

# Stochastic space-time and quantum theory: Part C: Five-dimensional space-time

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This is a continuation of Parts A[1] and B[2], which described a stochastic, granular space-time model. In this, Part C, in order to tessellate the space-time manifold, it was necessary to introduce a fifth dimension which is 'rolled up' at the Planck scale. The dimension is associated with mass and energy (in a non-trivial way). Further, it addresses other problems associated with the granular space-time model.

## I. INTRODUCTION

A previous paper, 'Stochastic space-time and quantum theory'[1], asserted that vacuum energy fluctuations imply mass fluctuations which then imply curvature fluctuations which implies fluctuations of the metric tensor. Taking the metric fluctuations as fundamental, a model of space-time was constructed where in the absence of mass space-time became not flat but stochastic. A derivation of the uncertainty principle for conjugate variables was obtained. A description of the spread of the wave function as well as the phenomenon of interference was also derived.

A stochastic space-time implies that the space-time manifold is fully and unambiguously covered (tessellated) by 'events' (points specified by an  $x, y, z,$  and  $t$ ). These events then migrate throughout the manifold. (In that model, there was no migration in the  $t$  coordinate.) But events are *points*. And since points have no extent, it is difficult to understand how the events cannot overlay other events, and how regions of the manifold cannot be left un-covered by events. 'Events' then, seemed the wrong model for stochasticity. What seemed to be needed was a model where the elements of the stochasticity had extent, that is to say, a granular space-time.

A subsequent paper, 'Stochastic space-time and quantum theory-Part B: Granular space-time'[2], gave a model for granularity. To make the model Lorentz invariant, the grains (which we call 'venues' to distinguish them from 'events') have a constant volume where the  $x, y,$  and  $z$  dimensions are, in the absence of mass, a Planck length. And for covariance reasons, migrations in time were also allowed. A venue then, is the smallest possible volume. Therefore it can hold only one indivisible particle (such as a quark, perhaps).

In our interpretation, a charged particle in stochastic motion does not radiate because it is the space-time rather than the particle which is stochastic. (This is in contrast to Nelson's formulation where the particle [when it's position is time-dependent] is simply posited not to radiate.)

Similarly, local to the particle, space-time is not stochastic. And there, a deterministic Lagrangian can be defined. That 'local to the particle space-time' coordinate system is covariant (as it is moving with the particle). From another coordinate frame (e.g. the laboratory frame) measurements on that local frame are subject to the intervening stochasticity (due to the stochasticity of the metric tensor), and because of that stochasticity, the measurements are also stochastic (and the measurements are contravariant [as can be seen by the raising of the covariant coordinates by the stochasticity of the metric tensor]).

For covariance one would like to treat time and space similarly. To do that, we then let the stochasticity apply to both space and time. This leads to an obvious problem: If a venue contains mass, then migrations can position the mass so it appears at multiple positions in space at the same time. Preventing this necessitates a change in how we view time. Consider the following graphic (showing successive  $x$  coordinate values of a moving particle represented by the black disk):



The particle initially at  $x=0$ , to  $x=1$ , then 2, then 3. (We consider space-time to be granular, hence the boxes.) There is a single instance of the particle.

Time is different:



The particle (at rest) at  $t=0$ , moves to  $t=1$ , etc. But when it goes from  $t=0$  to  $t=1$ , it also remains at  $t=0$ . There are now two instances of the particle, etc. So a particle at a particular time is still there as time advances, and the particle is also at the advanced time. We would like there to be only a single instance of the particle.

In the previous paper, we defined a new quantity to represent time,  $\tau$  (tau-time), that acts much like the usual time, but in accord with the first graphic, above. That is to say when the particle advances in time, it erases the previous instance. (' $\tau$ -time Leaves No Tracks').

The Von Neuman-Wigner interpretation of quantum mechanics[3] asserts that consciousness causes the collapse of the wave function. It is a viewpoint to which we do not subscribe. However, we do suggest that conscious-

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ness (particularly memory) informs our interpretation of time. Were it not for memory, the 'τ-time Leaves No Tracks' idea might seem more compelling.

There are still problems with the granular model (which are addressed in this paper). Perhaps the most fundamental problem with the granular hypothesis is the question of rigid body rotations: i.e. if a particle is rotating, any position at the radius of the particle must migrate at a (rotation) rate no slower than one Planck length per Planck time. If the the particle is rotating slowly enough that the radial migration rate is indeed one Planck length per Planck time, then for a position closer to the center, the geometry demands that that position is moving at less than a Planck length per unit Planck time. But by hypothesis there *is* (generally) no length less than the Planck length. And that would seem to make rigid rotations impossible.

Another issue is about what happens when the universe (or a local region within it) expands or shrinks. How can venues increase or decrease in number? Related to this is the question of the constancy of the infinitesimal volume element. The field equations of General Relativity state that interior to a mass ( $T_{\mu\nu} \neq 0$ ) the Ricci tensor is not zero, and so the volume element,  $\sqrt{-g}$ , is not conserved. This creates a problem with the tessellation of space-time.

To handle these (and other problems) we postulate a fifth dimension. It is 'rolled-up' at the Planck scale. Depending on circumstances, it is either space-like or time-like. When time-like, we call it 'υ' (upsilon). The Greek letter indicates that υ-time, like τ-time, 'Leaves No Tracks'. When space-like, we call it 'u'.

## II. TIME-LIKE OR SPACE-LIKE

We postulate a fifth dimension associated with mass. But is it space-like or time-like? The Space-Time-Matter consortium asserts it is space-like and the dimension *is* mass[4]. Were it time-like there would be difficulties. A characteristic of time is that it progresses. An identification of mass as the coordinate wouldn't work as the coordinate couldn't progress (unless the mass were continuously increasing).

But there are advantages to a time-like, mass-related dimension (see section III). And we can implement it by postulating that mass sets the relative rate of progression between the 4th and the 5th dimensional time (See III B). So, for example, if a venue contains no mass, the 5th dimensional time is (and remains at) zero. Increasing amounts of mass in a venue increases the rate of 5th dimensional time with respect to the forth. Since we regard the 5th dimension, υ, as rolled-up, one might visualize the situation by imagining a clock face. The more mass in a venue, the more rapidly the clock hands go around.

The different rates of the two times is analogous to Special Relativity where the rates of time between two

inertial frames in relative motion are dependent on their relative speed V (whereas the rates of the fourth and fifth dimensional time are dependent on the mass M).

