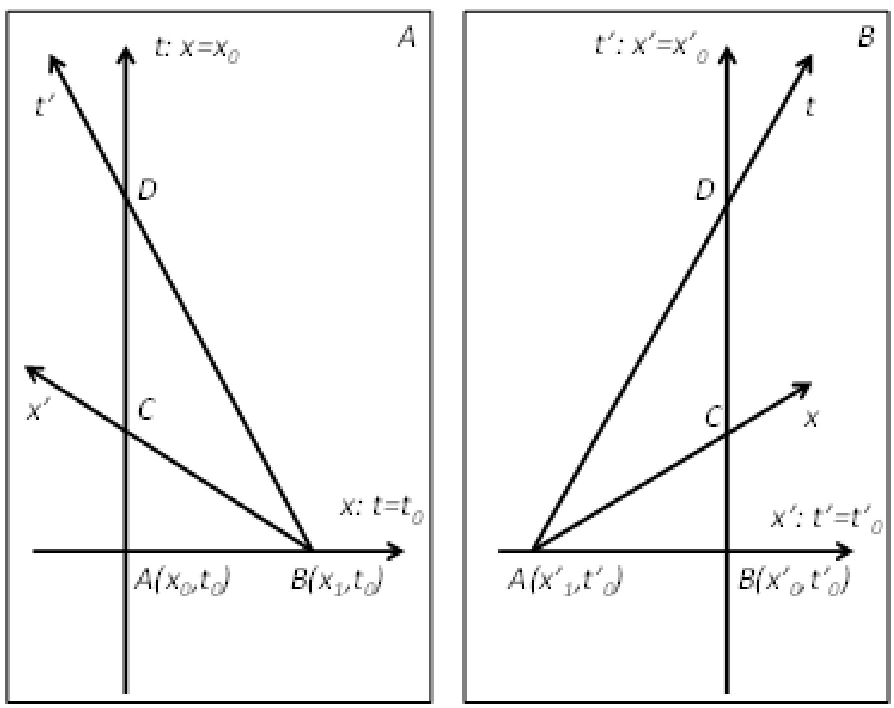

Fig.2 Minkowski diagrams representing the worldlines of observers A and B. Observer A's worldline is along a line parallel to the t-axis, $t|x = x_0$, and its line of simultaneity at its time $t = t_0$, $x|t = t_0$, is line AB in Fig.2A and line AC in Fig.2B. Observer B's wordline is along a line parallel to the t'-axis, $'|x' = x'_0$, and its line of simultaneity at its time $t' = t'_0$, $x'|t' = t'_0$, is line BC in Fig.2A and line AB in Fig.2B.

According to the reviewer, the values of the distance measured by the two observers are different, and that this does not involve any kind of violation of the relativity principle. It is actually a consequence of Einstein's two principles taken together, along with Einstein's redefinition of the concept of distant simultaneity, that spatial distances are frame-relative quantities. The reviewer acknowledges that the diagram is drawn from the perspective of the observer A-rest frame, and notes that the observer B-rest frame diagram is equally simple.

Since there is no privileged frame according to special relativity, there are a few questions to be asked with respect to the reviewer's view in terms of the relationship between results obtained from the Minkowski diagrams drawn from the perspective of observer A and observer B respectively. As implied by the reviewer, the distance $d_{AB,B}$ is shorter than $d_{BA,A}$ in Fig.2A, but we can also draw Fig.2B from the perspective of the observer B. Because Fig.2A and Fig.2B are symmetric, $d_{BA,A}$ must be shorter than $d_{AB,B}$ in Fig.2B. Then we may ask

Question 1: Does the distance $d_{BA,B}$ in Fig.2A have the same value as $d_{AB,B}$ in Fig.2B?

