Measurement Quantization Describes Galactic Rotational Velocities, Obviates Dark Matter Conjecture

Jody A. Geiger

Informativity Institute
30 E Huron Street
Chicago, IL 60611

E-mail: jodygeiger@informativity.org
Phone: (312) 898-7988
ORCID: 0000-0001-5389-0447

(Submission Date: November 25, 2018)

Abstract

A physical description of the orbital mechanics of stars around a galactic core has proved difficult. Notably, there is insufficient mass to account for observed stellar velocities. The mystery is one of few in modern science that defy the known laws of physics. In response, it has been conjectured that there exists a new form of matter that interacts gravitationally while otherwise remains undetectable. In this paper we resolve the mystery. The expressions do not modify the known laws of physics, contain no free variables or fitting and are entirely classical in nature. Using the notion of counts of the fundamental measures – length, mass and time – it is shown that our current understanding of mass is distorted. Accounting for this distortion reveals that the conjecture is unnecessary thus resolving the dark matter mystery.

1. INTRODUCTION

The initial focus of this paper will be a discussion of galactic rotation. It will be shown that physically significant bounds to measurement not only describe a smallest unit of measure, but also imply a greatest mass count per time count of those measures (i.e. commonly known as Planck's Units: length l_f, mass m_f and time t_f). Measurement bounds, in turn, are shown to constrain the effects of gravity at galactic distances. Thus, if the mass of a system is sufficiently large, the corresponding mass frequency bound will constrain the mass count of distinguishable events. The resulting gravitational effect is an invariant bounded velocity for stars at or greater than a specific distance from the galactic center. The radial crossover point is a function of both the total mass and how it is distributed.

With the evidence presented, the presence of dark or additional matter is no longer needed in what is a classical description of matter. Nevertheless, an analysis of each mass distribution will be discussed. The approach used will not use ΛCDM [1] to resolve the distribution values. Instead Informativity [2] is used; nevertheless the distributions are the same. The advantage of this approach is that Informativity allows us to inspect the individual distributions and gain a concise understanding of their physical traits and differences. Establishing an understanding of these distributions is paramount to providing a foundation with which to describe galactic rotation.

By integrating the effects of a mass frequency bound into Newton's expression we may then use a classical approach to describe the rotational velocity of stars around a galactic core. It will be shown that the predicted velocity curve matches the observational data with an average difference of 0.39% of the peak velocity.

The expressions are also modeled with an even mass distribution to demonstrate what an average of hundreds or thousands of galaxies would look like. As expected, the curve flat-lines. The magnitude of that velocity is directly correlated to the excess mass above and beyond the mass frequency bound.

Finally an analysis is presented demonstrating that the mathematical correlation between the dark matter distribution and universal expansion are equivalent, but not properly interpreted. A thorough exploration of the physical meaning of each of these phenomena is presented separating the two while at the same time bringing understanding as to how they became associated. We conclude by demonstrating that the principles of Informativity are sufficient to properly describe galactic rotational dynamics within the existing framework of classical mechanics.

2. METHODS

2.1. Fundamental Measures

The principles of Informativity rest on evidentiary support for the physical significance of fundamental units of measure. This property of observation differs significantly from what might be understood with respect to modern theory. That is, the fundamental units of measure do not imply that nature is discrete, only that measure – a property of observation – is discrete. Thus, while nature is infinitely divisible in length, mass and time, there are physically significant bounds to measure. And those bounds constrain the behavior of matter.

We will discuss the evidence only briefly and refer the reader to the paper *"Measurement Quantization Unites Classical and Quantum Physics"* [2] for a more complete treatment of the subject. We also refer the reader to the paper *"Measurement Quantization Unifies Relativistic Effects, Describes Inflation/Expansion Transition, Matches CMB Data"* [3] for examples of the application of measurement quantization to the distortion of measure, quantum inflation, the transition event that ends quantum inflation, initiates expansion and marks the formation of a Cosmic Microwave Background (CMB). For those familiar with these papers you may skip directly to Section 3.

We cite Heisenberg's Uncertainty Principle where applied to the position and momentum of a particle as one example of the physical significance of fundamental measure, in this case the significance of fundamental length. The expression, when reduced in its traditional form to the fundamental units of length l_f, mass m_f, time t_f, counts of those measures n_L, n_M, n_T and the length between a target and a center of mass n_{Lr} demonstrate that

$$n_M n_{Lr} n_L \geq l_f \ . \tag{1}$$

Thus, where we find physical support for the Heisenberg Uncertainty Principle, we also find l_f to be of physical significance, defining the threshold.

Evidence for the physical significance of quantized measure does not rest on one or even several examples in the experimental literature. There are, at present, more than 20 measurable, verifiable predictions of the model [2,3] in disciplines that include quantum physics, quantum gravity, classical physics, measurement (i.e. also described by relativity), quantum inflation, expansion and cosmology. One example, the measure of θ_{si} – an important new constant to physics that may be used to describe most of the existing constants – has been measured by Shwartz and Harris and published in their 2011 paper, *'Polarization Entangled Photons at X-Ray Energies'* in Physical Review Letters [4]. Using Informativity, their measures can be described to the same precision as shown in Table 1.

TABLE 1. Angle setting in radians of the **k** vectors of the pump, signal and idler for maximally entangled states at the degenerate frequency with corresponding Shwartz and Harris values (Ref. [5]).

Bell's State	k vector angle		
	θ_p	θ_s	θ_i
$(\lvert H_s, V_i> + \lvert V_s, H_i>)/\sqrt{2}$	$(l_f c^3/2G)-\pi$ (0.1208)	π - $(l_f c^3/2G)$ (-0.1208)	π - $(l_f c^3/2G)$ (-0.1208)
	2π - $(l_f c^3/2G)$ (3.02079)	$(l_f c^3/2G)$ (3.26239)	$(l_f c^3/2G)$ (3.26239)

With this and disciplines described in the first paper with respect to the formulation of expressions describing quantum gravity it may also be shown that the fundamental measures are related by the *fundamental expression*

$$l_f m_f = 2\theta_{si} t_f \ . \tag{2}$$

And with this and the associated nomenclature embraced in the Informativity approach, phenomena (exclusive of quantum mechanics) may be described with quantum accuracy.

2.2. Nomenclature

Informativity uses a distinct nomenclature to describe length, mass, time, unit counts of those measures and the measure of several other quantities that are valuable in the description of phenomena. Let us take this moment to discuss that nomenclature.

Where demonstrated in the initial publication of Informativity [2] that the fundamental measures are physically significant, the description of fundamental units with respect to the three measures are denoted as l_f for length, m_f for mass and t_f for time. In turn, a description of counts of the fundamental measures is denoted with the symbol n, each measure recognized by a corresponding capitalized subscript, L for length, M for mass and T for time. To avoid confusion between length descriptions of motion and those of gravitational fields, a subscript r (i.e., n_{Lr}) is used when describing a count of l_f between a static frame of reference and a center of gravity. Similarly, a subscript m (i.e., n_{Lm}) is used when describing a change in the count of l_f with respect to a target in motion to the observer.

With respect to those mass distributions associated with the universe there are several categories commonly discussed. The total mass of the universe, for instance, is represented with the symbol M_{tot}. The total may be divided into two parts, dark mass M_{dkm} and observable mass M_{obs}. The dark mass distribution is more commonly attributed to dark energy, but for reasons more clearly described in the first paper [2], a new term, dark mass, is used. Subtracting the visible M_{vis} from the observable gives you that which will be observed, the unobserved mass M_{uobs}, a distribution typically attributed to dark matter. There is also one more term, the fundamental mass M_f. This mass is associated with the mass frequency bound

$$\frac{1}{m_f} = \frac{1}{2.17643\ 10^{-8}} = 4.59468\ 10^{7} \text{ units s}^{-1}. \tag{3}$$

and is instrumental to the calculation of mass distributions in Informativity. While the distribution values are the same as those resolved with ΛCDM, the two approaches differ significantly. The Informativity approach is an outcome, a prediction of Informativity implicit to physically significant quantized measure.

Lastly, the expansion of the universe can be described with respect to two different measures. Stellar expansion, the measure of increasing distance between galaxies, follows the traditional understanding in modern theory. When discussing stellar expansion, we describe the effect using Hubble's constant H_o which is quoted in kilometers per second per megaparsec. Conversely, universal expansion H_U describes the expansion of the universe when defined with respect to the universe. SI units are used, but the reference is always fixed with respect to the age A_U and diameter D_U of the universe.

A listing of symbols used and there definitions may be found in Section 7.