Mass seems to come in chunks. Perhaps the smallest chunk is a quark. In any case, mass does not seem to have a continuous range of values. And if mass is a discrete quantity, then so too is υ-time. And it is not unreasonable to assume τ-time is discrete as well. This time discreteness could be the reason an orbital electron can transition 'instantaneously' between orbits.

However, the venue creation/annihilation mechanism (III D) only works with a space-like fifth dimension. How can we reconcile this?

First, we note that the fifth dimension is an indicator of mass and energy. But though inter-convertable, mass and energy are different. It would be good if the fifth dimension could indicate that difference.

We note also that in the Schwarzschild metric, as one goes through the event horizon, r becomes time-like and t, space-like.

So we postulate that in a venue containing mass, the fifth dimension (here υ) is time-like and continuously increases (as time, either τ or υ always increases). In a venue not containing a mass, the fifth dimension (here u) is space-like. If that venue contains energy, u is greater than zero. If not, u is zero.

## III. THE UTILITY OF A FIFTH DIMENSION

So then, a fifth dimension helps the model in many ways. In particular, it gives us:

- A- a way of achieving consistency with quantum mechanics and relativity,
- B- an argument for the constancy of the volume element,
- C- a mechanism for waves (without the need for forces) (and the 720 degree symmetry for Fermions),
- D- a way of creating and annihilating venues,
- E- a geometric interpretation of mass,
- F- the Kaluza-Klein[5] formalism for bringing electromagnetism into the stochastic space-time model,
- G- uniqueness of tessellation of the space-time by a regular honeycomb,

Below, we briefly expand on these topics.

### A. Consistency of the model with Relativity and Quantum Mechanics

In differential geometry, Loveridge[6] has pointed out that the Ricci tensor governs the evolution of a small volume element (i.e.  $\sqrt{-g}$ ) as it travels along a geodesic.

Following Loveridge, assume a very small spherical volume of dust  $o$  centered on point  $x^\mu(0)$  moving along a direction  $T^\mu$ . ( $T^\mu \equiv \frac{dx^\mu}{d\tau}$ ). One has that  $\frac{D^2}{d\tau^2}o - \frac{D_{flat}^2}{d\tau^2}o = -oR_{\mu\nu}T^\mu T^\nu$ , where  $D$  is the covariant derivative along the path. The equation applies for both three and four

dimensional volumes. The reason for subtracting the second term is that the choice of coordinates could give an apparent (not intrinsic) change of volume.

But in Special Relativity, the Ricci tensor is zero. Which means that the volume element,  $\sqrt{-g}$ , is invariant. (For Special Relativity, this is easy to see: In a Lorentz transformation, as the length shrinks, time expands to leave the volume unchanged.) In General Relativity, in empty space-time, while the Riemann tensor is not zero, the Ricci tensor is. So, in empty space (i.e. exterior to a mass), the volume element is also invariant.

But for quantum mechanics, our stochastic space-time model postulates that the volume element,  $\sqrt{-g}$ , is *non*-constant and proportional to  $\Psi^*\Psi$ . This would seem to be a contradiction.

To address this, what we would like is a mechanism where, in a region of space-time where the wave function  $\Psi$  does not vanish, the volume element is not constant. And, ideally, that mechanism would be a function of the space-time geometry.

We propose then, that there is a fifth dimension and that the five-dimensional volume element is *everywhere* constant (see **B** below). So when the fifth-dimension is not zero, the 4-volume element is different from what it would be if the coordinate *were* zero. One might expect that the fifth dimension coordinate is presumed to be zero except when in a mass or where the wave function  $\Psi$  is non-zero or where an electro-magnetic field is present. However, because of vacuum energy, all venues contain at least a very small mass equivalent.

The wave function itself is postulated to be proportional to the 4-dimensional volume element. And external to a mass, the General Relativity field equations, i.e.  $R_{\mu\nu} = 0$ , still hold (for both 4 and 5 dimensions, i.e.  $\mu$  and  $\nu$  range from 0 to 3, or from 0 to 4).

In order that the 5-dimensional line element not be observably different from the 4-dimensional line element, we adopt the Kaluza-Klein idea that the fifth dimension is 'rolled-up'. (Unlike with Kaluza-Klein, we posit a time-like dimension when a venue contains a mass, and otherwise, a space-like dimension. See **F** below.)

## B. Constancy of the Volume Element

While the global invariance of the five-dimensional volume could be seen as a simplifying and/or unifying concept, the idea seems important for being able to tessellate space-time with leaving holes in the space-time fabric.

It is difficult to fully cover space-time with venues where the venues do not have a constant volume; if a venue's volume were less than a neighbor's, the neighboring volumes would need to become larger. But, while this is possible, this is a stochastic space time model; i.e. the venues migrate throughout the space-time (and indeed they *are* the space-time). It is difficult to see how venues of different volumes could migrate while still tiling space-time.

In order that venues continue to be able to tessellate the space-time manifold, we postulate a fifth dimension where the volume element is *always* conserved, but in five dimensions. As in **A** above, that fifth dimension is not zero only where mass or energy (or  $\Psi$ ) is present. This is similar to the idea that the fifth dimension *is* mass, as proposed by Mashhoon & Wesson[7] and the Space-Time-Matter consortium[4]. Our model differs in that the fifth dimension is an *indicator* of mass (and energy), where their's *is* mass.

We need the fifth dimension to hold the 'overflow' from a single venue's 4-dimensional contraction. And as a venue's volume is at the Planck scale, the dimension needs to hold very little. It can be well represented by a rolled up dimension at a Planck length 'circumference'. E.g., if, say, the x coordinate were to go from 1 to zero, the fifth dimension would go from 0 to 1.

We could not find any other method for venues to tessellate space-time other than the idea of an invariant volume element made possible by a fifth dimension. In that sense, it was forced on us.

The invariant volume element has a value of 1 Planck length (for the x coordinate) times 1 Planck length (for the y coordinate) times 1 Planck length (for the z coordinate) times 1 Planck time (for the t [or  $\tau$ ] coordinate) times 0 Planck time (for the  $\nu$  or u coordinate). It is 0 for the  $\nu$  or u coordinate because it is a rolled up dimension with a circumference of 1 Planck time, and a coordinate value of 1 Planck time is the same as 0 Planck time.