To avoid any confusion caused by ambiguity in notation, we can rewrite $d_{AB,A}$ and $d_{AB,B}$ in Fig.2A as $d^{A}_{BA,A}$ and $d^{A}_{AB,B}$ respectively, and write $d_{AB,A}$ and $d_{AB,B}$ in Fig.2B as $d^{B}_{BA,A}$ and $d^{B}_{AB,B}$ respectively (Ma 2014). The distance $d^{A}_{AB,A}$ is the distance between A and B measured by A in the frame where A is stationary, and $d^{A}_{AB,B}$ is the distance between A and B measured by B in the frame which is viewed by A to be moving. The distance $d^{B}_{AB,B}$ is the distance between A and B measured by B in the frame where B is stationary, and

$d^B_{BA,A}$ is the distance between A and B measured by A in the frame which is viewed by B to be moving.

If $d^A_{AB,B}$ has the same value as $d^B_{AB,B}$,

$$d^A_{AB,B} = d^B_{AB,B} \tag{3}$$

which is obviously the claim of the reviewer, then we may ask

Question 2: Does the distance $d^A_{BA,A}$ in Fig.2A have the same value as $d^B_{BA,A}$ in Fig.2B?

If $d^A_{AB,B}$ has the same value as $d^B_{AB,B}$, since there is no privileged frame in special relativity, there is no reason why $d^A_{BA,A}$ should not have the same value as $d^B_{BA,A}$, then

$$d^A_{BA,A} = d^B_{BA,A} \tag{4}$$

From Fig.2A we know $d^A_{BA,A} > d^A_{AB,B}$, which is implied by the reviewer, if $d^A_{AB,B} = d^B_{AB,B}$, we must have

$$d^A_{BA,A} > d^B_{AB,B} \tag{5}$$

From Fig.2B we know $d^B_{AB,B} > d^B_{BA,A}$, if $d^B_{BA,A} = d^A_{BA,A}$, we must have

$$d^B_{AB,B} > d^A_{BA,A}\ . \tag{6}$$

Between Eqs. (5) and (6), we have a contradiction here, for logical consistency $d^A_{BA,A} > d^B_{AB,B}$ and $d^A_{BA,A} < d^B_{AB,B}$ cannot be both true. Therefore, we cannot assume $d^B_{AB,B} = d^A_{AB,B}$ for the distance between two inertial observers A and B in relative motion, for logical consistency we must have

$$d^B_{AB,B} \neq d^A_{AB,B} \tag{7}$$

The Minkowski diagram used by the reviewer (Fig.2A) can only show that in the Minkowski diagram drawn from the perspective of observer A the distance $d^A_{BA,A} > d^A_{AB,B}$, which does not mean that $d^A_{BA,A} > d^B_{AB,B}$ at all. The primary aim of this study is to find the relationship between $d^A_{BA,A}$ and $d^B_{AB,B}$, for this objective the reviewer's Minkowski diagram (Fig.2A) has proved nothing. Obviously the reviewer is not even aware of the difference between $d^A_{AB,B}$ and $d^B_{AB,B}$.

Since $d^B_{AB,B} \neq d^A_{AB,B}$, so even if $d^A_{BA,A} > d^A_{AB,B}$ in Fig.2A, we may still have $d^B_{AB,B} = d^A_{BA,A}$ or $d^B_{AB,B} > d^A_{BA,A}$. This situation is just like time dilation during the outward journey in the twin paradox, where both twins will find the other person being younger due to the principle of relativity and the non-existence of a privileged frame in special relativity. In Fig.2A, the distance measured by observer A is longer than that measured by observer B is because Fig.2A is drawn from the perspective of observer A, i.e. observer A is designated as the "stationary" observer by Fig.2A. According to special relativity, Fig.2B in which the distance measured by observer B is longer than that measured by observer A has the same status as Fig.2A. Therefore, the reviewer is completely wrong in claiming that the Minkowski diagram or Einstein's relativity of simultaneity can show that the distance (1907m) measured by observer A in observer A's rest frame is longer than that measured by observer B in observer B's rest frame, with the relationship between the values of the distance measured by the two observers being

$$d^B_{AB,B} = d^A_{BA,A}\sqrt{1 - v^2/c^2} \, .$$

3. The relationship between $d^A_{BA,A}$ and $d^B_{AB,B}$

From our analysis in section 2, it is wrong to claim $d_{AB,B}^{B} \equiv d_{BA,A}^{A}\sqrt{1 - v^2/c^2}$ although