2.2. Terminology

In our discussion, there are several terms that we should more thoroughly define with respect to the description of galactic properties. As we have recently introduced the nomenclature for describing expansion, consider now the expression. Specifically, such that universal expansion describes increasing space with respect to the universe ([2], Eq. 87) then

$$D_U = 2\theta_{si} A_U = 2 \bullet 3.26239 \bullet 13.799 = 90.035 \text{ bly}. \tag{4}$$

The expression can be modified to demonstrate that the rate of expansion in the local frame is

$$H = \frac{{}_{\text{km/Mpc}}}{A_U} = \frac{3.08567758\ 10^{19}{}_{\text{km/Mpc}}}{13.799\ 10^{9}{}_{\text{y}} \bullet 3.15576\ 10^{7}{}_{\text{s/y}}} = 70.860 \text{ km s}^{-1} \text{ Mpc}^{-1}. \tag{5}$$

And where defined with respect to the universe, expansion is an invariant measure and an innate property of the universe

$$H_U = 2\theta_{si} . \tag{6}$$

Resolving a description of phenomena with respect to the universe can often provide a perspective that is straight-forward and thus easy with which to build a cohesive understanding of many presently unsolved physical interactions on the cosmic scale.

While not as central to our discussion, it should also be noted that the *system constant* $2\theta_{si}$ is often present in physical description. The value is fundamental to the description of matter, an important feature regardless of the phenomenon under consideration. Whether quantum or cosmological in scale, $2\theta_{si}$ will often be found. In its most reduce form we may also describe the expansion of the universe with respect to measure or as a function of $2\theta_{si}$,

$$\left(\left(\frac{t_f}{l_f m_f}\right)^{1/3}\right)^2 + \left(\frac{n_{Lm}}{\mathrm{n_{Lc}}}\right)^2 = 1 , \tag{7}$$

$$\frac{1}{(2\theta_{si})^{2/3}} + \frac{n_{Lm}^2}{\mathrm{n_{Lc}^2}} = 1 . \tag{8}$$

Terminology in Informativity is often recognition of the many different forms of this expression, the *unity expression*. There are two classes; *relations* are expressions that may be reduced to the *fundamental expression* and *boundary expressions* are expressions that describe bounds between the fundamental measures (i.e. $c = l_f/t_f$) or counts thereof. A more thorough discussion of these two classes is provided in section 3.9 of the second paper [3].

If you are new to Informativity it should not go unnoticed as to what anchors measure, the fundamental measures – $(l_f m_f/t_f)=2\theta_{si}$ – or the corresponding rate of universal expansion $n_{Lm}/\mathrm{n_{Lc}}$. This is naturally a difficult inquiry as our understanding of measure is relatively defined. But their relation is fixed, marking $2\theta_{si}$ as perhaps the most fundamental of all constants. As demonstrated in the prior cited papers, many if not all of the known constants may be reduced to include only θ_{si}, the fundamental measures or counts thereof. By example, several are ([2], Eqs. 36, 49, 81)

$$\hbar = 2\theta_{si} l_f , \tag{9}$$

$$E_f = 2\theta_{si} l_f / t_f , \tag{10}$$

$$H_U = \frac{n_{Lu}}{n_{Tu}} c = 2\theta_{si} . \tag{11}$$

Conversely, the speed of light and the gravitational constant are *boundary expressions*, not *relations*,

$$\mathrm{c} = l_f / t_f , \tag{12}$$

$$G = \frac{l_f}{t_f}\frac{l_f}{t_f}\frac{l_f}{t_f}\frac{t_f}{m_f} . \tag{13}$$

Lastly, we commonly use the terms quantum and quantized throughout this paper. Neither should be understood as having a relation with respect to quantum mechanics. Rather, the term quantum is intended to mean small as in a few tens, hundreds or thousands of fundamental units of measure. The term quantized is intended to mean that expressions are composed of terms that are whole-unit counts of the fundamental units and that those units are physically significant.

A quantized expression possesses qualities that are immensely valuable in our effort to describe nature. For one, quantized expressions are defined for the entire measurement domain. Second, quantized expressions are nondimensionalized. Nondimensionalization is not in itself a valuable endeavor but demonstrating that all phenomena may be expressed entirely with nondimensionalized whole-unit counts of the fundamental measures contributes to a new understanding of measure that is finite and discrete.

A listing of terms used in Informativity may be found in Section 6.

3. RESULTS

In the sections that follow we will use Informativity to present expressions describing stellar motion within galaxies. As noted at the outset, an average of stellar velocities about the center of hundreds or thousands of their respective galaxies is invariant at a given radius and outward. The resulting velocity curve is in conflict with Newton's law of gravitation which describes a decreasing velocity with increasing distance.

A second anomaly concerns the magnitude of these velocities, a value that is significantly higher than expected. To describe these phenomena, incorporation of the effects of expansion and a new constraint to the behavior of matter will be entertained. While expansion is a seemingly straight-forward application, the constraint – mass frequency – is a new concept to modern theory. Like length frequency, $c=l_f/t_f$, mass frequency describes that bound where mass events may no longer be distinguished, greater than $1/m_f$. Even more applicable is the relation of mass to length, m_f/l_f, which can be used to describe the upper bound of mass events with respect to a three dimensional space.

Mass frequency bounds are physically significant and cannot be exceeded any more than a length frequency bound greater than 1 (i.e. $n_L/n_T>1 \triangleq c$). As we work through an understanding of mass frequency we will demonstrate how events in the local frame above and beyond this bound correspond to measure smaller than the fundamental units. Not only does a mass frequency above a frequency bound (i.e. a smaller value for m_f in the expression $1/m_f$) describe a point in space-time subject to indistinguishable events, it also describes a faster-than-light relationship between length and time, identifiable using the *fundamental expression*, $l_f m_f=2\theta_{si} t_f$ (i.e. a smaller value for m_f implies a larger value for l_f where $c=l_f/t_f$ then a faster-than-light relation).

3.1. Mass Distribution

Galactic star rotation follows classical theory with adjustments made for the effects described by relativity, the *Informativity differential* and universal expansion. To simplify the expressions, the first two effects will not be integrated into the results. The third effect, expansion, is a significant consideration with respect to galactic rotation and will be a part of the presentation. We begin with a brief review of expansion as described in the first paper [2].

Stellar expansion – the modern understanding of expansion – which is a function of universal expansion plus those forces of interaction since the earliest epoch will not be discussed. Universal expansion, conversely, describes only the increasing space in the universe defined with respect to the universe. The most significant quality of expressions defined with respect to the universe is that they are often invariant. The rate of universal expansion H_U for example is ([2] Eq. 81)

$$H_U = 2\theta_{si} . \tag{14}$$

The constant $2\theta_{si}$ is referred to as the *system constant*. With it universal expansion may be described using familiar terms ([2], Eq. 87) such as the diameter D_U of the universe in billions of light-years and the age A_U of the universe in billions of years.

$$D_U = 2\theta_{si} A_U = 2 \; 3.26239 \; 13.799 = 90.035 \text{ bly.} \tag{15}$$

A second expression ([2], Eq. 116) where the *system constant* appears follows from this axiom.

> $\boldsymbol{O_1}$**:** The same laws of motion apply to galaxies as apply to the universe.

Thus, the observable M_{Gobs} and the visible M_{Gvis} mass of a galaxy follow the same ratio as that which describes the universe

$$\frac{M_{Gobs}}{M_{Gvis}} = \frac{M_{obs}}{M_{vis}}. \tag{16}$$

In that the universal mass distribution ratio is equal to the *system constant* $2\theta_{si}$ as described in the first paper $M_{obs}=2\theta_{si}M_{vis}$ ([2], Eq. 116), it follows that the ratio of observable to visible galactic mass is

$$2\theta_{si} = \frac{M_{obs}}{M_{vis}} = \frac{M_{Gobs}}{M_{Gvis}}, \tag{17}$$

$$M_{Gobs} = 2\theta_{si} M_{Gvis}. \tag{18}$$

In more general terms, what we see is skewed by a factor of $2\theta_{si}$ relative to the universe. For a more complete list of conversions refer to Appendix 5.1.

There are several metrics that may be used to describe stellar rotation while at the same time incorporating mass and the effects of expansion. In this presentation, mass distributions appear the most straight-forward means of describing the relation between mass, galactic rotation and expansion. But, before we begin, we will need to regress briefly to discuss fundamental mass M_f, which is as instrumental to galactic rotation as it is to resolving mass distribution.

Understanding fundamental mass is all that is needed to resolve each of the mass distributions modern theorists presently use ΛCDM to resolve. Even more notable, Informativity does not depend on any free variables or experimental data to resolve these distributions. Each distribution is resolved mathematically as a necessary outcome of the physical significance of quantized measure.

