Research and report

Another way of expressing ζ (odd number)

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Abstract

Last time, I expressed ζ (odd number), such as ζ(3), ζ (5), ζ(7), ζ(9), ζ(11), ζ(13), ζ (15), ζ(17), ζ (19), ζ (21), ζ(23) and made an official.
Another way of expressing ζ (odd number), such as ζ(3), ζ (5), ζ(7) ζ(9), ζ(11), ζ(13), ζ(15), ζ(17), ζ(19), ζ(21), ζ(23) and made an official.

Introduction

in ζ (3).
Previously it represented as follows.

$$\zeta(3)=\frac{8}{7}(1+\sum_{n=1}^{\infty}\frac{1}{(2n+1)^3})$$

But, now, in ζ (3)

$$\zeta(3)=\frac{224}{189}(1+\frac{27}{28}\sum_{n=1}^{\infty}\frac{1}{(2n+3)^3})$$

or

$$\zeta(3) = \frac{8}{7}(\frac{28}{27} + \sum_{n=1}^{\infty} \frac{1}{(2n+3)^3})$$

This is merely another expression.

in ζ (5).
Previously it represented as follows.

$$\zeta(5) = \frac{32}{31}(1 + \sum_{n=1}^{\infty} \frac{1}{(2n+1)^5})$$

But, now, in ζ (5)

$$\zeta(5) = \frac{7808}{7577}(1 + \frac{243}{244}\sum_{n=1}^{\infty} \frac{1}{(2n+3)^5})$$

or

$$\zeta(5) = \frac{32}{31}(\frac{244}{243} + \sum_{n=1}^{\infty} \frac{1}{(2n+3)^5})$$

This is merely another expression.

Discussion

$$\zeta(3) = \sum_{n=1}^{\infty} \frac{1}{n^3} = 1 + \sum_{n=1}^{\infty} \frac{1}{(2n)^3} + \sum_{n=1}^{\infty} \frac{1}{(2n+1)^3}$$

calculated

$$\sum_{n=1}^{\infty}\frac{1}{n^3}=1+\frac{1}{8}\sum_{n=1}^{\infty}\frac{1}{n^3}+\sum_{n=1}^{\infty}\frac{1}{(2n+1)^3} \tag{1}$$

$$\frac{7}{8}\sum_{n=1}^{\infty}\frac{1}{n^3}=1+\sum_{n=1}^{\infty}\frac{1}{(2n+1)^3} \tag{2}$$

$$\zeta(3)=\frac{8}{7}(1+\sum_{n=1}^{\infty}\frac{1}{(2n+1)^3})$$

n=1, ζ (3)≒ 8/7*(1+1/27)= 1.185185185185185185185185185185185185185……
n=1,2, ζ (3)≒ 8/7*(1+1/27+1/125)= 1.194328042328042328042……
n=1,2,3 ζ (3)≒ 8/7*(1+1/27+1/125+1/343)= 1.1976599873509494500748……
n=1,2,3,4 ζ (3)≒ 8/7*(1+1/27+1/125+1/343+1/729)=
1.199227692622358425……
n=1,2,3,4,5 ζ (3)≒ 8/7*(1+1/27+1/125+1/343+1/729+1/1391)=
1.20004930092060…..
n=1,2,3,4,5,6 ζ (3)≒ 8/7*(1+1/27+1/125+1/343+1/729+1/1391+1/2197)=
1.2005694907899045327877…..
…….
ζ (3)=1.2020569031595942853997381615114499907649862923404988 81.....

or

$$\zeta(3)=\frac{224}{189}(1+\frac{27}{28}\sum_{n=1}^{\infty}\frac{1}{(2n+3)^3})$$

or

$$\zeta(3)=\frac{8}{7}(\frac{28}{27}+\sum_{n=1}^{\infty}\frac{1}{(2n+3)^3})$$

n=1 ζ (3)≒ 1.194328042328042328042……

n=1,2 ζ (3)≒1.197659987350949450074 81…….
n=1,2,3 ζ (3)≒1.1992276926223584251……
n=1,2,3,4 ζ (3)≒ 1.20004930092060…..
n=1,2,3,4,5 ζ (3)≒1.2005694907899045327877…..
n=1,2,3,4,5,6 ζ (3)≒1.200908115128528871412 1…..
n=1,2,3,4,5,6,7 ζ(3) ≒1.2011407341276856275401908 5……..
…….
ζ (3) = 1.20205690315959428539973816151144 9……..