$$d_{AB,B}^{A} = d_{BA,A}^{A}\sqrt{1 - v^2/c^2}\ . \qquad (8)$$

Since $d_{AB,B}^{B} \neq d_{AB,B}^{A}$, given Eq. (8), we may still have $d_{AB,B}^{B} = d_{BA,A}^{A}$ or $d_{AB,B}^{B} > d_{BA,A}^{A}$. Then the question is what the relationship between $d_{BA,A}^{A}$ and $d_{AB,B}^{B}$ is. At the beginning of this paper, we have stated that a naïve application of the principle of relativity will give us the relationship between the two distances

$$d_{AB,B}^{B} = d_{BA,A}^{A} \qquad (9)$$

This is Assertion 2, i.e. the distance between A and B measured by observer B in the frame where observer B is stationary is the same as the distance between A and B measured by observer A in the frame where observer A is stationary.

Since the reviewer has objected to this naïve application of the principle of relativity, we will use the fundamental principles of special relativity to prove the correctness of Assertion 2, the naïve application of the principle of relativity. According to special relativity, the space-time interval in the Minkowski space is invariant, that is, the space-time interval is independent of the inertial reference frame chosen (Minkowski 1909; Landau and Lifshitz 2002). Therefore, the space-time interval between A and B is constant in all the inertial reference frames.

The space-time interval between A and B is

$$s_{AB}{}^2 = \left(x_{B,A}^{A} - x_{A,A}^{A}\right)^2 + \left(y_{B,A}^{A} - y_{A,A}^{A}\right)^2 + \left(z_{B,A}^{A} - z_{A,A}^{A}\right)^2 - \left(t_{B,A}^{A} - t_{A,A}^{A}\right)^2$$

$$= (x'^{B}_{B,B} - x'^{B}_{A,B})^2 + (y'^{B}_{B,B} - y'^{B}_{A,B})^2 + (z'^{B}_{B,B} - z'^{B}_{A,B})^2 - (t'^{B}_{B,B} - t'^{B}_{A,B})^2 \quad (10)$$

In Eq. (10), $x^{A}_{*,A}$, $y^{A}_{*,A}$, $z^{A}_{*,A}$ and $t^{A}_{*,A}$ are space-time coordinates in the frame where observer A is at rest and which is the "stationary" one in the Minkowski diagram; $x'^{B}_{*,B}$, $y'^{B}_{*,B}$, $z'^{B}_{*,B}$ and $t'^{B}_{*,B}$ are space-time coordinates in the frame where observer B is at rest and which is the "stationary" one in the Minkowski diagram.

Since $y^{A}_{*,A}$, $z^{A}_{*,A}$, $y'^{B}_{*,B}$ and $z'^{B}_{*,B}$ are all zero in the present setup, Eq. (10) can be simplified to

$$s_{AB}{}^2 = \left(x^{A}_{B,A} - x^{A}_{A,A}\right)^2 - \left(t^{A}_{B,A} - t^{A}_{A,A}\right)^2 = (x'^{B}_{B,B} - x'^{B}_{A,B})^2 - (t'^{B}_{B,B} - t'^{B}_{A,B})^2 \quad (11)$$

In Fig.2A and Fig.2B, since $t^{A}_{A,A} = t^{A}_{B,A}$ and $t'^{B}_{A,B} = t'^{B}_{B,B}$, so Eq. (11) leads to

$$\left(x^{A}_{B,A} - x^{A}_{A,A}\right)^2 = (x'^{B}_{B,B} - x'^{B}_{A,B})^2 \quad (12)$$

Since

$$x^{A}_{B,A} - x^{A}_{A,A} = d^{A}_{BA,A},$$

$$x'^{B}_{B,B} - x'^{B}_{A,B} = d^{B}_{AB,B} \quad (13)$$

therefore,

$$d^{B}_{AB,B} = d^{A}_{BA,A}$$

This proves that the naïve application of the principle of relativity is the correct result. The distance between A and B measured by observer B is equal to the distance between A and B measured by observer A. The reviewer's opinion is completely wrong.