As described in the first paper ([2], Eq. 93), the expression for fundamental mass is

$$M_f = A_U \theta_{si} \frac{m_f}{t_f}. \tag{19}$$

And finally the distributions for dark mass M_{dkm}, observable mass M_{obs}, visible mass M_{vis} and unobserved mass M_{uobs} are as described in the first paper ([2], Eqs. 109, 110, 113 and 115):

$$M_{dkm} = \frac{\theta_{si}^2 - 2}{\theta_{si}^2 + 2} = 68.3624\,\%, \tag{20}$$

$$M_{obs} = \frac{4}{\theta_{si}^2 + 2} = 31.6376\ \%, \tag{21}$$

$$M_{vis} = \frac{1}{2\theta_{si}} \frac{M_{obs}}{M_{tot}} = \frac{M_{obs}}{2\theta_{si}} = 4.84884\ \%, \tag{22}$$

$$M_{uobs} = M_{obs} - M_{vis} = 63.3624 - 4.84884 = 26.7888\ \%. \tag{23}$$

The graphical representation in Figure 1 provides additional detail on mass such that the age of the universe. 13.799 billion light-years [4] is taken as our most accurate measure of the universe and is the only value needed to resolve each mass distribution.

In the sections ahead we will present a more in-depth understanding of observable mass M_{obs} and what it means with respect to the behavior of galactic rotation, but for now note that M_{obs} describes the mass that can be observed but has not yet been observed because sufficient time has not elapsed for light from that mass to reach the observer.

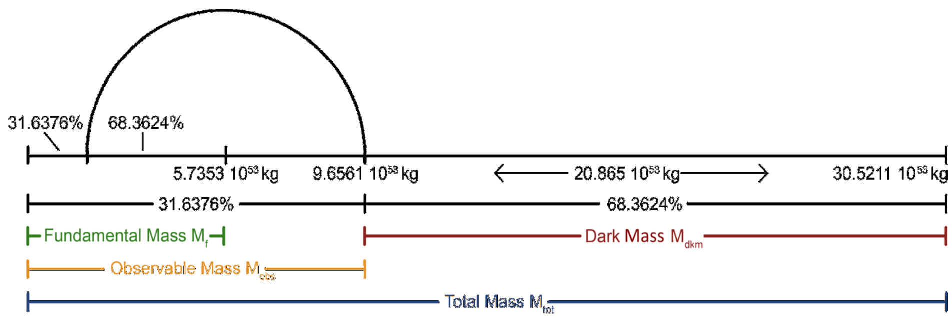

FIG 1. Relative measure of mass.

To provide a reference for galactic expressions presented throughout this paper we will use the Milky Way as an example. We will consider only the mass within the first 84,000 light-years. Corresponding values for each of the mass distributions are then

$$M_{Gvis} = 1.31201\ 10^{41}\ \text{kg}, \tag{24}$$

$$M_{Gobs} = 2\theta_{si} M_{Gvis} = 8.56060\ 10^{41}\ \text{kg}. \tag{25}$$

All mass, density and velocity data for the Milky Way comes from Stacy McGaugh's 2018 Milky Way mass model on which he bases several MOND investigations in the following cited paper [9].

3.2. Rotational Velocity Bounds

Count bounds are an important and physically significant attribute in describing the behavior of matter. The most well-known count bound we call length frequency $c = l_f/t_f$; for each count of fundamental time there can be at most one count of fundamental length. Any count of l_f greater than t_f would correspond to a velocity greater than the speed of light. The physical significance of fundamental units of measure is what distinguishes measurement quantization from an arbitrarily bounded mathematical framework. In light of the Shwartz and Harris results [5] (Table 1) among other predictions [2] measurement quantization has found six-sigma correlations in disciplines ranging from quantum physics to cosmology. The data has thus far supported the quantization of measure, physically significant references for which no smaller measure can be made.

Importantly, measurement quantization may be expanded to include m_f/t_f and m_f/l_f. We call these bounds respectively mass frequency and mass-to-length frequency. The rotational velocity of a star is subject to all three bounds in addition to the effects of expansion. A description may be reduced in several steps starting with the classical expression for rotational velocity,

$$v = \sqrt{\frac{GM}{R}} = \sqrt{\frac{l_f^3}{t_f^3}\frac{t_f}{m_f}\frac{n_M m_f}{n_{Lr} l_f}} = \sqrt{\frac{l_f^2}{t_f^2}\frac{n_M}{n_{Lr}}} = c\sqrt{\frac{n_M}{n_{Lr}}} \quad . \tag{26}$$

The expression reveals that rotational velocity is a product of c and the square root of mass count with respect to radial length count, the third bound mentioned above. You may recall from the second paper ([3], Eq. 80) that the upper mass-to-length count bound is

$$2n_M < n_{Lr} \,. \tag{27}$$

Therefore the count n_M of fundamental units of mass per count n_L of fundamental unit of length cannot exceed the ratio $n_M/n_{Lr}<1/2$. With this, consider now that the smallest count of m_f with respect to l_f may not be less than the fundamental measure m_f=2.17647 10^{-8} kg in SI units. In that fractional counts are in conflict with the definition of a reference we will need to convert this to a whole-unit count ratio. Multiply the numerator and denominator by the same value such that

$$\frac{2.17643\ 10^{-8}\ \text{units}\, m_f}{1\ \text{unit}\ l_f} = \frac{\text{units}\ m_f}{4.59468\ 10^{7}\ \text{units}\ l_f} = \mathrm{n_{Mb}} = m_f \,. \tag{28}$$

Also note, the final substitution of m_f for a count is a non-dimensionally valid assignment when defined with respect to a *self-defining* frame of reference (i.e. the universe). We refer the reader to Section 3.9 of the first paper [2] as a prerequisite to non-dimensionalized unit analysis for a description of the differences between *self-referencing* and *self-defining* frames of reference.

Accounting only for the mass frequency bound, Eq. (26) may be reduced such that the upper count bound of mass to length is 2 to 1 and for every count of m_f there may not be more than 4.59468 10^7 units of l_f (i.e. Eq. (28), $n_M/n_{Lr<}=m_f$). The latter bound is straight-forward while the prior would benefit from explanation.

Boundary expressions are *self-defining*, defined with respect to the diameter of the universe. Rotational velocity defines mass n_M with respect to 1/2 of the length count n_{Lr} (i.e. the radius). Thus, the mass frequency bound must be divided by two to reflect a radial frame of reference. We may expose that with the bound expression from Eq. (27) or by way of discussion here. Thus, the classical rotational velocity is then

$$v = c\sqrt{\frac{n_M}{n_{Lr}}} = c\sqrt{2m_f} \quad . \tag{29}$$

The expression is static with respect to the local frame. It must be adjusted to accommodate universal expansion, $H_U=2\theta_{si}$. The traditional form of H_U describes expansion with respect to the universe, again not a radial distance as is defined with respect to an orbital velocity. A rate of expansion inclusive of the diameter of the universe $2\theta_{si}$ needs to be divided by 2 to reflect a radial expansion.

Next, we emphasize that the rate of expansion θ_{si} is with respect to the *self-defining* frame (i.e. the universe). This does not imply that the view of a galaxy is skewed by θ_{si}. Rather the *self-defining* frame, the universe is expanding by θ_{si} in the radial direction. While the radial bound velocity expression may be applied to any scope, the principle used is a function of count bounds. It is a *self-defining* expression describing velocity in the local frame as an upper count bound between m_f and l_f. Thus, the expansion effect is a product of θ_{si} with respect to the static expression. It may be written in two ways; the *fundamental expression* may be used to convert between them:

$$v = \theta_{si} c\sqrt{2m_f} = 204.054 \text{ km/s}, \tag{30}$$

$$v = cm_f \sqrt{\theta_{si} c} = 204.054 \text{ km/s}. \tag{31}$$

This is the radial bound velocity corresponding to the upper bound frequency of mass events that may be discerned at a point in space.

To resolve where the radial bound and observable velocities intersect set the two expressions equal to one another. The intersection is instrumental, necessary to then apply adjustments, such as the effects of invariant mass densities respective of a target galaxy. Notably, we use the observable mass as opposed to the visible because the light from galaxies is often presented already reflecting the expansion effect.

> ***O_2*:** Visible mass is reflective of the light from mass that has arrived at a point in space. Observable mass also includes mass reflective of light that will arrive at some point in the future.

Reduced with the *fundamental expression* $m_f l_f = 2\theta_{si} t_f$, then

$$\theta_{si} c\sqrt{2m_f} = \sqrt{\frac{GM_{Gobs-f(R)}}{R_{Gobs}}}, \tag{32}$$

$$R_{Gobs} = \frac{GM_{Gobs-f(R)}}{2\theta_{si}^2 c^2 m_f} = c^3 \frac{t_f}{m_f} M_{Gobs-f(R)} \frac{1}{2\theta_{si}^2 c^2 m_f}, \tag{33}$$

$$R_{Gobs} = M_{Gobs-f(R)} \frac{l_f}{2\theta_{si}^2 m_f^2} = M_{Gobs-f(R)} \frac{t_f}{l_f m_f} \frac{l_f}{\theta_{si} m_f^2}, \tag{34}$$

$$R_{Gobs} = M_{Gobs-f(R)} \frac{t_f}{\theta_{si} m_f^3}. \tag{35}$$

Keep in mind, the galactic observable mass $M_{Gobs-f(R)}$ is a function of the mass within the considered orbital radius $f(R)$. Informativity introduces the additional constraint describing orbital motion with respect to the mass-to-length frequency bound m_f/l_f.