when

$$\zeta(5)=\sum_{n=1}^{\infty}\frac{1}{n^5}=1+\sum_{n=1}^{\infty}\frac{1}{(2n)^5}+\sum_{n=1}^{\infty}\frac{1}{(2n+1)^5}$$

calculated

$$\zeta(5)=\frac{32}{31}(1+\sum_{n=1}^{\infty}\frac{1}{(2n+1)^5})$$

n=1, ζ (5)≒ 32/31*(1+1/243)= 1.03650604009026948095048……
n=1,2, ζ (5)≒ 32/31*(1+1/243+1/3125)=1.0368363626709146422408 0
…….
n=1,2,3, ζ (5)≒ 32/31*(1+1/243+1/3125+1/16807)=
1.0368977810123507182229 72……
n=1,2,3,4, ζ (5)≒ 32/31*(1+1/243+1/3125+1/16807+1/59049)=
1.0369152623933142591640938344…….
……
…….
ζ (5) = 1.0369277551433699263313654864570

or

$$\zeta(5)=\frac{7808}{7533}(1+\frac{243}{244}\sum_{n=1}^{\infty}\frac{1}{(2n+3)^5})$$

or

$$\zeta(5)=\frac{32}{31}(\frac{244}{243}+\sum_{n=1}^{\infty}\frac{1}{(2n+3)^5})$$

n=1 ζ (5)≒1.0368363626709146422408O.......
n=1,2 ζ (5)≒1.036897781012350718222972......
n=1,2,3 ζ (5)≒1.03691526239331425916409383 44.......
....
ζ (5) = 1.036927755143369926331365486457O.....

when

In the same way, ζ (7) or more is as follows.

$$\zeta(7)=\sum_{n=1}^{\infty}\frac{1}{n^7}=1+\sum_{n=1}^{\infty}\frac{1}{(2n)^7}+\sum_{n=1}^{\infty}\frac{1}{(2n+1)^7}$$

calculated

$$\zeta(7)=\frac{2^7}{2^7-1}(1+\sum_{n=1}^{\infty}\frac{1}{(2n+1)^7})$$

$$\zeta(7)=\frac{128}{127}(1+\sum_{n=1}^{\infty}\frac{1}{(2n+1)^7})$$

n=1, ζ (7) ≒ 128/127*(1+1/2187)= 1.0083348634918577.....
n=1,2, ζ (7) ≒128/127*(1+1/2187+1/78125)= 1.00834776427925933126 67......

n=1,2,3,　ζ (7) ≒128/127*(1+1/2187+1/78125+1/823543)=
1.008348988106085311247…

…….

…….

ζ (7)= 1.008349277381922826839…….

or

$$\zeta(7) = \frac{280064}{277749}(1 + \frac{2187}{2188}\sum_{n=1}^{\infty}\frac{1}{(2n+3)^7})$$

or

$$\zeta(7) = \frac{128}{127}(\frac{2188}{2187} + \sum_{n=1}^{\infty}\frac{1}{(2n+3)^7})$$

n=1,　ζ (7) ≒1.0083477642792593312667……

n=1,2　ζ (7) ≒1.008348988106085311247…

…….

ζ (7)= 1.008349277381922826839…….

when

$$\zeta(9) = \sum_{n=1}^{\infty}\frac{1}{n^9} = 1 + \sum_{n=1}^{\infty}\frac{1}{(2n)^9} + \sum_{n=1}^{\infty}\frac{1}{(2n+1)^9}$$

calculated

$$\zeta(9) = \frac{2^9}{2^9-1}(1 + \sum_{n=1}^{\infty}\frac{1}{(2n+1)^9})$$

or

$$\zeta(9) = \frac{512}{511}(1 + \sum_{n=1}^{\infty} \frac{1}{(2n+1)^9})$$

n=1, ζ (9)≒512/511*(1+1/3^9)= 512/511*(1+1/19683)=

1.0020078518490680001......

n=1,2, ζ (9)≒512/511*(1+1/3^9+1/5^9)=

512/511*(1+1/19683+1/1953125)=
1.002008364851024948963577597284.........
…….
ζ (9) =1.002008392826082214417852769 2……

or

$$\zeta(9) = \frac{1439744}{1436859}(1 + \frac{19683}{19684} \sum_{n=1}^{\infty} \frac{1}{(2n+3)^9})$$

or

$$\zeta(9) = \frac{512}{511}(\frac{19684}{19683} + \sum_{n=1}^{\infty} \frac{1}{(2n+3)^9})$$

when

$$\zeta(11) = \sum_{n=1}^{\infty} \frac{1}{n^{11}} = 1 + \sum_{n=1}^{\infty} \frac{1}{(2n)^{11}} + \sum_{n=1}^{\infty} \frac{1}{(2n+1)^{11}}$$

calculated

$$\zeta(11)=\frac{2^{11}}{2^{11}-1}(1+\sum_{n=1}^{\infty}\frac{1}{(2n+1)^{11}})$$

or

$$\zeta(11)=\frac{2048}{2047}(1+\sum_{n=1}^{\infty}\frac{1}{(2n+1)^{11}})$$

n=1,

ζ (11)≒ 2048/2047*(1+1/3^11)= 2048/2047*(1+1/177147)=

1.00049416757202925667272........

n=1,2,

ζ(11)≒2048/2047*(1+1/3^11+1/5^11)=2048/2047*(1+1/177147+1/48828125)=

1.00049418806203414187057225..........