## C. A Mechanism for Waves (without the need of forces), and the 720 Degree Fermion Symmetry (without recourse to spinors)

In the stochastic space-time model, the motion of an object can be broken down into two parts: motion *through* the space-time, and motion due to the migrations *of* the space-time. Analogously to the way a Brownian pollen grain moves under the collective collisions with water molecules, the more the object is at the quantum scale the more the motion is due to motions *of* the space-time. We expect then, that the *rotational* motion of an elementary particle is due mainly to the rotations of venues in the space-time manifold.

We begin by seeking a mechanism for generating a particle's rotational frequency (as a function of mass) with respect to  $\nu$  time (the fifth dimension time) and then seek a mechanism for  $\tau$ -time (the fourth dimension time).

From the laboratory frame, consider observing a distant venue containing a small mass.  $\nu$ -time at a venue is, by hypothesis, associated with the mass in a venue. In particular, the *increment* of  $\nu$  is so associated.

Masreliez[8] and Mukhopadhyay[9] among others have suggested that a mass oscillates at its Compton frequency, (and without such oscillation, there would be no DeBroglie wave, or indeed a  $\Psi$ ). We accept that suggestion. The Compton frequency,  $f_c$ , is defined as  $f_c = \frac{mc^2}{h}$

Hz.

We first convert Hz to cycles/Planck time.

$\frac{f_c}{\sqrt{\frac{khG}{c^5}}} = \frac{mc^2}{h}$  where  $k$  is a constant that equals either  $\frac{1}{2\pi}$  for reduced Planck units, or 1 for *unreduced* Planck units (The reason we do this will be seen soon.)

Now we'll convert  $m$  from kilograms to Planck mass,  $m_p$ .

$$\frac{f_c}{\sqrt{\frac{khG}{c^5}}} = \frac{mc^2}{h} \sqrt{\frac{khc}{G}}$$

Simplifying, we have  $f_c = km_p$ .

Depending on  $k$ , this gives either  $f_c = \frac{m}{2\pi}$  or  $f_c = m_p$ .

The  $k = 1$  case,  $f_c = m_p$ , is compelling. It implies that if the mass in a venue is zero, (from the viewpoint of the laboratory observer) the  $\upsilon$  time does not advance. The more mass in a venue, the more 'rapidly'  $\upsilon$  advances until at a maximum venue mass of one Planck mass, the frequency has increased to one cycle per Planck time. This also argues that the Planck units should be defined using Planck's constant rather than (as is usually done) the reduced Planck's constant,  $h/2\pi$ . Henceforth, we'll then use the unreduced Planck units.

'Time' then can be considered made up of two characteristics: a coordinate ( $t$ ) going from minus to plus infinity, and  $\upsilon$ , the fifth dimensional time, representing an ordering schema as described by H. Reichenbach[10]. Feasible time-like extra dimensions (in the context of Kaluza-Klein theory) have been discussed by Aref'eva[17] and Quiros[16] (among others).

The rolled-up fifth dimension has proven quite useful. As mentioned in Section II, the 5th dimension was invoked for the idea that the volume element is constant in five dimensions.

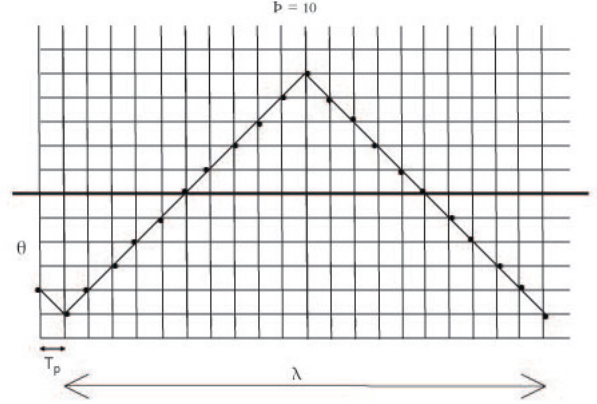
[Note: The idea of a constant volume element has an interesting corollary: In the presence of mass, external to the mass, the 4-dimensional volume element is also constant. And the evolution of the 4-dimensional volume element is governed by the Ricci tensor. So, if the volume element is constant, the Ricci tensor is zero, i.e.  $R_{\mu\nu} = 0$ , which are the mass-free General Relativity field equations. If then, Einstein had never achieved his theory of relativity, we would still have the empty-space field equations. Karl Schwarzschild could have taken it from there.]

We are used to the concept of complex phase. Perhaps that fifth dimension is represented by the complex phase in quantum wave equations.

We consider a space-time occupied by a single (indivisible) mass. We can impose a coordinate system centered on the mass. We consider its rotation. We take migrations (coin flips) for rotations about the three coordinate axes and (optionally) for time. Note that the particle is (in this model) rotating in all directions (reminiscent of spin).

In a mass, migrations in time allow an easy interpretation: A 'flip' from forward to backward in  $\tau$  time is evidenced by the space 'measures' ( $m_{x,y,z}$ ) (probabilities of moving clockwise) going from  $m$  to  $1-m$ .

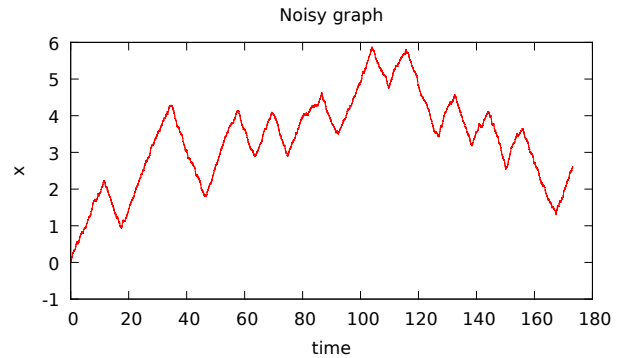
Consider the following idealized graph:



There is no reason the rate of space flips (here angle flips) and time flips must be the same. We define  $\mathbb{P}$  (Icelandic/Old-English 'thorn') as the ratio of space vs. ( $\tau$ ) time flips. In the graph, we, for example, have set the time measure  $m_t=0.5$ ,  $\mathbb{P}$  equal to 10 and the angle measure equal 1. The graph shows a typical cycle produced as follows: Assuming that initially,  $\tau$  time is progressing forward, at successive time steps  $t_p$  (unit of Planck time), the angle increases. When ten ( $\mathbb{P}$ ) steps have occurred, time has a 50 percent chance of flipping backwards. Here we assume it has done so. At that point, the angle measure goes to 0 (meaning that the angle continually decreases) until another 10 steps occur. This is an idealized case which we'll generalize further below.