Given the low mass density of the universe, the universe will also not approach the bound. The mass total and corresponding density is considerable and for that reason solar systems also do not approach the bound.

$$M_{Gobs-f(R)} = \theta_{si} \frac{m_f^3}{t_f} R_{Gobs}. \tag{36}$$

By example, a system with a bulk of the mass in a radius of 84,000 light-years R=7.94157 10^{20} m would need more than

$$M_{Gobs-f(R)} = 4.95454 \; 10^{41} \text{ kg} \tag{37}$$

of mass, 2.49 10^{11} solar masses to display behavior associated with a measurement quantization bound. Such a mass reflects 2.49 10^{11}/8.5 10^{11}=29.3% of the mass of the Milky Way. Specifically,

$$\frac{M_{Gobs-f(R)}}{R_{Gobs}} = \theta_{si} \frac{m_f^3}{t_f} \tag{38}$$

describes the upper bound to measurable mass events unadjusted for total mass and density variation. If mass density exceeds this bound, the number of mass events will exceed the mass frequency m_f/t_f making those events indistinguishable.

Now consider what a higher or lesser frequency bound velocity implies. For one, given that $v_{Gobs}=(n_L l_f)/(n_T t_f)$, when the expression is organized such that

$$\frac{n_L l_f}{n_T t_f} = c\sqrt{\frac{n_M}{n_{Lr}}}\ , \tag{39}$$

$$\frac{n_M}{n_{Lr}} = \frac{n_L^2}{n_T^2}\ , \tag{40}$$

we see that the radial distance n_{Lr} is inversely proportional to the square of the length frequency n_L/n_T. Likewise, given that $v_{Gobs}=\theta_{si}c(2m_f)^{1/2}$ from Eq. (30) which is also $v_{Gobs}=\theta_{si}c(n_M/n_{Lr})^{1/2}$, then

$$\theta_{si} c\sqrt{\frac{n_M}{n_{Lr}}} = \theta_{si}\mathrm{c}\sqrt{2m_f}\ , \tag{41}$$

$$\frac{n_M}{n_{Lr}} = 2m_f = \frac{2}{1/m_f}\ , \tag{42}$$

$$\frac{n_M}{n_{Lr}} = \frac{2}{1/m_f} = \frac{2}{4.59460\ 10^7}\frac{\text{units}\ m_f}{\text{units}\ l_f}\ . \tag{43}$$

We recognize that the mass to radial distance ratio is constrained by the mass frequency bound (i.e. m_f=2.17647 10^{-8}). So, what does a greater or lesser velocity mean?

While any count of a fundamental unit must be a whole-unit count, it is possible to have fractional ratios, in this case the count of l_f with respect to the count of m_f. To give some context to this ratio, note that the count value n_{MGobs} is less than the mass frequency bound $1/m_f$=4.59468 10^7. Thus the physically significant count range is [1 – 4.59468 10^7]. Thus, a count ratio 100 units greater than this implies a corresponding speed of

$$l_f m_f = 2\theta_{si} t_f\ , \tag{44}$$

$$\frac{l_f}{t_f} = c = 2\theta_{si}\frac{1}{m_f} = 2\theta_{si}\left(4.59468\ 10^7 + 100\right) = 299{,}793{,}110\ \text{m/s}, \tag{45}$$

a 652 m/s increase above the speed of light. A same count increase above mass frequency would correspond to a radial bound velocity of

$$v = \theta_{si} c\sqrt{2\mathrm{n}_{\mathrm{Mb}}} = \theta_{si} c\sqrt{\frac{2}{\left(\left(1/m_f\right)+100\right)}} = 204.053\ \text{km/s}, \tag{46}$$

a decrease of 0.001 km/s. This does not mean that the rotational speed of a star may not fall below 204.054 km/s. The expression describes an upper bound with which to discern mass events in the local frame and as such an upper bound to the gravitational pull on a star. When the mass count of a galaxy exceeds the mass frequency bound, the observer is unable to distinguish additional events and as such the gravitational effect of mass on a star reaches a maximum.

Notably, this investigation also does not imply that stars cannot have velocities greater than 204.054 km/s. While the expressions are thus far invariant, we have not investigated the effects of different galactic mass totals or the effects of uneven mass distribution. This will be the subject moving forward.

3.3. Galactic Rotation Applied to the Milky Way

Given our current understanding of mass frequency bounds with respect to stellar rotational velocity, a formal expression may now be developed specific to the Milky Way galaxy. The relation follows the form of a mass distribution result a function of what we see, the visible mass. The relation must be adapted to our point-of-view. By example, a relation describing the unobserved mass distribution M_{uobs} (aka dark matter) with respect to the visible mass distribution M_{vis} follows this form, Eq. (A6), Appendix 5.1,

$$M_{uobs} = M_{vis}\left(2\theta_{si} - 1\right) . \tag{47}$$

Replacing the speed parameter θ_{si} provides us the final expression. But, a clear understanding of the physical characteristics of the replacement is difficult. For that reason, we will follow a longer algebraic solution that resolves the relative percent difference $\Delta\%_{a\text{-}b}$ between the actual v_a and the bound velocity v_b and then uses that to resolve Eq. (47) in terms of the effective and bound mass.

Take note, because universal mass distributions are defined with respect to the system diameter and radial velocity is defined with respect to a system radius, the percentage change expression $\Delta\%_{a\text{-}b}$ needs to be multiplied by 2. To reduce the expression, we will also need the mass corresponding to the mass frequency bound $M_{b\text{-}f(R)}$ from Eq. (36) and the bound velocity v_b from Eq. (30).

$$\Delta\%_{a-b} = 2\frac{\left(v_a - v_b\right)}{v_b} , \tag{48}$$

$$M_{b-f(R)} = \theta_{si}\frac{m_f^3}{t_f}R , \tag{49}$$

$$v_b = \theta_{si}c\sqrt{2m_f} . \tag{50}$$

With this we may resolve an expression for a mass density sensitive mass frequency bound, what we will hereafter refer to as the effective mass $M_{e\text{-}f(R)}$. Also, note that we will use the symbol *-f(R)* in subscript to indicate that the mass considered is only the mass within the target radius R from a galactic center. The effective mass is then

$$M_{e-f(R)} = M_{b-f(R)} + 2M_{b-f(R)}\Delta\%_{a-b} = M_{b-f(R)}\left(1 + 2\frac{\left(v_a - v_b\right)}{v_b}\right), \tag{51}$$

$$M_{e-f(R)} = M_{b-f(R)}\left(2\frac{v_a}{v_b} - 1\right). \tag{52}$$

When incorporating expansion we realize that the observer's view of the universe is skewed; the effect creates the appearance of more mass than is actually present. In Figure 2 actual $M_{a\text{-}f(R)}$, bound $M_{b\text{-}f(R)}$ and effective mass $M_{e\text{-}f(R)}$ are displayed.

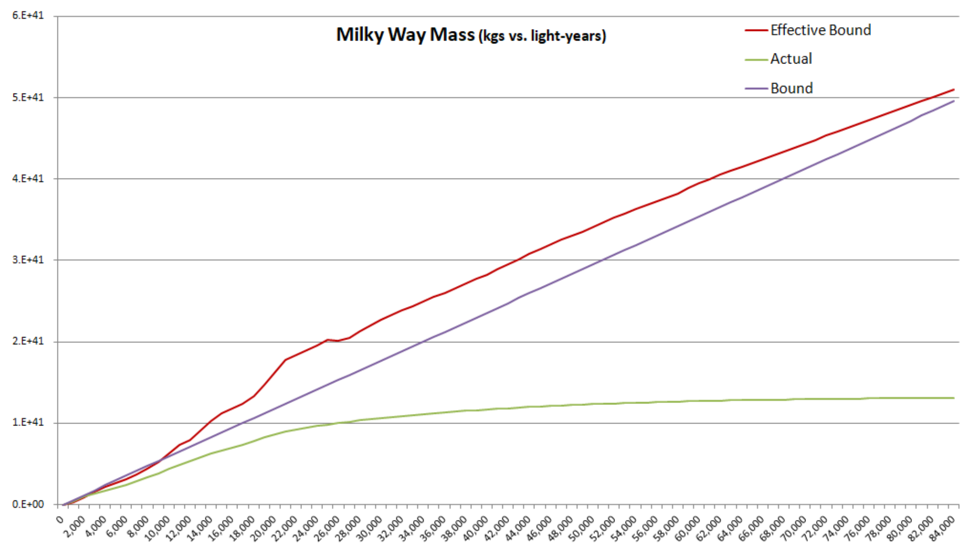

FIG 2. Galactic mass corresponding to actual (green), mass frequency bound (red) and relative mass frequency bound (purple).