4. The meaning of $d^A_{AB,B}$ and the relationship between $d^A_{BA,A}$ and $d^A_{AB,B}$

In Fig.2A we have a distance measured by observer B in a Minkowski diagram drawn from the perspective of observer A, $d^A_{AB,B}$, what is the meaning of $d^A_{AB,B}$? The distance $d^A_{AB,B}$ is the observer A's perception or observation of the distance between A and B measured by observer B who is at rest in the moving frame, from the perspective of observer A. This perception of observer A is affected by the velocity between observers A and B, which is described by the Minkowski diagram Fig,2A.

From our analysis in section 3, it seems appropriate to rewrite the Lorentz transformation when the velocity between two inertial frames A and B, *v*, is in the positive direction of the *x*-axis as

$$x'^A_{t,B} = \frac{x^A_{t,A} - vt^A_{x,A}}{\sqrt{1-v^2/c^2}} \tag{14}$$

$$t'^A_{x,B} = \frac{t^A_{x,A} - vx^A_{t,A}/c^2}{\sqrt{1-v^2/c^2}} \tag{15}$$

In Eqs. (14) and (15), $x'^A_{t,B}$ indicates the frame A's perception of the x'-coordinate in frame B at time *t* (frame A's time); $x^A_{t,A}$ is the *x*-coordinate in frame A at time *t* (frame A's time); $t^A_{x,A}$ is the *t*-coordinate in frame A when the event's spatial coordinate in frame A is *x* (frame A's spatial coordinate); $t'^A_{x,B}$ indicates the frame A's perception of the t'-coordinate in frame B when the event's spatial coordinate in frame A is *x* (frame A's spatial coordinate).

Given Eqs. (14) and (15), we obtain the relationship between $d^A_{BA,A}$ and $d^A_{AB,B}$ with the accepted standard approaches for deriving length contraction (Kittel et al. 1973; Schutz 1985):

$$d_{AB,B}^{A} = d_{BA,A}^{A}\sqrt{1 - v^2/c^2}$$

This result is what is described by Fig.2A.

Similarly, the Lorentz transformation from the primed coordinate system to the unprimed coordinate system can be rewritten as

$$x_{t\prime,A}^{B} = \frac{x\prime_{t\prime,B}^{B} - vt_{x\prime,B}^{B}}{\sqrt{1-v^2/c^2}} \tag{16}$$

$$t_{x\prime,A}^{B} = \frac{t\prime_{x\prime,B}^{B} - vx\prime_{t\prime,B}^{B}/c^2}{\sqrt{1-v^2/c^2}} \tag{17}$$

In Eqs. (16) and (17), $x_{t\prime,A}^{B}$ indicates the frame B's perception of the *x*-coordinate in frame A at time t' (frame B's time); $x'^{B}_{t\prime,B}$ is the x'-coordinate in frame B at time t' (frame B's time); $t'^{B}_{x\prime,B}$ is the *t'*-coordinate in frame B when the event's spatial coordinate in frame B is x' (frame B's spatial coordinate); $t_{x\prime,A}^{B}$ indicates the frame B's perception of the *t*-coordinate in frame A when the event's spatial coordinate in frame B is x' (frame B's spatial coordinate).

Since $d_{AB,B}^{B} = d_{BA,A}^{A}$ which has been proved in section 3 and $d_{AB,B}^{A} = d_{BA,A}^{A}\sqrt{1 - v^2/c^2}$, we have

$$d_{AB,B}^{A} = d_{AB,B}^{B}\sqrt{1 - v^2/c^2} \neq d_{AB,B}^{B} \tag{18}$$

The observer A's perception of the distance measured by observer B is not the same as the measurement by observer B. This point has rarely been spelt out or emphasized in the relativity literature. The cause of the reviewer's incorrect view is the lack of understanding of the difference between A's perception of B's measurement and B's measurement per se.

Using the accepted standard approaches for deriving length contraction (Kittel et al. 1973; Schutz 1985), we can also obtain

$$d^B_{BA,A} = d^B_{AB,B}\sqrt{1 - v^2/c^2} = d^A_{AB,B} = d^A_{BA,A}\sqrt{1 - v^2/c^2} \neq d^A_{BA,A} \qquad (19)$$

These relationships are partly reflected in Fig.2B. Symmetric to $d^A_{AB,B}$, $d^B_{BA,A}$ is the observer B's perception of the distance between A and B measured by observer A.