……...

ζ (11) =1.00049418860411946455870228252......

or

$$\zeta(11)=\frac{2048}{2047}(\frac{177148}{177147}+\sum_{n=1}^{\infty}\frac{1}{(2n+3)^{11}})$$

n=1, ζ(11)≒ 1.00049418806203414187057225..........

……...

ζ (11) =1.00049418860411946455870228252......

when

$$\zeta(13)=\sum_{n=1}^{\infty}\frac{1}{n^{13}}=1+\sum_{n=1}^{\infty}\frac{1}{(2n)^{13}}+\sum_{n=1}^{\infty}\frac{1}{(2n+1)^{13}}$$

calculated

$$\zeta(13) = \frac{2^{13}}{2^{13}-1}(1+\sum_{n=1}^{\infty}\frac{1}{(2n+1)^{13}})$$

or

$$\zeta(13) = \frac{8192}{8191}(1+\sum_{n=1}^{\infty}\frac{1}{(2n+1)^{13}})$$

or

$$\zeta(13) = \frac{8192}{8191}(\frac{1594322}{1594321}+\sum_{n=1}^{\infty}\frac{1}{(2n+3)^{13}})$$

n=1　 ζ (13)≒1.0001220703125
n=1,2　ζ (13)≒1.000122697537974386……
ζ (13)= 1.000122713347578489146751836526357……

$$\zeta(15) = \sum_{n=1}^{\infty}\frac{1}{n^{15}} = 1+\sum_{n=1}^{\infty}\frac{1}{(2n)^{15}}+\sum_{n=1}^{\infty}\frac{1}{(2n+1)^{15}}$$

calculated

$$\zeta(15) = \frac{2^{15}}{2^{15}-1}(1+\sum_{n=1}^{\infty}\frac{1}{(2n+1)^{15}})$$

or

$$\zeta(15) = \frac{32768}{32767}(1 + \sum_{n=1}^{\infty} \frac{1}{(2n+1)^{15}})$$

n=1,　ζ (15)≒ 32768/32767*(1+1/3^15)= 32768/32767*(1+1/14348907)=
1.0000305882033222608468038……..

…….

ζ (15) =1.0000305882363070204935517…….

or

$$\zeta(15) = \frac{32768}{32767}(\frac{14348908}{14348907} + \sum_{n=1}^{\infty} \frac{1}{(2n+3)^{15}})$$

…….

ζ (15) =1.0000305882363070204935517…….

when

$$\zeta(17) = \sum_{n=1}^{\infty} \frac{1}{n^{17}} = 1 + \sum_{n=1}^{\infty} \frac{1}{(2n)^{17}} + \sum_{n=1}^{\infty} \frac{1}{(2n+1)^{17}}$$

calculated

$$\zeta(17) = \frac{2^{17}}{2^{17}-1}(1 + \sum_{n=1}^{\infty} \frac{1}{(2n+1)^{17}})$$

or

$$\zeta(17) = \frac{131072}{131071}(1 + \sum_{n=1}^{\infty} \frac{1}{(2n+1)^{17}})$$

n=1,　ζ (17)≒ 131072/131071*(1+1/3^17)=131072/131071*(1+1/129140163)=

=2^17/2^17-1*(1+1/3^17)=131072/131071*(1+1/129140163)=
1.0000076371963228089989……

…….

ζ (17) = 1.00000763719763789976227……

or

$$\zeta(17)=\frac{131072}{131071}(\frac{14348908}{14348907}+\sum_{n=1}^{\infty}\frac{1}{(2n+3)^{17}})$$

ζ (17) = 1.0000076371976378997622736002935630292...

when

$$\zeta(19)=\sum_{n=1}^{\infty}\frac{1}{n^{19}}=1+\sum_{n=1}^{\infty}\frac{1}{(2n)^{19}}+\sum_{n=1}^{\infty}\frac{1}{(2n+1)^{19}}$$

calculated

$$\zeta(19)=\frac{2^{19}}{2^{19}-1}(1+\sum_{n=1}^{\infty}\frac{1}{(2n+1)^{19}})$$

or

$$\zeta(19)=\frac{524288}{524287}(1+\sum_{n=1}^{\infty}\frac{1}{(2n+1)^{19}})$$

n=1, ζ (19)≒ 524288/524287*(1+1/3^19)= 524288/524287*(1+1/1162261467)=
1.00000190821266403655359801……..

