An alternative way of writing the Riemann zeta function

Abstract: In this paper, I will be proposing another legitimate way of writing the Riemann zeta function using Euler's constant, e.

1. Logarithms

There are several rules of logarithms, but the two that we will use to re-write the Riemann zeta function will be:

Law 1.1: $b^{log_b x} = x$

Law 1.2: $\log_b x^y = y \log_b x$

2. Riemann zeta function

The Riemann zeta function is defined as the following:

Definition 2.1: $\zeta(\mathrm{s}) = \sum_{\mathrm{n}=1}^{\infty} \mathrm{n}^{\text{-s}} = \sum_{\mathrm{n}=1}^{\infty} \frac{1}{\mathrm{n^s}} = 1 + \frac{1}{2^{\mathrm{s}}} + \frac{1}{3^{\mathrm{s}}} + \frac{1}{4^{\mathrm{s}}} +\ldots$

When we look at the way the Zeta function is defined in its expanded form we can first apply Law 1.1 (with base e) so that the expanded form becomes:

Equation 2.2: $\zeta(s) = 1 + \frac{1}{e^{\ln 2^{\mathrm{s}}}} + \frac{1}{e^{\ln 3^{\mathrm{s}}}} + \frac{1}{e^{\ln 4^{\mathrm{s}}}} +\ldots$

Now applying Law 1.2 to equation 2.2, we get:

Equation 2.3: $\zeta(\mathrm{s}) = 1 + \frac{1}{se^{\ln 2}} + \frac{1}{se^{\ln 3}} + \frac{1}{se^{\ln 4}} +\ldots$

Now, if we compacted Equation 2.3 into sigma notation, we get:

Notation 2.4: $\zeta(\mathrm{s}) = \sum_{\mathrm{n}=1}^{\infty} \frac{1}{\mathrm{s}e^{\ln \mathrm{n}}} (1+x)^n = 1 + \frac{nx}{1!} + \frac{n(n-1)x^2}{2!} + \cdots$

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