Where the effective mass $M_{e\text{-}f(R)}$ is less than the bound $M_{b\text{-}f(R)}$, the rotational velocity of stars will follow a classical behavior. Conversely, an effective mass greater than the bound presents a number of mass events greater than the mass-to-length frequency bound. Some events will be indistinguishable leading to a constraining effect on gravity and corresponding stellar velocities. You will notice the crossover between these two behaviors occurs at 9 10^3 light-years.

Using Newton's expression for velocity, $v=(GM_{f(R)}/R)^{1/2}$, we may resolve effective stellar velocities v_e as a function of the effective mass,

$$v_e = \left(\frac{GM_{e-f(R)}}{R}\right)^{1/2} = \left(\frac{GM_b}{R}\left(2\frac{v_a}{v_b}-1\right)\right)^{1/2} , \tag{53}$$

$$v_e = \left(\frac{G}{R}\theta_{si}\frac{m_f^3}{t_f}R\left(2\frac{v_a}{\theta_{si}c\sqrt{2m_f}}-1\right)\right)^{1/2} , \tag{54}$$

$$v_e = 2\theta_{si}\left(2\frac{v_a}{\sqrt{2m_f}}-c\theta_{si}\right)^{1/2} . \tag{55}$$

While it may seem more appropriate to use a mass or mass density dataset the choice is irrelevant. One may modify the expression to enter velocity, mass or mass density and still arrive at the same expression. For example, written in terms of the actual mass $M_{a\text{-}f(R)}$ the expression becomes

$$v_e = 2\theta_{si}\left(2\sqrt{\frac{GM_{a-f(R)}}{2m_f R}}-c\theta_{si}\right)^{1/2} . \tag{56}$$

More importantly, using Newton's expression for velocity does not produce the observed velocity curve. Informativity succeeds because the expression for effective velocity is a function of the mass frequency bound, Eq. (30), an invariant expression with no free variables. To highlight that fact, we retain the corresponding bound velocity v_b in Figure 3 (purple) to demonstrate the natural tendency for stars to approach the bound when the number of mass events reaching a star exceeds the effective bound.

- $\boldsymbol{v_e}$: Effective Mass Frequency Bound Velocity *(red)*
- $\boldsymbol{v_b}$: Mass Frequency Bound Velocity *(purple)*
- $\boldsymbol{v_a}$: Actual Measured Velocity *(green)*
- $\boldsymbol{v_c}$: Classical Velocity *(blue)*

The remaining curves are defined as follows. The green line plots the measured stellar velocity of stars, what we actually see. The red line plots the effective velocity v_e. And the blue line plots the classical velocity as calculated with Newton's expression.

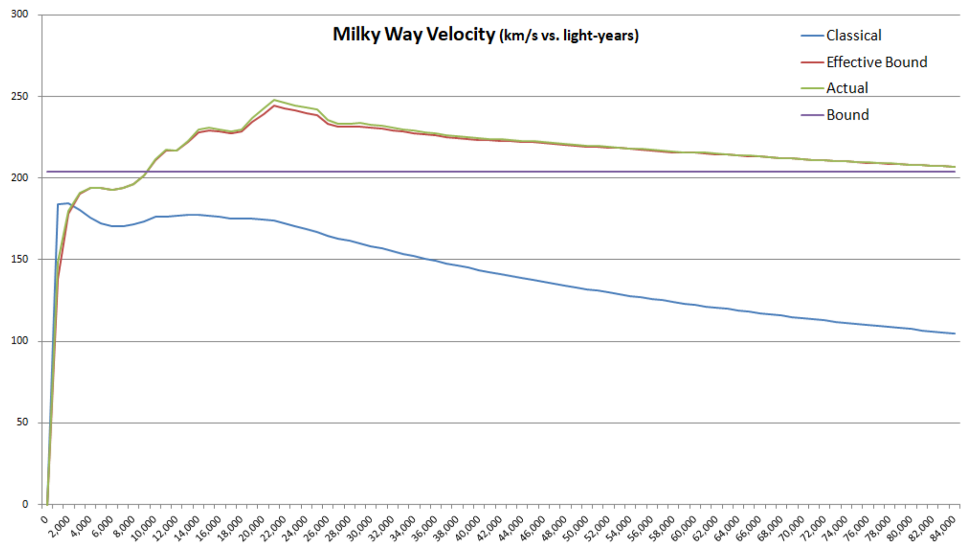

FIG 3. Stellar velocities corresponding to actual (green), relative mass frequency bound (red), mass frequency bound (purple) and Newton's expression (blue).

There are two points of view in conflict. That is, the classical velocity v_c implies that what we measure v_a is moving too fast. Stars should fly outward. Newton's expression suggests that there is a missing dark matter holding the stars in orbit. At the same time, the actual velocity v_a also suggests correspondence to variations in mass density.

The Informativity approach resolves the discrepancy describing an effective velocity v_e that follows the bound v_b when the effective mass $M_{e\text{-}f(R)}$ exceeds the bound mass $M_{b\text{-}f(R)}$. When $M_{e\text{-}f(R)}$ does not exceed the bound, orbital velocities follow a classical behavior.

Although the bound is invariant – 204.054 km/s – our point of view and variations in galactic mass density do affect the gravitational pull on a star. These effects may be mitigated when taking an average of thousands of galaxies. Except near the galactic core where the crossover between classical and Informativistic behavior varies from one galaxy to the next, rotational velocity flat lines canceling the individual variation in mass density between galaxies.

Notably, an unexpected effect of mass frequency is apparent between 4 and 8 thousand light-years where stellar velocities flat line until otherwise affected by increasing mass density. The exact cause of this effect is the subject of further investigation, but may favor a preference for classical behavior at the mass frequency crossover bound.

That said, the bound mass clearly delineates two distinct behaviors. Recall from Eq. (36), $M_{Gobs\text{-}f(R)} = \theta_{si} m_f^3 R_{Gobs}/t_f$ that the mass frequency bound is a function of how much mass is within a given radius. Variations in mass density imply increases or decreases in the spherical space described by R_{Gobs} for a fixed amount of mass. If we fix R in consideration of a region of greater mass density, then the effective velocity v_e will be higher, describing measured velocities that rise above the bound (i.e. 204.054 km/s). The opposite effect applies for less dense regions such that velocities lesson.

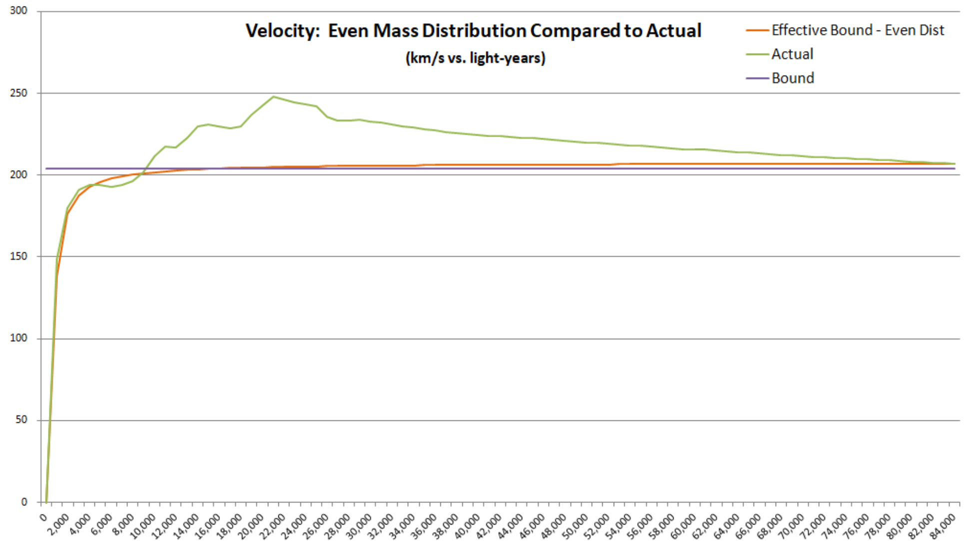

FIG 4. Stellar velocities corresponding to actual (green), an even mass distribution (orange) and the mass frequency bound (red).

To further demonstrate this effect, consider Figure 4 where a model galaxy with the same mass as the Milky Way is presented, but mass distribution has been evened as though we were averaging the mass distribution of thousands of galaxies. To be clear, a mass equal to that within the first thousand light-years of the center of the Milky Way is taken. Then the remaining mass (where the total considered is only the mass in the first 84 thousand light-years) is evenly divided between each of the remaining 83 thousand light-years. The corresponding effective velocity v_e (orange) is drawn. As expected, the curve levels out just above the bound velocity (purple) with a magnitude that increases in proportion to the excess mass above the bound mass. Thus, an average of thousands of galaxies will demonstrate a flat line velocity curve with a magnitude that corresponds to the average mass in excess of the bound mass.