…….

ζ (19) =1.00000190821271655393892…….

or

$$\zeta(19)=\frac{524228}{424227}(\frac{1162261468}{1162261467}+\sum_{n=1}^{\infty}\frac{1}{(2n+3)^{19}})$$

ζ (19) =1.0000019082127165539389 2…….

when

$$\zeta(21)=\sum_{n=1}^{\infty}\frac{1}{n^{21}}=1+\sum_{n=1}^{\infty}\frac{1}{(2n)^{21}}+\sum_{n=1}^{\infty}\frac{1}{(2n+1)^{21}}$$

calculated

$$\zeta(21)=\frac{2097152}{2097151}(1+\sum_{n=1}^{\infty}\frac{1}{(2n+1)^{21}})$$

or

$$\zeta(21)=\frac{2^{21}}{2^{21}-1}(1+\sum_{n=1}^{\infty}\frac{1}{(2n+1)^{21}})$$

n=1, ζ (21)≒ 2^21/(2^21-1)*(1+1/3^21)= 2097152/2097151*(1+1/10460353203)= 1.0000004769329846888538………

…….

ζ (21) =1.0000004769329867878064 6……

or

$$\zeta(21)=\frac{2096912}{2016911}(\frac{10460353204}{10460353203}+\sum_{n=1}^{\infty}\frac{1}{(2n+3)^{21}})$$

ζ (21) =1.0000004769329867878064 6……

when

$$\zeta(23)=\sum_{n=1}^{\infty}\frac{1}{n^{23}}=1+\sum_{n=1}^{\infty}\frac{1}{(2n)^{23}}+\sum_{n=1}^{\infty}\frac{1}{(2n+1)^{23}}$$

calculated

$$\zeta(23)=\frac{8388608}{8388607}(1+\sum_{n=1}^{\infty}\frac{1}{(2n+1)^{23}})$$

or

$$\zeta(23)=\frac{2^{23}}{2^{23}-1}(1+\sum_{n=1}^{\infty}\frac{1}{(2n+1)^{23}})$$

n=1, ζ (23)≒ 2^23/(2^23-1)*(1+1/3^23)=
8388608/8388607*(1+1/ 94143178827)=
1.000000119219925881388331 06………
…….
ζ (23) =1.0000001192199259653110 73…….

or

$$\zeta(23)=\frac{8387648}{8387647}(\frac{14143178828}{94143178827}+\sum_{n=1}^{\infty}\frac{1}{(2n+3)^{23}})$$

ζ (23) =1.0000001192199259653110 73…….

And the formula will be

$$\zeta(2m+1)=\sum_{n=1}^{\infty}\frac{1}{n^{2m+1}}=1+\sum_{n=1}^{\infty}\frac{1}{(2n)^{2m+1}}+\sum_{n=1}^{\infty}\frac{1}{(2n+1)^{2m+1}}$$

calculated

$$\sum_{n=1}^{\infty}\frac{1}{n^{2m+1}}=1+\frac{1}{2^{2m+1}}\sum_{n=1}^{\infty}\frac{1}{n^{2m+1}}+\sum_{n=1}^{\infty}\frac{1}{(2n+1)^{2m+1}} \quad (1)$$

$$\frac{2^{2m+1}-1}{2^{2m+1}}\sum_{n=1}^{\infty}\frac{1}{n^{2m+1}}=1+\sum_{n=1}^{\infty}\frac{1}{(2n+1)^{2m+1}} \quad (2)$$

$$\zeta(2m+1)=\frac{2^{2m+1}}{2^{2m+1}-1}(1+\sum_{n=1}^{\infty}\frac{1}{(2n+1)^{2m+1}})$$

n and m are positive integer.

$$\zeta(2n+1)=\frac{2^{2m+1}}{2^{2m+1}-1}(\frac{3^{2m+1}+1}{3^{2m+1}}(1+\sum_{n=1}^{\infty}\frac{1}{(2n+3)^{2m+1}})$$

n and m are positive integer.

【*References*】

1) https://en.wikipedia.org/wiki/Riemann_hypothesis

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☃～☃～☃～☃～☃～

I am a psychiatrist now and also a doctor of brain surgery before.

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I would like to receive an email. I will not answer the phone.

Currently 56 years old
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（I am very poor of English. Almost all document are google-translation.）
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But, I read letter by google translation.