Refutation of a modal logic for partial awareness from published example

Abstract: We evaluate a modal logic for partial awareness from a published example. The definitions and conjectures are *not* tautologous. We show how to exclude a priori logical clauses to promote a perhaps unintended tautology for the example. However, our evaluation does not rely on modal operators, suggesting that the system as proffered should be renamed to a logic for awareness, without the word modal.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated *proof* value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables for more variables. (See ersatz-systems.com.)

> **Remark 4.3**: Since Ex. 4.1 and 4.2 are related with total verbiage greater than Ex 4.3, we select Ex. 4.3 to evaluate.

LET p, q, r, s, t, u, v, w, x, y, z:
P, Q, R, A_1, A_2, d_1, d_2, w, w_1, w_2, w_3.
~ Not; + Or ; & And; > Imply; = Equivalent; (p@p) contradiction, null, zero 0.

From: Halpern, J.Y.; Piermont, E. (2018). Partial awareness.
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> **Remark 4.3**: We evaluate Example 4.3 because its verbiage is less than that for the related Examples 4.1 and 4.2.

$P^I_{w1} = d_1$ (4.3.1.1)

(p&x)=u ; TTTT TTTT TTTT TTTT, **FFFF FFFF FFFF FFFF**,
T**F**T**F** T**F**T**F** T**F**T**F** T**F**T**F**, **F**T**F**T **F**T**F**T **F**T**F**T **F**T**F**T (4.3.1.2)

$Q^I_{w3} = d_2$ (4.3.2.1)

(q&z)=v ; TTTT TTTT TTTT TTTT, **FFFF FFFF FFFF FFFF**,
TT**FF** TT**FF** TT**FF** TT**FF**, **FF**TT **FF**TT **FF**TT **F**FTT (4.3.2.2)

$P^I_{w2} = P^I w_3 = Q^I_{w^2} =$ zero (4.3.3.1)

(p&y)=((p&z)=((q&y)=(p@p))) ;
TTTT TTTT TTTT TTTT, T**FF**T T**FF**T T**FF**T T**FF**T,
T**F**T**F** T**F**T**F** T**F**T**F** T**F**T**F**, TT**FF** TT**FF** TT**FF** TT**FF** (4.3.3.2)

$R^I_w = d_1$ (4.3.4.1)

(r&w)=u ; TTTT TTTT TTTT TTTT, **FFFF FFFF FFFF FFFF**,
TTTT **FFFF** TTTT **FFFF**, **FFFF** TTTT **FFFF** TTTT 4.3.4.2)

A_1w = null [or] (P or Q) [or] zero (4.3.5.1)

(s&w)=((p@p)+((p+q)+(p@p))) ;
T**FFF** T**FF**F T**FFF** T**FFF**, T**FFF** T**FFF** **F**TTT **F**TTT (4.3.5.2)

A_2w = null [or] (Q or R) [or] zero (4.3.6.1)

(t&w)=((p@p)+((q+r)+(p@p))) ;
TT**FF** **FFFF** TT**FF** **FFFF**, **FF**TT TTTT **FF**TT TTTT (4.3.6.2)

"agent$_1$ wants d_1 only when it has property P (to trade in states w_2 [or] w_3), and
agent$_2$ wants d_2 only when it has property Q (to trade in states w_1 and w_2"
with
"for agent$_1$, w_2 and w_3 are equivalent, and (4.3.7.1)
for agent$_2$, w_1 and w_2 are equivalent." (4.3.8.1)

((p&(y+z))>(s>u))>(y=z) ;
TTTT TTTT TTTT TTTT, **FFFF** **FFFF** **FFFF** **FFFF**,
FFFF **FFFF** **F**T**F**T **F**T**F**T (4.3.7.2)

((q&(w&x))>(t>v))>(x=y) ;
TTTT TTTT TTTT TTTT, **FFFF** **FFFF** **FFFF** **FFFF** (4.3.8.2)

"However, neither agent can propose an acceptable contract." (4.3.9.1)

Remark 9.1: To evaluate Eq. 4.3.9/10 we process Eqs. 4.3.7.2 or 4.3.8.2 respectively as the consequent of the definitions in Eqs. 4.3.1.2/6.2.

(((((p&x)=u)&((q&z)=v))&(((p&y)=((p&z)=((q&y)=(p@p))))&
((r&w)=u)))& (((s&w)=((p@p)+((p+q)+(p@p))))&
((t&w)=((p@p)+((q+r)+(p@p))))))
>
((((p&(y+z))>(s>u))>(y=z))+(((q&(w&x))>(t>v))>(x=y))) ;
TTTT TTTT TTTT TTTT, **F**TTT TTTT **F**TTT TTTT,
FTTT TTTT TTTT TTTT, TTTT TTTT TT**F**T TTTT (4.3.9.2)

Eqs. 4.3.9.2 as rendered is *not* tautologous, and hence as presented "neither agent can propose an acceptable contract."

Remark 4.3.10: To rehabilitate Eq. 4.3.9.2, we exclude the agent clauses from Eqs. 4.3.7/8 for w-equivalences as potential a priori commentary. (4.3.10.1)

(((((p&x)=u)&((q&z)=v))&(((p&y)=((p&z)=((q&y)=(p@p))))&
((r&w)=u)))& (((s&w)=((p@p)+((p+q)+(p@p))))&
((t&w)=((p@p)+((q+r)+(p@p))))))
>
(((p&(y+z))>(s>u))+((q&(w&x))>(t>v))) ;
TTTT TTTT TTTT TTTT (4.3.10.2)

Eq. 4.3.10.2 is tautologous, hence without the injected agent w-equivalences, the agents can propose an acceptable contract. We do not guess what that contract is.

Excepting Eq. 4.3.10, the others are *not* tautologous. This means the example does not support a modal logic for partial awareness. We note that modal operators were not used by us here at all.

Our conclusion is not to refute the notion of a partial awareness in semantics. This can be construed as a newly coined academic term for VŁ4, where the four-valued logic purposely codifies falsity and truthity based on exact truth table results in the range from contradiction to tautology. Because of that, VŁ4 is better suited for the *exact* analysis of partial awareness with or without modal operators.