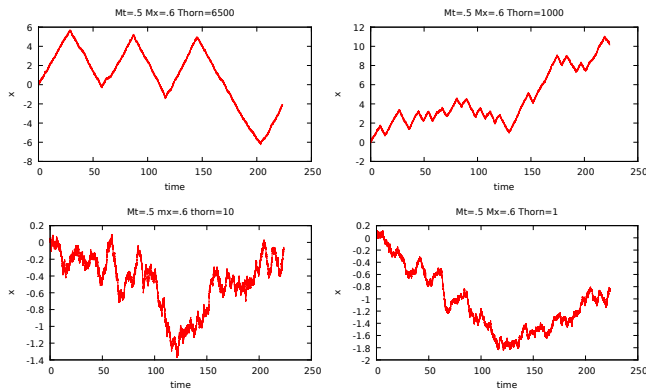
In the above graph, note that the wavelength is  $2\mathbb{P}$ . (1 cycle =  $2\mathbb{P}$ ) the frequency  $f$  is then  $1/(2\mathbb{P})$  cycles/Planck time.

In the graph below, we've let the angle measure be less than unity. This, of course has resulted in a noisier graph, but the above dominant wavelength is still evident.



As  $\mathbb{P}$  increases, the frequency of this (torsional) oscillation decreases until, as the mass goes to zero, the oscillations cease, leaving only the migrations of the center of mass. As  $\mathbb{P}$  decreases, the frequency increases and the graph gets noisier. We ask now when will the dominant frequency get lost in the noise. We expect that will happen when  $\mathbb{P}$  is at or slightly above 2 (because wavelength= $2\mathbb{P}$ ).

Below are some graphs showing how the frequency gets 'lost in the noise' as  $\mathbb{P}$  decreases:



As in the case of  $\nu$  time, we also assume that with  $\tau$  time the frequency is proportional to the mass.

The  $\nu$  time case had  $f_c = m_p$ . And that fit very well to the stochastic space-time model.

But the  $\nu$  time case is very different than that of  $\tau$  time: the  $\nu$  time argument is for rotations in a one degree of freedom plane whereas with  $\tau$  time, one has oscillations of a sphere, which is two degrees of freedom (latitude and longitude). And as rotations don't commute, it is hard to see how the rotations in the two degrees of freedom can occur at the same time. This might suggest that the  $\nu$  rotations go at twice the rate of the  $\tau$  oscillations, i.e. there are two cycles for  $\nu$  for every one of  $\tau$ . In this scenario, the Planck mass is the limiting mass for quantum mechanics i.e. particles heavier than the Planck mass behave classically, and, the  $\nu$  frequency is half the  $\tau$  frequency which says that the particle must rotate through 720 degrees before returning to its initial state.

This allows us to claim:

the Planck length is the smallest possible length,

the Planck time is the smallest possible time, and

the Planck mass is the smallest possible purely classical (i.e. not subject to quantum mechanics) mass. Further, it supports the argument that waves are intrinsic to this model.

As the mass increases, the waves' sub-harmonics grow in intensity until the composite wave is indistinguishable from the foam—resulting in a vanishing of the wave. So in contrast to conventional QM where a massive particle is presumed to have a wave function (that is perhaps beyond the current measurement threshold), if  $\Psi$  is intimately related to the Compton wave, our model predicts that there isn't a wave function for a sufficiently large, spherical (non-interacting) mass. And that limiting mass is the (very close to the) Planck mass. So, even in principle, the two slit experiment cannot be done with bee-bees, or marbles, or cannonballs.

The above was a simplification where time changed direction every  $\mathbb{P}$  Planck times. Our model though, says that every  $\mathbb{P}$  Planck times, a coin flip determines if time changes direction. The argument still holds as the signal still gets lost in the noise, but less smoothly.

The above takes the Planck length and time as the smallest possible in free space, i.e. the quantum of space and time. But what is the smallest possible mass, the

quantum of mass? The Planck mass is the upper bound for quantum masses. What is the lower bound? Our model can't say. But that mass must be smaller than anything in nature. The mass of a neutrino isn't known, but it is in the order of  $10^{-36}$ kg. The mass difference between the types of neutrinos will be smaller still. We can say then, that the quantum of mass is less than  $10^{-28}$  Planck masses.

If, reflective of venues, masses can oscillate and rotate, it is reasonable to expect that volume-preserving pulsations (like squeezing jelly-babies) also occur. But whereas rotations of a mass do not necessarily disturb nearby venues, pulsations propagate through space-time. Each mass in the space-time manifold can generate pulsations, causing a stochastic/chaotic foam in the space-time. This may well be the genesis of the vacuum energy.

As to forces, this model doesn't currently consider forces. It is concerned with 'stuff', which is composed of Fermions. The force carriers, i.e. Bosons, will be addressed in a future paper.

#### D. Creating and Annihilating Venues

A problem with a space-time of granules (with constant volumes) rather than of points is how to handle an expanding or contracting space-time or region of space-time. We need a mechanism to create and annihilating empty venues (venues not containing mass) without leaving gaps in the space-time manifold. Either the constants  $c$ ,  $G$ , and/or  $h$  depend on the size of the universe and so change the Planck units in such a way as to preserve the number of venues, or the Susskind[11] landscape model is applicable and in addition the overall volume of the multiverse is constant and venues can migrate between universes, or there is a mechanism for the creation and annihilation of venues. We suggest the following:

The 'rolled-up' (here, space-like since we're considering venues that do not hold mass) 5th dimension provides that mechanism as follows: Consider a space-time contraction in one direction ( $x$ ).

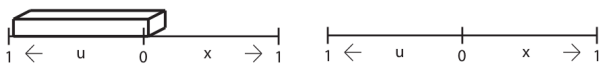
Initially the 5th dimension is zero as there is no mass in the venue. And as a result, there is no 5th dimension frequency.

The following diagrams show coordinates  $x$  and  $u$ , and a rectangular solid representing a venue.



As the  $x$  coordinate of the venue contracts, to preserve the 5-volume, the 5th dimension must expand.

At some point, the contraction coordinate,  $x$ , approaches zero while the 5th dimension,  $u$ , approaches its circumference.



