

Refutation of the Frauchiger-Renner thought paradox as a quantum model

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Abstract: We use modal logic to evaluate a quantum model of the Frauchiger-Renner thought experiment as not a contradiction (paradox) *and* not a tautology (theorem). The example misapplies the Born rule which we refute elsewhere.

We assume the method and apparatus of Meth8/VL4 with Tautology as the designated *proof* value, **F** as contradiction, **N** as truthity (non-contingency), and **C** as falsity (contingency). The 16-valued truth table is row-major and horizontal. The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables for more variables. (See ersatz-systems.com.)

LET $p, q, r, s, t, u, v, w, x, y, z$:
 Alice, $|1\rangle$, R, S, memory, Ursula, Bob, Wigner, $\sqrt{1}, \sqrt{2}, \sqrt{3}$,
 $\sim q \quad |0\rangle$;
 \sim Not; + Or; & And; \ Not And; > Imply; = Equivalent; @ Not Equivalent;
 % possibility, for one or some; # necessity, for all or every;
 (p=p) ok, tautology, ordinal three 3; (p@p) fail, contradiction, zero 0;
 (%p>#p) truthity, ordinal one 1; (%p<#p) falsity, ordinal two 2;
 a=0 ((p&t)=(~q&p)); a=1 ((p&t)=(q&p));
 b=0 ((v&t)=(~q&v)); b=1 ((v&t)=(q&v)).

From: Nurgalieva, N.; del Rio, L. (2018). Inadequacy of modal logic in quantum settings.
arxiv.org/pdf/1804.01106.pdf delrio@phys.ethz.ch

Initial settings:

$$R = ((\sqrt{1/3} |0\rangle_R) + (\sqrt{2/3} |1\rangle_R)) \quad (1.0.1.1)$$

$$r = (((x \setminus y) \& (\sim q \& r)) + ((y \setminus z) \& (q \& r))) ;$$

TTTT	TTTT	TTTT	TTTT	TTTT	FFTT	TTTT	FFTT
TTTT	TTF	TTTT	TTF	TTTT	FFFF	TTTT	FFFF

(1.0.1.2)

$$S = |0\rangle_S \quad (1.0.2.1)$$

$$s = (\sim q \& s) ;$$

TTTT	TTF	TTTT	TTF
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(1.0.2.2)

$$\text{Alice memory} = |0\rangle_A \quad (1.0.3.1)$$

$$(p \& t) = (\sim q \& p) ;$$

TFTT	TFTT	TFTT	TFTT	TTTT	TTF	TTTT	TTF	TTTT	TTF
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(1.0.3.2)

$$\text{Bob memory} = |0\rangle_B \quad (1.0.4.1)$$

$$(v \& t) = (\sim q \& v) ;$$

TTTT	TTTT	TTTT	TTTT	TTTT	(4)	FFTT	FFTT	FFTT	FFTT
TTF	TTF	TTF	TTF	TTF	TTF	TTF	TTF	TTF	TTF

(1.0.4.2)

$$\text{Ursula memory} = |0\rangle_u \quad (1.0.5.1)$$

$$\begin{aligned} (u\&t)=(\sim q\&u); \\ \text{TTTT TTTT TTTT TTTT}(2), \text{ FFTT FFTT FFTT FFTT,} \\ \text{TTF\textbf{F} TTF\textbf{F} TTF\textbf{F} TTF\textbf{F}} \end{aligned} \quad (1.0.5.2)$$

$$\text{Wigner memory} = |0\rangle_w \quad (1.0.6.1)$$

$$\begin{aligned} (w\&t)=(\sim q\&w); \\ \text{TTTT TTTT TTTT TTTT}(8), \text{ FFTT FFTT FFTT FFTT,} \\ \text{TTF\textbf{F} TTF\textbf{F} TTF\textbf{F} TTF\textbf{F}} \end{aligned} \quad (1.0.6.2)$$

T1: **Remark 1:** We map agent $T = \langle 1, 2, 3, 4, 5 \rangle$ using instructions in the text as best as we can follow.

$$R = |0\rangle_R, |1\rangle_R \quad (1.1.1.1)$$

$$\begin{aligned} r=((\sim q\&r)+(q\&r)); \\ \text{TTTT TTTT TTTT TTTT}(128) \end{aligned} \quad (1.1.1.2)$$

Alice records the result in her memory A . (1.1.2.1)

$$\begin{aligned} (p\&t)=((\sim q\&p)+(q\&p)); \\ \text{T\textbf{F}T\textbf{F} T\textbf{F}T\textbf{F} T\textbf{F}T\textbf{F} T\textbf{F}T\textbf{F}, TTTT TTTT TTTT TTTT} \end{aligned} \quad (1.1.2.2)$$