As a final note, given such a close match to observational data, a difference of 0.39% of the peak velocity, additional review of the approach is prudent to rule out the possibility that the expressions are not a reflection of the mass density data.

To this end we note that separation of the velocity expression from the data can be challenging. Notably, it is the mass density data that characterizes the galaxy under consideration. By example, Newton's expression for rotational velocity, $v_c=(GM_{a\text{-}f(R)}/R)^{1/2}$ is a reflection of the radial mass $M_{a\text{-}f(R)}$. Given a radius R, the classical velocity v_c will always be an invariant result of the measured value for $M_{a\text{-}f(R)}$.

The argument may be extended to demonstrate that it is also irrelevant what dataset is chosen: mass, mass density or velocity. As each measure is mathematically related, an argument for data independence by favoring any dataset over another cannot be made.

But, there are two remaining properties that do support data independence. Notably, an expression must describe a phenomenon with the correct magnitude. The Informativity expression is in fact Newton's expression modified to accommodate the effects of a mass frequency bound in an expanding universe. Where Newton's expression does not provide the observed magnitude in describing rotational velocity, the Informativity expression does.

Also providing support is the bound itself, the purple line denoting an invariant velocity of 204.054 km/s. The bound expression contains no measurement data, no free variables and no 'fitting', $v_b=\theta_{si}c(2m_f)^{1/2}$. That is, the bound is a composite of constants. Referring to Figure 3, observed stellar velocities favor the bound. But, that will not always be clearly evident. What is clear is that the bound is the baseline measure from which the magnitude of Newton's expression is calculated. If the bound were not physically significant, the magnitude would be incorrect and the resulting curve would not match the observational data.

Finally, while the second expression to follow will not be discussed in detail until the next section, a comparison to the first equation as derived in Eq. (52) would lose much of its impact if not discussed presently. That said, the latter expression may be resolved with existing distribution data; M_{uobs}/M_{obs} is equivalent to $(2\theta_{si}-1)$. Using the *fundamental expression* $2\theta_{si}=l_f m_f/t_f$ along with $M_{obs}=2\theta_{si}M_{vis}$ reveals the latter, Eq. (58).

Thus, returning to our initial discussion, our goal was to develop a mass expression defined with respect to a bound. To this we can compare the effective mass expression with respect to the visible M_{vis} and unobserved mass M_{uobs} distributions. They match, each taking the form $M_1=M_2\,(2 \cdot$ speed parameter - 1).

$$M_{e-f(R)} = M_{b-f(R)}\left(2\frac{v_a}{v_b}-1\right), \tag{57}$$

$$M_{uobs} = M_{vis}\left(2\theta_{si}-1\right). \tag{58}$$

The details of the speed parameter depend on the masses being compared. In the case of rotational velocity, the parameter is found on the right-side of this relativistic relation ([3], Eq. 68)

$$\frac{n_{Lm}^2}{\mathrm{n}_{\mathrm{Lc}}^2} = 2\frac{n_M}{n_{Lr}} \tag{59}$$

which predicts and demonstrates equivalence in the distortion of measure between motion and gravity. Notably, multiplying the numerator and denominator of $n_{Lm}^2/\mathrm{n}_{\mathrm{Lc}}^2$ by $l_f/n_T t_f$ gives you the speed parameter $\beta^2=v^2/c^2$. Both the left and right terms anchor their denominator with respect to a count bound. We also note that β is a common construct notable within many expressions describing measurement change between an inertial frame and target.

Lastly, as an important reminder, M_{uobs} is the mass distribution commonly referred to as dark matter.

3.4. What Does the 26.7888% Distribution Describe?

Several observations over the years have been attributed to the phenomenon 'dark matter' [7]. It should be noted that there is no specific evidence linking a new form of matter – dark matter – to galactic rotational velocities. Nevertheless, the term has been retained in recognition of galactic rotational velocities that are inconsistent with classical physics. The idea that there may be missing dark matter has remained a viable candidate because stellar velocities are far greater than what the mass we can measure would explain.

In this section, we will discuss how and why the dark matter phenomenon has been so closely associated with the ΛCDM distribution also distinguished by the same name. We will not be using the ΛCDM approach or its expressions to discuss mass distribution but instead use the respective Informativity expressions which are measure independent and have no free variables.

With a model describing galactic rotation we ask, what does the 26.7888% mass/energy distribution represent? And, how did this distribution become so strongly associated with galactic rotation?

We may approach the subject using only the quantization of measure. Their definitions are presented in Eqs. (20-23). We begin with the unobserved mass

$$M_{uobs} = M_{obs} - M_{vis} = 31.6376 - 4.84884 = 26.7888\ \%, \tag{60}$$

which describes that mass which will be observed M_{uobs} ([2], Eq. 110) in the future, but is not presently visible M_{vis} ([2], Eq. 113). Subtracting the visible from the observable gives the unobserved distribution. The M_{uobs} distribution is not a new form of undetected matter. But the distribution is a function of expansion which is important to describing orbital dynamics. When comparing the dark to observable mass ratio with the inverse of expansion $H_U = 2\theta_{si}$, subtracted from the whole, we find the two are equivalent. This may be demonstrated algebraically using Eq. (21) $M_{obs}=4/(\theta_{si}^2+2)$, Eq. (60) $M_{uobs}=M_{obs}-M_{vis}$ and Eq. (22) $M_{obs}=2\theta_{si}M_{vis}$,

$$\frac{M_{uobs}}{M_{obs}} = \frac{M_{obs} - M_{vis}}{M_{obs}} = 1 - \frac{M_{vis}}{M_{obs}} = 1 - \frac{M_{vis}}{2\theta_{si} M_{vis}} = 1 - \frac{1}{2\theta_{si}}. \tag{61}$$

As discussed in Eq. (58), substituting $M_{obs}=2\theta_{si}M_{vis}$ from Eq. (17) reveals that

$$M_{uobs} = M_{obs}\left(1 - \frac{1}{2\theta_{si}}\right), \tag{62}$$

$$M_{uobs} = M_{vis}\left(2\theta_{si} - 1\right). \tag{63}$$

The unobserved mass M_{uobs} is the dark matter distribution, equal to what we see M_{vis} times the rate of expansion $H_U - 1$. Using the *fundamental expression* $(l_f m_f/t_f)=2\theta_{si}=H_U$, this may be written explicitly as

$$M_{uobs} = M_{vis}\left(H_U - 1\right). \tag{64}$$

The details as to how these two phenomena have been confused are specific to each case, but the effects of expansion may be safely separated from the measure of dark matter which is just matter that has not yet been observed. The association, while mathematically equivalent has led to a misunderstanding of the physical processes at work in galactic orbital dynamics.

Note also, Informativity does not imply that the mass we can measure in a galaxy is all the mass present. There are studies that suggest there exists additional non- or low-light-absorbing fine dust [8]. Gravitational lensing studies also demonstrate mass above that which has been measured in the visible spectrum. And while there is a great deal to learn about galactic mass, Informativity constrains the magnitude of this mass to the observable mass distribution M_{obs}.

3.5. Kinetic Energy

As a follow up to mass frequency, we may further confirm our understanding of $n_M/n_{Lr}=2m_f$ by reducing the classical expression to demonstrate the equation for kinetic energy. Notably, the classical expression does not include the radial expansion parameter θ_{si}. So, we start with the static radial form. Such that $m_f=2\theta_{si}/c$ from the *fundamental expression* and the fundamental unit of energy $E_f=2\theta_{si}c$ ([2], Eq. 49) which describes the energy of one fundamental unit of mass, then the static bound velocity is

$$v = c\sqrt{2m_f} = c\sqrt{\frac{4\theta_{si}}{c}} = \sqrt{4\theta_{si}c} = \sqrt{2E_f} \ . \tag{65}$$

The generalized expression such that $n_M <= (1/m_f)$ is

$$v = c\sqrt{2m_f} = c\sqrt{\frac{2}{1/m_f}} = c\sqrt{\frac{2}{n_M}} = \sqrt{\frac{2c^2}{n_M}} \ , \tag{66}$$

$$v = \sqrt{\frac{2\theta_{si}}{c}\frac{2c^2}{n_M m_f}} = \sqrt{\frac{4\theta_{si}c}{n_M m_f}} = \sqrt{\frac{2E}{m}} \ , \tag{67}$$

and may then be reduced to resolve the kinetic energy associated with any mass,

$$E = \frac{mv^2}{2} \ . \tag{68}$$

Naturally, one may compare the first and last velocity expressions and wonder why the latter has a mass value in the denominator. The mass value is what generalizes the expression for any mass, velocity and energy. The initial expression is invariant, a description of the smallest unit of energy E_f corresponding to a mass count bound $n_M=1/m_f$. That ratio is precisely 1 leaving us with $2E_f$ under the square root operator.