5. Discussions

The present study has proved that two inertial observers A and B in relative motion will obtain the same value in measuring the distance between them. Given that special relativity rejects any privileged inertial reference frame and that the space-time interval is frame-independent, the present result is logically obvious and natural. From the non-existence of privileged frame, the Minkowski diagram depicted in Fig.2B has the same status as that depicted in Fig.2A. With the two Minkowski diagrams, the frame-independent space-time interval in the Minkowski space leads directly to the conclusion that the distance between A and B measured by observer B when B's frame is the "stationary" frame has the same value as the distance between A and B measured by observer A when A's frame is the "stationary" frame.

The view that the distance measured by observer B is shorter than that measured by observer A, $d^B_{AB,B} = d^A_{BA,A}\sqrt{1 - v^2/c^2}$, is obviously contradictory to special relativity that rejects the existence of privileged inertial reference frames. If the distance between A and B measured by observer B, $d^B_{AB,B}$, is shorter than that measured by observer A, $d^A_{BA,A}$, the two reference frames would be unequal. Therefore, the reviewer and people with similar views actually advocate an interpretation that designates one frame being more privileged than the other. Moreover, how they can decide that the distance measured by observer A is longer than that measured by observer B seems to be a mystery.

As we have shown in this study, it is not possible to have two reference frames unequal within the framework of special relativity. The equality between the two frames and the frame-independence of the space-time interval in the Minkowski space ensure the equality between the values of the distance measured by A and B. Since Lorentz ether theory can also explain relativistic phenomena and there is a privileged ether frame in Lorentz ether theory, it might provide a mechanism for Assertion 1, i.e. $d_{AB,B}^{B} = d_{BA,A}^{A}\sqrt{1 - v^2/c^2}$, to be true.

Interestingly, the reviewer also strongly objects to the remark that Lorentz ether theory might provide a mechanism for $d_{AB,B}^{B} = d_{BA,A}^{A}\sqrt{1 - v^2/c^2}$. According to the reviewer, because Lorentz ether theory and special relativity are fully equivalent in their predictions (Dorling 1968; Janssen 1995; Acuña 2014), if special relativity cannot provide a mechanism for inequality between observers A and B, nor can the Lorentz ether theory. This seems strange, because insisting on $d_{AB,B}^{B} = d_{BA,A}^{A}\sqrt{1 - v^2/c^2}$ is equivalent to alleging that observer A's stationary frame is more privileged than observer B's stationary frame, which is a kind of Lorentzian claim. Moreover, to show that Lorentz ether theory might provide a mechanism for $d_{AB,B}^{B} = d_{BA,A}^{A}\sqrt{1 - v^2/c^2}$, we only need to use a tautology: a theory with more privileged frames allows the existence of more privileged frames.

People insisting on $d_{AB,B}^{B} = d_{BA,A}^{A}\sqrt{1 - v^2/c^2}$, the Assertion 1 supporters, are unwitting supporters of Lorentz ether theory, despite their stated support for special relativity. Why do many people on one hand insist on unequal inertial frames (a Lorentzian view) and on the other hand claim support for special relativity? One cause of such phenomena is their failure in truly understanding special relativity. Lorentz ether theory emerged before Einstein's special relativity and it appears a bit more consistent with people's everyday experience than special relativity. Another cause might be that they find Lorentzian explanations conforming to their expected results and mistake Lorentzian explanations as relativistic ones.

In conclusion, from the non-existence of privileged inertial reference frames in special relativity and the frame-independence of the space-time interval in the Minkowski space, it is obvious that the distance between two observers A and B measured by A is the same as that measured by B. This finding might have important implications for understanding the foundation of special relativity.

Acknowledgements: I would like to thank the anonymous reviewer of another manuscript for explaining why the observer A on the earth should obtain a value larger than that obtained by observer B in measuring the distance between A and B.

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