At the point where the contraction reaches zero, the

5th dimension component 'rolls over' to zero. The 5-volume is then zero and the venue blinks out of existence.

Creation of venues is similar: When a venue (outside a mass) expands, it subtracts from the fifth dimension (which is slightly greater than zero due to vacuum energy).

When it subtracts that small vacuum energy mass, the fifth dimension 'rolls over' to its maximal value. This pushes the venue's 5-volume volume to an even larger value which violates the constant 5-volume hypothesis. The venue then splits in two. Each half contains half the maximal 4-volume and half the maximal dimension five value, again preserving the constant 5-volume of each new venue. The new venues equilibrate by a reduction of the fifth dimension and increase of the 4-volume thus accommodating the expansion (that required a new venue).

The vacuum energy is convenient as it provides a buffer against instantaneous venue creation in the case that fluctuations at an instant of time indicate the need for another venue.

At no point then, is the space-time manifold not fully tessellated.

In a sense, during an expansion of the space-time manifold, mass is converted to 4-volume.

However, expansion/contraction would occur not in a single coordinate direction, but in all four. But because venues are in a constant state of volume-preserving pulsations, at any instant, the expansion/contraction would be effectively in a single (albeit random) direction, so the above, single direction argument would still hold.

Although the mechanism allows for both venue creation and annihilation, perhaps only creation is relevant; Since venue migration is a diffusion process, if the universe is not infinite, the diffusion will increase the size of the universe. Which is to say, the granular, stochastic space-time model predicts an ever expanding universe.

### E. A Geometric Interpretation of Mass

Our stochastic space-time model doesn't attempt to say what mass is, but instead examine the geometric properties pertaining to the mass.

The original reason for associating mass with a fifth dimension was the argument in **A**: To maintain a constant 5-volume in (and only in) a venue with mass, the coordinate value of the fifth dimension would need to be non-zero. So the fifth dimension is *indicative* of mass. But unlike with the Space-Time-Matter Consortium[4, 7], we do not contend that the fifth dimension *is* mass.

The principal function of mass is (in the model) the stabilization of space-time, i.e, one would like the fluctuations in/of space-time not to rip apart masses. In particular, a mass causes adjacent mass-containing venues, because of stabilization, to act as a single larger venue (which is why  $E(\text{mass})=hf$  works).

In empty space, in particular, the venues' dimension coordinates fluctuate (and this is required for the cre-

ation and annihilation of empty venues). The fluctuating mass can be associated with vacuum energy fluctuations and metric tensor fluctuations. The idea of metric tensor fluctuations was the initial idea behind (Part A) stochastic space-time theory.

Since the fifth dimension is associated with mass and energy, venue fluctuations imply mass(energy) fluctuations. That is to say that energy is not strictly conserved. But since the fluctuations occur at Planck-time time scales, on short-time average energy effectively *is* conserved. However, because of the venue creation/annihilation mechanism (**D** above) during space-time expansion or contraction (either of the universe or a region thereof), energy is not conserved. It is conceivable that some of this energy of expansion/contraction can be extracted.

In (the usual interpretation of) General Relativity, mass causes 'curvature'. But what is curvature? Arguably, it is merely an artifact of describing space-time with one too few dimensions. For example, if a (two dimensional) ant were wandering on the surface of a sphere, he could measure curvature and determine that his environment was non-Euclidean. A three-dimensional (ignoring time) being would say the space(-time) *was* Euclidean and the ant was not able to see that third dimension. G. 't. Hooft[12] has made a similar argument, as has J. Beichler[13]. (And, of course, 'Campbell's Embedding Theorem'[14] states that any n-dimensional Riemannian manifold can be embedded locally in an n+1-dimensional Ricci-flat manifold.) But for us, rather than using a full extra dimension to explain curvature, we describe curvature as an artifact of the four-dimensional contraction or expansion of venue volumes, the 5-volume constancy provided by a rolled-up fifth dimension.

The idea that in a mass dimension 5 is time-like and associated with that mass is an attractive notion. If, as with  $\tau$ -time,  $\nu$ -time, goes forward, this gives a frequency proportional to the mass in the venue. This would explain  $e=hf$ .

The problem is that it works only with an isolated venue (and a venue is about 20 orders of magnitude smaller than the size of a proton). If adjacent venues have mass, each venue would have a frequency proportional to the mass in that venue. What we need is for the frequency to be the sum of the frequencies in each venue. We postulate then, that adjacent venues containing mass act as one larger venue (except that the venues may still be able to migrate). This encapsulates the idea that mass stabilizes space-time. The postulate might be a result of forces (the strong force, in particular) between the mass-containing venues. (Forces are outside the scope of this paper.)

Note: As for the translatory motion of a mass (as opposed to the rotational), the mass doesn't become 'fuzzy', but (because of migration external to the mass) its location does begin to blur as the mass decreases below the Planck mass. This results in an effectively larger mass diameter.

Two effects: like a smaller pollen grain in Brownian motion: the smaller the grain, the more it stochastically moves. But as the effective grain radius increases, the movement decreases as there is a larger circumference over which the movements can average.

Note then that the effective radius *rate* of increase decreases as the effective radius increases. To reiterate, this is because, as the particle grows in *effective* size the average effect of the venue migrations against the particle surface begin to average out (analogous to the case of Brownian motion where the jitter of a large pollen grain is less than that of a smaller grain).

We maintain that where quantum mechanics uses the radius, it should use the *effective* radius.  $r = r_c + r_{qc}$ .

One might consider the 'actual' radius as the covariant (and hence unobservable radius) whereas the effective radius is the contravariant (in principle, observable) radius.

A quantum particle moving with venues effectively spreads. So, in some sense, the mass is spread through the space-time. And the field equations act on the spread mass. And (since inside a mass, the Ricci tensor is not zero) the space-time near a *quantum* particle has a non-constant 4-volume element.

In short then, there is relationship between a particle's mass and its radius; the higher the mass, the shorter the radius.

#### F. A Note on the Geometric Interpretation of Electromagnetism (Kaluza-Klein)

Theodor Kaluza and Oskar Klein's not entirely unsuccessful unification of General Relativity and electromagnetism is well-trod territory (e.g. see Beichler[13], Dongen[15], Halpern[5], and Quiros[16]). Kaluza-Klein theory is not, per se, part of our Stochastic, Granular Space-time model. But the Kaluza-Klein model and our Stochastic Space-time model both employ a fifth dimension that is rolled-up at the Planck scale. So, for free so to speak, our Stochastic Space-time model can take in electromagnetism.