Alice prepares S :
if outcome $a = 0$ then her memory is $|0\rangle_A$ and S is $|0\rangle_S$; or
if outcome $a = 1$ then her memory is $|1\rangle_A$ and S is
 $(1/\sqrt{2})(|0\rangle_S + |1\rangle_S)$ (1.1.3.1)

$$\begin{aligned} (((p\&t)=(\sim q\&p))>(((p\&t)=(\sim q\&p))\&(s=(\sim q\&s)))) + \\ (((p\&t)=(q\&p))>(((p\&t)=(q\&p))\&(s=((x\y)\&((\sim q\&s)+(q\&s)))))); \\ \text{TTTT TTTT TTF\textbf{T} TTF\textbf{T}, TTTT TTTT TTTT TTTT} \end{aligned} \quad (1.1.3.2)$$

Alice replaces Bob's system S with her own. (1.1.4.1)

$$\begin{aligned} s=(((p\&t)=(\sim q\&p))>(((p\&t)=(\sim q\&p))\&(s=(\sim q\&s)))) + \\ (((p\&t)=(q\&p))>(((p\&t)=(q\&p))\&(s=((x\y)\&((\sim q\&s)+(q\&s)))))); \\ \text{F\textbf{F}F\textbf{F} F\textbf{F}F\textbf{F} TTTT TTTT, F\textbf{F}F\textbf{F} F\textbf{F}F\textbf{F} TTF\textbf{T} TTF\textbf{T}} \end{aligned} \quad (1.1.4.2)$$

T2: **Remark 2:** We follow *T1* by replacing Alice with Bob and R with S , but exclude Eq. 1.1.3.1 for Bob.

$$S = |0\rangle_S, |1\rangle_S \quad (2.1.1.1)$$

$$\begin{aligned} s=((\sim q\&s)+(q\&s)); \\ \text{TTTT TTTT TTTT TTTT}(128) \end{aligned} \quad (2.1.1.2)$$

Bob records the result in his memory B. (2.1.2.1)

$$(v\&t)=((\sim q\&v)+(q\&v));$$

$$\text{TTTT TTTT TTTT TTTT(4), FFFF FFFF FFFF FFFF} \quad (2.1.2.2)$$

Bob prepares R :

if outcome b = 0 then his memory is $|0\rangle_B$ and R is $|0\rangle_R$; or

if outcome b = 1 then his memory is $|1\rangle_B$ and R is

$$(1/\sqrt{2}) (|0\rangle_R + |1\rangle_R) \quad (2.1.3.1)$$

$$(((v\&t)=(\sim q\&v))>(((v\&t)=(\sim q\&v))\&(r=(\sim q\&r))))$$

$$+$$

$$(((v\&t)=(q\&v))>(((v\&t)=(q\&v))\&(r=((x\backslash y)\&((\sim q\&r)+(q\&r))))));$$

$$\text{TTTT TTTT TTTT TTTT, TTTT TTF F TTTT TTF F} \quad (2.1.3.2)$$

T3: Ursula measures and records the result of Alice's lab as:

RA = $|ok\rangle_{RA}, |fail\rangle_{RA}$ where

$$|ok\rangle_{RA} = \sqrt{1/2} (|0\rangle_R |0\rangle_A - |1\rangle_R |1\rangle_A)$$

$$|fail\rangle_{RA} = \sqrt{1/2} (|0\rangle_R |0\rangle_A + |1\rangle_R |1\rangle_A) \quad (3.1.1.1)$$

$$(((r\&p)\&(p=p))=((x\backslash y)\&(((\sim q\&r)\&(\sim q\&p))-((q\&r)\&(q\&p)))) +$$

$$(((r\&p)\&(p@p))=((x\backslash y)\&(((\sim q\&r)\&(\sim q\&p))+((q\&r)\&(q\&p)))));$$

$$\text{TTTT TTF F TTTT TTF F, TTTT TTTT TTTT TTTT} \quad (3.1.1.2)$$

T4: Wigner measures and records the result of Bob's lab as:

SB = $|ok\rangle_{SB}, |fail\rangle_{SB}$ where

$$|ok\rangle_{SB} = \sqrt{1/2} (|0\rangle_S |0\rangle_B - |1\rangle_S |1\rangle_B)$$

$$|fail\rangle_{SB} = \sqrt{1/2} (|0\rangle_S |0\rangle_B + |1\rangle_S |1\rangle_B) \quad (4.1.1.1)$$

$$(((s\&v)\&(p=p))=((x\backslash y)\&(((\sim q\&s)\&(\sim q\&v))-((q\&s)\&(q\&v)))) +$$

$$(((s\&v)\&(p@p))=((x\backslash y)\&(((\sim q\&s)\&(\sim q\&v))+((q\&s)\&(q\&v)))));$$

$$\text{TTTT TTTT TTTT TTTT, TTTT TTTT FFFF FFFF} \quad (4.1.1.2)$$

T5: Ursula and Wigner compare their recorded measurements.

