Refutation of modal forms on Vietoris space

Abstract: We use modal logic to evaluate the defined modal forms on Vietoris space to find them *not* tautologous.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated *proof* value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, , or repeating fragments of 128-tables for more variables. (See ersatz-systems.com.)

LET p, q, r, s: C, U, V, X;
~ Not; & And, ∩; > Imply; < Not Imply, less than, ∈;
= Equivalent; @ Not Equivalent, ≠;
% possibility, for one or some, ◇; # necessity, for all or every, □;
(q@q) null, ∅; ≤ ⊆; ~(y < x) (x ≤ y); (y > x) (x | y).

From: Borlido, C.; Gehrke, M. (2018). A note on powers of Boolean spaces with internal semigroups. arxiv.org/pdf/1811.12339.pdf cborlido@unice.fr

The power construction on compact Hausdorff spaces is the so-called Vietoris space. Vietoris is a covariant endofunctor on compact Hausdorff spaces which restricts to the category of Boolean spaces. At the level of objects it assigns to a space X the set of all its closed subsets, denoted V(X) and called the Vietoris space of X, equipped with the topology generated by the sets of the form [where U ⊆X ranges over all open subsets of X]

$$\Diamond U = \{C \in V(X) \mid C \cap U \neq \varnothing\} \text{ and} \quad (3.1.1)$$

~(s<q)>(%q=(((p&q)@(q@q))>(p<(r&s)))) ;
CCTT CCTT TTTT TTTF (3.1.2)

$$\Box U = \{C \in V(X) \mid C \subseteq U\}, \quad (3.2.1)$$

~(s<q)>(#q=(~(q<p)>(p<(r&s)))) ; TFNN TFNN TFNN TTNC (3.2.2)

Eqs. 3.1.2 and 3.2.2 as rendered are *not* tautologous. This means the defined modal forms on Vietoris space are refuted.