There is a difference, however. The (usual) Kaluza-Klein fifth dimension is space-like whereas ours is sometimes time-like. Researchers have considered a time-like rolled up fifth dimension, but rejected it due to the prediction of tachyons and (bad-)ghosts (unphysical solutions, e.g. negative probability states).

Other more recent work (e.g. Aref'eva & Volovich[17], Quiros[18], Kociński & Wierzbicki[19]) suggests that a time-like dimension can work after all, if one has constraints imposed on the dimension.

We have also had the tachyon problem when we allowed both space and time migrations (See the Part B paper). The 'time leaves no tracks' interpretation of time was postulated to prevent particles to be at different places at the same time (and to prevent tachyons). The time-like Kaluza-Klein ghosting problem also seems

resolved with the 'time leaves no tracks' hypothesis. So we postulate that both  $\tau$ -time and  $\upsilon$ -time each leave no tracks.

#### G. Uniqueness of Tessellation by a Regular Honeycomb

Far from any masses, we would expect the venues tessellating 5-dimensional space-time, on average (temporal and positional), to be identical.

In two dimensions (i.e. the plane) regular tessellation can be accomplished with squares, equilateral triangles, or regular hexagons. In four dimensions, there are three regular tessellations (honeycombs). But in five dimensions, there is only one: the 5-cubic honeycomb[20] (the order-4 penteractic honeycomb).

It would be awkward were there more than a single regular honeycomb possibility. Different types of regular honeycombs can not be mixed (for complete tessellation). And the stochastic space-time model would be hard put to assure that different regions of space-time would use the same type of honeycomb. Five-dimensional space-time then, can be tessellated only with five-dimensional cubes. This is an argument (albeit a weak one) for the existence of five rather than four dimensions.

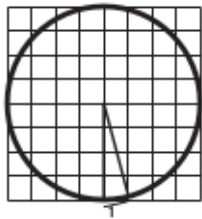
In the following sections, we discuss other details of the model (not necessarily related to the fifth dimension.

#### IV. ROTATIONS IN AND OF THE SPACE-TIME MANIFOLD

In general, with a stochastic space-time, there are two ways in which a particle can move: it can move through the space-time manifold, or it can move as a result of the venues at the particle's position migrating. We consider that for an elementary particle, its rotations are entirely the result of venue migration. This is somewhat akin to frame-dragging in the Kerr metric, except rather than the rotating particle dragging the space-time, the rotating space-time is dragging the particle. And, if the particle were charged, it would not radiate since not the particle but the space-time is rotating.

There is however a particular issue with (rigid) particle rotations in a granular manifold. We are mindful however, that in Special Relativity, there are no rigid objects[36]. We are interested in whether granular space-time theory also does not allow rigid objects.

The diagram below shows a mass with the dark circle indicating its circumference. It is sitting on a background of venues (the squares). It also shows an arc length of one Planck length.



If the particle, rather than the space-time were rotating, then when the particle rotates through the one Planck length arc, a point close to the origin would rotate far less than a Planck length, leaving the point in the same venue. But, by hypothesis, a venue has no internal structure; e.g. there can't be two distinct points in a venue.

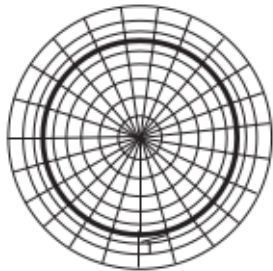
So with the above diagram, rigid rotations of the particle cannot happen.

In the case of the space-time itself rotating, it is also difficult to see how the grid of venues could rotate.

We'll address this by assuming the particle is subject to General Relativity.

In the previous paper (Part B[2], section VI) curvature was not a property of space-time, but merely an expression of the compression of venues. (E.g. a venue could compress in space dimensions while expanding in the conventional time dimension).

Consider the diagram below.



Here the particle (circle) is set against a grid of venues distorted by the particle's mass.

We consider that the space-time exterior to the particle is described by the usual Schwartzschild metric and the interior by the Schwartzschild, perfect fluid interior solution:

$$c^2 ds^2 = \frac{1}{4} \left( \left( 1 - \frac{2Gm}{c^2 r_m} \right)^{\frac{1}{3}} - \left( 1 - \frac{r^2 2Gm}{c^2 r_m^3} \right) \right)^2 c^2 dt^2 - \left( 1 - \frac{2Gm r^2}{c^2 r_m^3} \right)^{-1} dr^2 - r^2 (d\Theta^2 + \sin^2 \Theta d\phi^2) \text{ where } r_m \text{ is the radius of the mass.}$$

Notice that the 'curvature' increases as one approaches the surface from the exterior, and decreases as one proceeds from the radius towards the center. And, as long as the Schwartzschild radius is less than the particle radius, there is no Swartzschild singularity. Further, because venues are not points, there is no singularity at the center either. The rotation now is like a 'pizza slice' or wedge, no point on any venue rotates into the same venue.

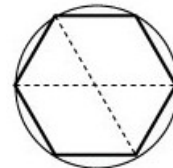
(Note that if we truly consider rotations, the Kerr metric would be more appropriate. But as we consider the

rotations as going in all directions at the same time, Kerr would also not be the appropriate metric.)

There are however (at least) two reasons why the above schema doesn't work: First, while the argument seems reasonable in two dimensions, it does not work in three. For a spherical particle rotating as above, a point on a venue on the surface on the axis of rotation would rotate in the same venue. Second, if the Swartzschild interior metric is roughly appropriate, then at the center of the particle, the 'curvature' would vanish. That is to say that the venues near the center would be minimally distorted. And, especially as in the schema, the number of venues circling the particle at any radius is the same, the venues near the center would be extremely distorted. We conclude then, that rotations are not rigid. Can we explain how even non-rigid rotations can be explained?

We can modify the above schema. First, (due to migrations) we can allow the number of venues circling the particle not to be the same at any radius. Further, the circling venues at any radius can circle the particle independently of the circling venues at any other radius. This, incidentally, would allow zero total angular momentum if the various rotating circles of venues were rotating in opposite directions. And second, we can allow rotations to occur simultaneously in any plane containing the center of the particle. This gives a geometric interpretation of particles rotating simultaneously in all directions and is suggestive of spin.