If both are ok, then the experiment ends, otherwise initial settings and timers are reset to repeat. (5.0)

Excepting the obvious theorems of Eqs. 1.1.1.2 and 2.1.1.2, Eqs. 1.2-4. as rendered are *not* tautologous. This means the model conjectured is not a contradiction (paradox) *and* not a tautology (theorem). Therefore the model is indeterminate.

The authors invoke the Born rule. (We refute the Born rule elsewhere in Everettian quantum mechanics (EQM) as the probability of the wave function squared.) The authors halt the experiment at

an injected $T 2.5$ to give a probability of $1/12$. In fact, the model cannot halt because at each iteration, initial values are reset .

We ask what is the logical table result of the entire system as rendered, combining each step as an antecedent to imply the next step as a consequent. This amounts to:

If Eqs.1.0, then if T 1 and T 2 then T 3 and T 4. (6.1)

$$\begin{aligned}
 &(((r=((x\backslash y)\&(\sim q\&r))+((y\backslash z)\&(q\&r))))\&(s=(\sim q\&s))\&(((p\&t)= \\
 &(\sim q\&p))\&(((v\&t)=(\sim q\&v))\&(((u\&t)=(\sim q\&u))\&((w\&t)= \\
 &(\sim q\&w)))))) \\
 &> \\
 &(((r=(\sim q\&r)+(q\&r))\>((p\&t)=(\sim q\&p)+(q\&p)))\>(s=(((p\&t)= \\
 &(\sim q\&p))\>(((p\&t)=(\sim q\&p))\&(s=(\sim q\&s))))+(((p\&t)=(q\&p))\> \\
 &(((p\&t)=(q\&p))\&(s=((x\backslash y)\&((\sim q\&s)+(q\&s)))))))))) \\
 &\& \\
 &(((s=(\sim q\&s)+(q\&s))\>((v\&t)=(\sim q\&v)+(q\&v))))\> \\
 &((((v\&t)=(\sim q\&v))\>(((v\&t)=(\sim q\&v))\&(r=(\sim q\&r))))+(((v\&t)= \\
 &(q\&v))\>(((v\&t)=(q\&v))\&(r=((x\backslash y)\&((\sim q\&r)+(q\&r)))))) \\
 &> \\
 &((((r\&p)\&(p=p))=((x\backslash y)\&(((\sim q\&r)\&(\sim q\&p))-((q\&r)\&(q\&p))))+ \\
 &(((r\&p)\&(p@p))=((x\backslash y)\&(((\sim q\&r)\&(\sim q\&p))+((q\&r)\&(q\&p)))))) \\
 &\& \\
 &(((s\&v)\&(p=p))=((x\backslash y)\&(((\sim q\&s)\&(\sim q\&v))-((q\&s)\&(q\&v))))+ \\
 &(((s\&v)\&(p@p))=((x\backslash y)\&(((\sim q\&s)\&(\sim q\&v))+((q\&s)\& \\
 &(q\&v))))))));
 \end{aligned}$$

FT**F**T **F**T**F**F TTTT TTTT, **F**F**F**T **F**F**F**T TTTT T**F**TT,
FF**F**T **F**F**F**T TTTT T**F**TT, **F**F**F**T T**F****F**T TTTT TTTT,
FF**F**T TTTT TTTT TTTT, **F**F**F**T **F**F**F**T **F**F**F**T **F**F**F**T,
FF**F**T **F**F**F**T TTTT T**F**TT, **F**F**F**T TTTT TTTT TTTT,
FT**F**T **F**TTT TTTT TTTT, **F**T**F**T T**F****F**T TTTT TTTT,
FT**F**T TTTT TTTT TTTT, T**F****F**T T**F****F**F TTTT TTTT,
T**F****F**T T**F****F**T TTTT TTTT, T**F****F**T TTTT TTTT TTTT

(317 steps) (6.2)

Eq. 6.2 is *not* tautologous, meaning the thought experiment is not a paradox (contradiction) *and* not a theorem (tautology).