4. Discussion

Within this paper are presented expressions that describe the rotational velocity of stars around a galactic core. A more concise description is provided delineating the classical behavior described by Newton's expression from the flat-line velocity curve more commonly attributed to dark matter. Up to this point, it has been conjectured that an undetected form of dark matter is needed to properly describe the orbital dynamics of galactic rotation. With Informativity, we present an understanding of mass frequency and how that bound should be applied to Newton's expression in turn resolving the correct velocities of stars.

In concluding remarks, these principles are then applied to existing Milky Way velocity models demonstrating a 0.39% difference from peak velocity. In later discussions it is also shown that the magnitude of the dark matter distribution is precisely one-and-the-same as a similar ratio describing galactic expansion. The correlation has led to an inappropriate association of the ΛCDM dark matter distribution with the galactic rotation phenomenon. Expressions are presented clearly defining the physical processes associated so that readers may separate each effect appropriately.

In conclusion, it is proposed that the principles of Informativity are sufficient to properly describe galactic rotational dynamics within the existing framework of classical mechanics.

5. Appendix

5.1. Mass Distribution Conversions

From time to time one may need to resolve a mass distribution from knowledge of another mass distribution. To facilitate that interest here are several conversions. Several are already resolved from the first paper ([2], Eqs. 113, 110, 109 and 108). Notably, many of the expressions in the first paper are percentage expressions of a total mass. To resolve distribution values in kilograms, multiple the distribution percentage by M_{tot}.

$$M_{obs} = 2\theta_{si} M_{vis} , \tag{A1}$$

$$M_{obs} = M_{tot} \frac{4}{\theta_{si}^2 + 2} , \tag{A2}$$

$$M_{dkm} = M_{tot} \frac{\theta_{si}^2 - 2}{\theta_{si}^2 + 2} , \tag{A3}$$

$$M_{tot} = M_{obs} + M_{dkm} , \tag{A4}$$

$$M_{uobs} = M_{obs} - M_{vis} . \tag{A5}$$

We may resolve these

$$M_{uobs} = M_{vis} \left(2\theta_{si} - 1 \right) , \tag{A6}$$

$$2M_{tot} = M_{vis} \theta_{si} \left(\theta_{si}^2 + 2 \right) , \tag{A7}$$

$$M_{dkm} = M_{obs} \frac{\left(\theta_{si}^2 - 2 \right)}{4} , \tag{A8}$$

$$M_{dkm} = M_{vis} \frac{\theta_{si} \left(\theta_{si}^2 - 2 \right)}{2} . \tag{A9}$$

And from the first paper ([1], Eq. 118) we may also resolve

$$2M_{tot} M_f = M_{obs} (M_{tot} + M_f) , \tag{A10}$$

$$M_{tot} M_f = \theta_{si} M_{vis} (M_{tot} + M_f) , \tag{A11}$$

$$M_f = \frac{M_{tot} \theta_{si} M_{vis}}{M_{tot} - \theta_{si} M_{vis}} . \tag{A12}$$

We may also derive the relationship between the total and fundamental mass using the following relation. Using the expression for total mass ([2], Eq. 134) and the expression for fundamental mass ([2], Eq. 128) from the first paper we may demonstrate the relation between each as

$$M_{tot} = n_{Tu} m_f \frac{\theta_{si}^3}{2} , \tag{A13}$$

$$M_f = n_{Tu} m_f \theta_{si} , \tag{A14}$$

$$M_{tot} = M_f \frac{\theta_{si}^2}{2} , \tag{A15}$$

$$\frac{M_f}{M_{tot}} = \frac{2}{\theta_{si}^2} . \tag{A16}$$

Before concluding it should be emphasized that distribution expressions may often take on a percentage value or mass depending on their frame of reference. An expression demonstrating percentages only may be converted to kilograms by multiplying the result by M_{tot}. To demonstrate the issue more clearly, consider Eq. (17) when modified into the following form,

$$2\theta_{si} = \frac{M_{obs}}{M_{vis}} , \tag{A17}$$

$$\frac{2}{\theta_{si}^2} = \frac{1}{\theta_{si}^3}\frac{M_{obs}}{M_{vis}} . \tag{A18}$$

Then set the two expressions equal to one another to get

$$\frac{M_f}{M_{tot}} = \frac{1}{\theta_{si}^3}\frac{M_{obs}}{M_{vis}} = \frac{1}{\theta_{si}^3}\frac{2\theta_{si}M_{vis}}{M_{obs}/2\theta_{si}} = \frac{4}{\theta_{si}}\frac{M_{vis}}{M_{obs}} , \tag{A19}$$

$$\theta_{si}M_{obs}M_f = 4M_{vis}M_{Tot} . \tag{A20}$$

And finally where

$$M_{tot} = n_{Tu}m_f\frac{\theta_{si}^3}{2} \tag{A21}$$

is a known function of time ([2], Eq. 134), we may reduce Eq. (A20) such that time is the only free variable.

$$M_{obs} = 2n_{Tu}m_f\frac{\theta_{si}^3}{\theta_{si}^2+2} . \tag{A22}$$

With elapsed time n_{Tu} one might assume that the observed mass distribution M_{obs} is increasing. This is not a complete picture. The observed and total mass (A21) are both increasing while the distributions remain invariant,

$$M_{obs} = \left(n_{Tu}m_f\frac{\theta_{si}^3}{2}\right)\frac{4}{\theta_{si}^2+2} , \tag{A23}$$

$$M_{obs} = M_{tot}\frac{4}{\theta_{si}^2+2} . \tag{A24}$$

The result was demonstrated in the first paper ([2], Eq. 110).

Lastly, we have the observable v_{obs} and visible v_{vis} velocity, the prior being $2\theta_{si}$ times the visible as described in Eq. (18). Each corresponds to a radial mass such that the observable mass is what we measure in whole.

$$\frac{v_{obs}}{v_{vis}} = \frac{\sqrt{GM_{Gobs-f(R)}/R}}{\sqrt{GM_{Gvis-f(R)}/R}} , \tag{A25}$$

$$\frac{v_{obs}}{v_{vis}} = \sqrt{\frac{M_{Gobs-f(R)}}{M_{Gvis-f(R)}}} = \sqrt{\frac{2\theta_{si}M_{Gvis-f(R)}}{M_{Gvis-f(R)}}} = \sqrt{2\theta_{si}} . \tag{A26}$$

The visible is what we see presently. In terms of mass visible corresponds to the 4.84884% distribution as described in Eq. (22). The observable corresponds to the 31.6376% distribution as described in Eq. (21). Although the difference between each is less clear on a galactic scale, their role in describing rotational velocity is important. Notably, the observable mass incorporates universal expansion, $M_{obs}=H_UM_{vis}$.

5.2. Observable Mass as a Function of Time

Where

$$M_{tot} = n_{Tu} m_f \frac{\theta_{si}^3}{2} \qquad \text{(B1)}$$

is a known function of time ([2], Eq. 134), we may reduce Eq. (A10) such that time is the only free variable,

$$2M_{tot}M_f = M_{obs}(M_{tot} + M_f)\ , \qquad \text{(B2)}$$

$$2M_{tot}2\frac{M_{Tot}}{\theta_{si}^2} = M_{obs}(M_{tot} + 2\frac{M_{Tot}}{\theta_{si}^2})\ , \qquad \text{(B3)}$$

$$\theta_{si}^2 M_{tot}M_{obs} + 2M_{Tot}M_{obs} - 4M_{Tot}^2 = 0\ , \qquad \text{(B4)}$$

$$\theta_{si}^2 n_{Tu} m_f \frac{\theta_{si}^3}{2} M_{obs} + 2n_{Tu} m_f \frac{\theta_{si}^3}{2} M_{obs} - 4n_{Tu}^2 m_f^2 \frac{\theta_{si}^6}{4} = 0\ , \qquad \text{(B5)}$$

$$n_{Tu} m_f \frac{\theta_{si}^5}{2} M_{obs} + n_{Tu} m_f \theta_{si}^3 M_{obs} - n_{Tu}^2 m_f^2 \theta_{si}^6 = 0\ , \qquad \text{(B6)}$$

$$\frac{\theta_{si}^2}{2} M_{obs} + M_{obs} - n_{Tu} m_f \theta_{si}^3 = 0\ , \qquad \text{(B7)}$$

$$M_{obs}\left(\frac{\theta_{si}^2}{2} + 1\right) = n_{Tu} m_f \theta_{si}^3\ , \qquad \text{(B8)}$$

$$M_{obs} = 2n_{Tu} m_f \frac{\theta_{si}^3}{\theta_{si}^2 + 2}\ . \qquad \text{(B9)}$$

6. Glossary of Terms

Boundary Expressions
Expressions define invariant upper and lower bounds to length, mass, time frequency, and combinations thereof. Examples include $c=l_f/t_f$, $G=(l_f/t_f)^3(t_f/m_f)$, $\hbar=2\theta_{si}l_f$, and $H_U=l_f m_f/t_f$.