We consider now the innermost circle of venues. The venues making up the circle are triangular wedges, and the venues are like those in free space. In free space, a venue is presumed to have dimensions of Planck length cubed by Planck time. And it is posited that in free space there is no length less than the Planck length, and all lengths are integral multiples of the Planck length. We need a triangular wedge where the side opposite the central point is one Planck length and the other two sides are an integer number of Planck lengths. As is easily proved using Niven's theorem[21] the only such wedge is



the regular hexagon. We deduce then that the number of venues in the inner circle is six. Incidentally, at the Planck scale there can be no circles as an arc cannot be made up of only Planck length lines. What then can we use as a circle at the Planck scale? Again by Niven's theorem, we can only use the regular hexagon. And the ratio of circumference to diameter, what would normally be called  $\pi$ , is three.



## V. QUIDDITY, ENTANGLEMENT, THE TWO-SLIT EXPERIMENT

### A. Information & Quiddity : Pilot-waves & Entanglement

There are two forms of information at play: one of which is restricted to travel at no greater than the speed of light and the other (e.g. collapse of the wave function, entanglement and the like) not so restricted. These are very different processes, and so using the word 'information' for the both of them is confusing. We'll reserve 'information' for the first case, and 'quiddity' for quantum information. (Quiddity means the inherent nature or essence of something. And the first three letters, qui, make it easy to remember QUantum Information.)

Information is carried by photons or mass (energy). Quiddity, as it travels faster than light (and therefore can also travel backward in time), can not be carried by energy. In Stochastic Granular Space-time theory then, what can carry quiddity? The only thing left is empty venues. While a venue has an invariant 5-dimensional volume, it can vary in its individual dimensions. As described earlier, the 4-dimensional volume is related to the probability density,  $\Psi^*\Psi$ . So that probability density is a type of quiddity.

The wave function acts as a 'pilot wave' (as proposed by Louis de Broglie), moving well in advance of a quantum particle. When the particle 'catches up' to a place where the pilot wave is, that wave then determines the particle's probability density.

Entanglement seems to work the same way: by the superluminal propagation of probability densities. Entanglement then is not an extremely strange peripheral property of quantum theory, but a necessary and central component of the theory. An entangled set of particles then could interact superluminally, but an observer in the laboratory frame could not observe the result of the interaction until a time later, when a classical (subluminal) signal could have reached the interaction.

Our aim in the following is not to provide a theory/mechanism for entanglement, but to argue that Stochastic Granular space-time Theory allows for it, within the confines of five dimensions.

Bell's theorem[22], however, requires that to have entanglement, we must abandon 'objective reality' and/or 'locality'. Dropping locality means that things separated in space can influence each other instantaneously. Dropping objective reality means that a physical state isn't defined until it is measured (e.g. is the cat dead or alive?).

Weak measurement experiments[24–26] building on the work of Yakir Aharonov and Lev Vaidman[27] imply that there *is* objective reality in quantum mechanics[27, 28] (in contradiction to the Copenhagen interpretation of quantum mechanics). By objective reality, we mean a particle does have a path (blurred somewhat by space-time fluctuations) regardless of whether it is being observed or not.

We're left then, with non-locality. SGST is non-local. The issue, of course, is how to have non-locality whilst not violating Einstein's prohibition of information traveling faster than light. We slightly re-interpret that prohibition by positing that it is energy (as opposed to information) that can't travel faster than light.

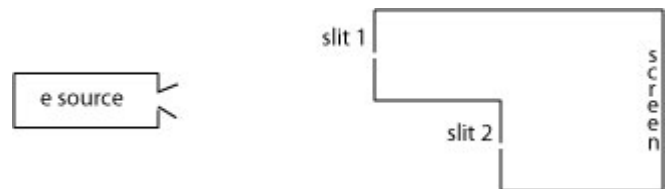
Empty venues carry no energy, and so (as we have seen) can migrate through space-time arbitrarily rapidly. The hope then is that we can find a way that empty venues can carry quiddity.

One empty venue seems not to fulfill that hope as a single empty venue's only quiddity is the fact of its existence, and since number of (empty) venues is not conserved, that fact doesn't seem to be able to explain entanglement.

We suggest though, that through some *unknown* mechanism (which is why this is a suggestion and not a theory) that a number of empty venues can be bound together can migrate collectively through space-time (e.g. spiraling through space-time) they would then carry a more complex quiddity. So, for instance, two created entangled particle would carry this quiddity with them as they spread out (as a link between them). And through another unknown mechanism, a measurement of one particle forces the state of the other and then dissolves the link.

Again, this discussion is not an *explanation* of entanglement, but just an attempt to show that SGST can contain such entangled states.

### B. The Delayed-choice Two-slit Experiment



The diagram shows the 'delayed choice two-slit experiment': A low-intensity source directs electrons to a box containing two slits (slit 1 and slit 2). The beam intensity is such that there is only one electron traveling in the box at any time. As expected, an interference pattern is gradually produced on the screen at the back of the box. If a particle detector is introduced at slit 2 to determine which slit an electron passed through, then there will be no interference produced. One can arrange that the detector is optionally turned on only when the electron has passed by slit 1. If the detector is on at that point, then again, there will be no interference pattern produced. So it seems that when the electron gets to slit 2 and finds that the detector is on, it goes back in time to tell the electron to go through, or not go through slit-1.

How does the Granular Stochastic Space-time model explain this?

First, we introduce the concept of an 'ephemeral' measurement: An electron has an associated electro-

magnetic field. As it goes through a slit, that field will interact with the electrons in the wall of the box at the slit. The box electrons then can tell if an electron has passed through a slit. And this could be considered a measurement; the box electrons could be considered a particle detector. But the interference pattern still occurs in this case. The difference is that the box electrons measurements are ephemeral; After the moving electron passes through the slit, the box electrons return to their undisturbed state, retaining no 'memory' of the measurement. The measurement is not preserved. The film can be run backward and it would be a valid physical situation. For there to be a true measurement then, there must be a mechanism to 'remember' the measurement – a latch or flip-flop of sorts. And that would mean the film could *not* be run backward. We regard measurement then, as a breaking of time-reversal symmetry. In the macro-world, everything is a measurement of sorts (viewing a scene gives an estimate of positions, etc.) and hence we can't run macro-world scenes backwards.