Crossover Radius
A specific radial distance from the center of a galaxy marking the point where the effects of mass exceed the mass frequency bound. Mass events in excess of this bound cannot all be distinguished as to do so would exceed the precision of the reference m_f. Some events appear as one limiting the effects of gravity.

Framework
A frame of reference against which a system of measure is applied. Frameworks are commonly discussed in Informativity and are typically either that of the observer's inertial frame, the observed target or that of the universe.

Fundamental Expression
The simplest expression correlating the three fundamental measures, $l_f m_f=2\theta_{si}t_f$.

Fundamental Mass

The fundamental mass of the universe distinguishes a specific amount of mass whereby from a point in space-time additional mass would cause overlapping mass events that could not be distinguished due to physically significant bounds to the measure of fundamental units of mass. Understanding and resolving fundamental mass in turn allows one to solve for all the mass distributions presently understood only with ΛCDM.

Fundamental Measure
One of the measures length l_f, mass m_f, and time t_f along with their correlation called the *fundamental expression*. Using measurement data from the Shwartz and Harris experiments in combination with Heisenberg's Uncertainty Principle, each are macroscopically defined and physically significant.

Informativity Differential
The Informativity differential $Q_L n_{Lr}$ describes a new form of length contraction associated with the lower bound to measure. The loss of immeasurable space at each increment of t_f describes gravity.

Measurement Distortion
A short-hand notation for the contraction and dilation of measure.

Observable Mass
The observable mass includes the mass which is visible in the present and the mass which will be visible at some point in the future. The observable mass represents all the mass that can be known in the universe. This is as opposed to mass that exists sufficiently distant that it is beyond the universal horizon and as such, due to the expansion of the universe, the light from that mass will never reach the observer.

Quantum
The term quantum is intended to mean a small measure such as a few tens, hundreds or thousands of fundamental units of measure.

Quantized
The term quantized is intended to mean that expressions are composed of terms that are whole-unit counts of the fundamental units and that those units are physically significant.

Relation Expressions
Any expression that may be reduced to the fundamental expression, $l_f m_f = 2\theta_{si} t_f$. Examples include universal mass distribution and the correlation of the diameter to the age of the universe.

Self-referencing
An expression defined with respect to the observer's inertial frame of reference.

Self-defining
An expression defined with respect to the universe as a frame of reference.

System Parameters
Any invariant value associated with a self-defining expression.

Visible Mass
The visible mass is that mass which is presently visible. In relation to the universe this would be the mass of those stars, dust or other forms of mass that are visible in the present as opposed to the mass corresponding to light that will be visible in the future.

Unity Expression

A self-defining Pythagorean expression with terms describing measurement bounds and a hypotenuse equal to 1.

7. Symbol Definitions

H_U is the expansion of the universe defined with respect to the universe (diameter). This differs slightly from stellar expansion (i.e. Hubble's description).
l_f, m_f and t_f are effectively Planck's Units for length, mass and time, but not precisely the same.
θ_{si}, is 3.26239 radians or kg m/s (momentum) or no units at all a function of the chosen frame of reference. This is a new constant to modern theory and exists in nearly every equation of the model. It may be measured macroscopically given specific Bell states necessary for quantum entanglement of X-rays such as those carried out by Shwartz and Harris.
$A_{s\text{-}ref}$ is the dilated age of the universe as measured from our point of view inside an expanding universe.
$A_{s\text{-}def}$ is the non-dilated age of the universe as would be measured if the universe were not expanding.
M_{vis} is the mass that is presently seen from a point in space.
M_{obs} is the mass that is presently or will eventually be seen from a point in space.
M_{dkm} is the mass that is beyond the observable mass, mass which will never be seen from a given point in space.
M_{uobs} is the mass that will eventually be seen from a point in space, but has not presently in view.
M_{tot} is all the mass in the universe.
M_f is the fundamental mass. Mass in excess of the fundamental mass exceeds the number of mass events per unit of time that can be distinguished at a point in space.
M_{acr} is the rate of mass accretion with respect to the universe.
$M_{a\text{-}f(R)}$ is the actual measured radial mass within a given radial orbit of a target galaxy
$M_{e\text{-}f(R)}$ is the Informativity effective radial mass within of a target galaxy. The value incorporates Newton's expression and the effects of universal expansion.
$M_{b\text{-}f(R)}$ is the Informativity mass frequency bound radial mass which corresponds to upper mass bound of mass events that equals but does not exceed the upper mass-to-length frequency bound.
$M_{\odot}$ is one solar mass.
V_U is the volume of the universe.
A_U is the age of the universe.
R_U is the radius of the universe.
D_U is the diameter of the universe.
H_U is the rate of universal expansion with units light-years per year.
n_{Mu} is a count of m_f equal to the total of mass/energy in the universe.
n_{Tu} is a count of t_f equal to the age of the universe.
n_{Lu} is a count of l_f equal to the diameter of the universe.
n_{Lo} is a count of l_f that is being observed.
n_{Lr} is a count of l_f from the observer to a center of gravity.
n_{Ll} is a count of l_f as measured in the local frame of reference.
n_{Tl} is the count of t_f as measured in the local frame of reference.
n_{To} is the count of t_f that is being observed.
n_{Lm} is the change in position of the target as a count of l_f as measured in the local frame of reference.
n_{Lc} is the change in position of light as a count of l_f as measured in the local frame of reference.
n_M is a count of m_f representing the mass corresponding to a gravitational field.
n_L is a count of l_f representing the length between an observer and the target.
n_T is a count of t_f representing the time elapsed between two events.
n_{Lf} is a known count of l_f typically used when describing distance with respect to an observer.
Q_{Lf} is the fractional portion of a count of l_f when engaging in a more precise calculation.
b_{Lf} is a known distance, a count of the reference $l_{f.}$
r_{Lf} is the unknown count of l_f between a center of gravity (the target) and the observer.

v_n is the radial velocity of a star plotted with respect to Newton's expression for gravity
v_a is the actual measured radial velocity of a star when accounting for all well-established effects
v_e is the Informativity effective velocity of a star in orbit around a galactic core. The expression may resolved using Newton's expression and the effective radial mass for a given radius.
v_b is the Informativity mass frequency bound velocity which corresponds to upper mass bound of mass events that equals but does not exceed the upper mass-to-length frequency bound.
G is Newton's gravitational constant.
S is the symbol assigned to the unknown constant when resolving a description of gravity. The symbol is replaced with θ_{si}.
c is the speed of light which may also be written as $c=l_f/t_f$.
v is velocity measured between an observer and a target.
r is some unknown distance between an observer and a target.
h is Planck's constant adjusted to reflect the quantum effects of the *Informativity differential*.
$\hbar$ is Planck's reduced constant adjusted for the Informativity differential as a function of distance to target.
σ_x is a description of the uncertainty in the position of a particle
σ_P is a description of the uncertainty in the momentum of a particle
k is the Boltzmann constant.
ρ is the energy density of mass/energy accumulated at a given age of the universe.
a is the total energy radiated as described with respect to blackbody radiation (i.e. the Stefan-Boltzmann law).
T is the temperature of the Cosmic Microwave Background

ACKNOWLEDGMENTS

We thank Edanz Group (www.edanzediting.com/ac) for editing a draft of this manuscript.

REFERENCES

[1] Lonappan, Anto, Ruchika, Sen, Anjan: Is it time to go beyond CDM universe? (2017) https://arxiv.org/abs/1705.07336
[2] Geiger, J.A.: Measurement Quantization Unites Classical and Quantum Physics, Journal of High Energy Physics Gravitation and Cosmology. Vol. 4, No. 2, 262–311 (Apr 2018). https://doi.org/10.4236/jhepgc.2018.42019
[3] Geiger, J.A.: Measurement Quantization Unifies Relativistic Effects, Describes Inflation/Expansion Transition, Matches CMB Data, Journal of High Energy Physics Gravitation and Cosmology. Vol. 4, No. 4 (Oct 2018). https://doi.org/10.4236/jhepgc.2018.44038
[4] Clabon Walter Allen; Arthur N. Cox, Allen's Astrophysical Quantities, Springer, p. 294 ISBN 0-387-98746-0
[5] S. Shwartz and S. E. Harris, Polarization Entangled Photons at X-Ray Energies, Phys. Rev. Lett. 106, 080501 (2011), arXiv: 1012.3499.
[6] McGaugh, S.S.: Milky Way Mass Models and MOND (2008) ApJ, 683, 137-148
[7] Hossenfelder, Sabine, McGaugh, Stacy, Is Dark Matter Real?, Scientific American (2018) https://www.scientificamerican.com/article/is-dark-matter-real/
[8] Mathew Madhavacheril, et. al., Evidence of Lensing of the Cosmic Microwave Background by Dark Matter Halos, Physical Review Letters 114, 151302 (April 13, 2015)