With quiddity (in this case, the pilot wave) able to move superluminally as well as to move backward in time, there isn't much to explain. The pilot wave precedes the electron going into the box. The pilot wave determines the probability of the electron being found at any point in the box at any time. If (at any time) the detector is switched on, that would change the geometry and hence the wave (at all points, future and past). The electron would continue its motion, catching up with the revised pilot wave and then moving accordingly. (This is much like the mechanism of entanglement).

## VI. DISCUSSION

General relativity is a theory relating the large scale structure of space-time to the masses in it. Similarly, the stochastic space-time model relates the micro-structure of space-time to the behavior of masses at the quantum level. One says for general relativity, mass tells space how to bend. Space tells mass how to move. And in the stochastic model, we say mass tells space how to jell. Space tells mass how to jiggle. The model is neither one of quantum mechanics nor General Relativity. It requires both theories in its development.

In the model, particles move (in an indeterminate manner) due to the space-time fluctuations exterior to the particle (similar to the way a Brownian Motion pollen grain moves). But unlike with Brownian motion, time (as well as space) fluctuates.

In free-space, there is no meaning in retracing a trajectory as, because of the space fluctuations, there is no well-defined 'place'. Inside a mass, it is different. In particular, going backwards in time doesn't mean a system goes to a well-defined earlier place in time. If there is a time reversal, it is evidenced only by the reversal of global quantities (such as direction of rotation).

A form of the metric tensor suggested that quantum

oscillations of particles could be described as torsional vibrations occurring simultaneously in all directions (and there don't seem to be that many symmetrical oscillations available). A previous paper[35] has presented a model whereby such oscillating particles could pass through a polarizer admitting fifty percent throughput rather than just those particles aligned perfectly with the polarizer.

The object of the present model is to provide a conceptual basis for quantum mechanics—to show that the 'quantum weirdness' can be explained in terms of the behavior of space-time. And indeed, the model has managed to replicate some of the fundamental processes in conventional quantum theory. Yes, it is only a model but because it has produced a prediction (i.e. no quantum effects for masses greater than  $m_p$ ) the model might now be elevated to a theory—or at least the start of a theory.

We recognize that this paper contains some unusual ideas. But they were forged in a chain (a long chain) of logic from the initial conjecture that space-time is stochastic. In addition, this paper jumps around between topics. For that, I apologize. The problem was that the concepts are so interconnected that I could not find a single, compelling, logical order in which to present them.

## VII. THE WAY FORWARD

For the better part of a century, researchers have sought an understanding of the physical nature of quantum mechanics. Perhaps then, a mathematical solution to the problem is impossible and it is an example of the Gödel incompleteness theorem[34]. Does Gödel imply that there is no mathematical solution, or perhaps there is one but it can not be derived? Roger Penrose, Herman, Weyl, among others, felt that the theorem shows that we can never know many, if not most, of the physical laws of the universe.

Perhaps we need another approach.

Our granular, stochastic space-time model is (at the moment) more phenomenological than mathematical; it is an assemblage of interconnected (hopefully self-consistent) phenomena—a scaffold onto which quantum phenomena can be attached. Each phenomenon attached to the scaffold might well be describable by mathematics, but the entire populated scaffold might not be.

While the mathematical description is thus far from complete, the model is specified sufficiently to allow (super)computer simulations. And perhaps, because of Gödel, computer simulations are the best we can do. Stay tuned for 'Stochastic space-time and quantum theory: Part D: computer simulations'.

## ACKNOWLEDGMENTS

I wish to acknowledge and thank Nicholas Taylor, Norman Witriol, and Nicolae Mazilu for helpful discussions of the ideas in this paper.

## APPENDIX

## Some Useful Numbers

Planck's constant ( $h$ )  $\approx 6.626 * 10^{-34} \text{ m}^2 \text{ kg s}^{-1}$   
 Constant of gravitation  $\approx 6.674 * 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$   
 Speed of light ( $c$ )  $\approx 2.99792458 * 10^8 \text{ m s}^{-1}$   
 Planck mass ( $m_p = \sqrt{\frac{hc}{2\pi G}}$ )  $\approx 2.176470 * 10^{-8} \text{ kg}$

Planck length ( $l_p = \sqrt{\frac{hG}{2\pi c^3}}$ )  $\approx 1.616229 * 10^{-35} \text{ m}$   
 Planck time ( $t_p = \sqrt{\frac{hG}{2\pi c^5}} = \frac{l_p}{c}$ )  $\approx 5.39116 * 10^{-44} \text{ s}$   
 Uranium 238 nucleus mass  $\approx 3.95 * 10^{-25} \text{ kg}$   
 Uranium 238 nucleus diameter  $\approx 11.7142 * 10^{-15} \text{ m}$   
 Uranium atom diameter  $3.50 * 10^{-10} \text{ m}$   
 Hydrogen atom diameter  $\approx 1.06 * 10^{-10} \text{ m}$   
 Electron mass  $\approx 9.10938356 * 10^{-31} \text{ kg}$   
 Neutrino mass  $\approx 2 * 10^{-36} \text{ kg}$  (very uncertain)  
 Range of the strong force  $\approx 0.8 * 10^{-15} \text{ m}$   
 Proton diameter  $\approx 0.84 * 10^{-15} \text{ m}$   
 Range of the weak force  $\approx 10^{-18} \text{ m}$   
*Unreduced Planck units (up)*  
 Planck mass ( $m_{up} = \sqrt{\frac{hc}{G}}$ )  $\approx 5.454 * 10^{-8} \text{ kg}$   
 Planck length ( $l_{up} = \sqrt{\frac{hG}{c^3}}$ )  $\approx 4.051 * 10^{-35} \text{ m}$   
 Planck time ( $t_{up} = \sqrt{\frac{hG}{c^5}} = \frac{l_{up}}{c}$ )  $\approx 1.351 * 10^{-43} \text{ s}$

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- [36] NOTE: One can see this from the well-known problem of a telephone pole of length  $2 * l$  sliding along a sidewalk in which is a hole of length  $l$ . If the pole is traveling fast enough, an observer on the sidewalk would observe the pole to be less than  $l$  long where an observer on the pole would observe a very short hole. Does the pole fall in the hole? The apparent paradox is resolved by noting that as the front edge of the pole goes over the hole, it will bend into the hole as the mass further back can't instantly effect the front edge to keep it from tipping.