Research and report

ζ(3), ζ (5), ζ (7), ζ (9)), ζ (11), ζ (13) are irrational numbers

Toshiro Takami
mmm82889@yahoo.co.jp

Abstract

Since ζ(3) could be represented by sin, cos and π, we report here. I spelled
In wolframAlpha,
sum_(n=1)^infty [sin((6n-4)*pi/3) -sin((6n-2)*pi/3) + sin(6n*pi/3)] / [sqrt(3)*n^3]
=1.2020569031595942853997381615114…...

and, ζ(5), ζ(7), ζ(9), ζ (11), ζ(13) considered.

From these equations, it can be said that ζ(3), ζ (5), ζ (7), ζ (9), ζ (11), ζ(13) are irrational numbers.
ζ (15), ζ (17) etc. can also be expressed by these equations.

discussion

$$\zeta(3) = \sum_{n=1}^{\infty} \frac{\sin\frac{(6n-4)\pi}{3} - \sin\frac{(6n-2)\pi}{3} + \sin\frac{6n\pi}{3}}{\sqrt{3}n^3}$$

be equivalent to

$$\zeta(3)=\sum_{n=1}^{\infty}\frac{-\cos\frac{(6n-4)\pi}{3}-\cos\frac{(6n-2)\pi}{3}+\cos\frac{6n\pi}{3}}{2n^3}$$

be equivalent to

$$\zeta(3)=\sum_{n=1}^{\infty}\frac{\sin\frac{(6n-4)\pi}{3}-\sin\frac{(6n-2)\pi}{3}}{\sqrt{3}n^3}$$

be equivalent to

$$\zeta(3)=\sum_{n=1}^{\infty}\frac{-\cos\frac{(6n-4)\pi}{3}-\cos\frac{(6n-2)\pi}{3}+1}{2n^3}$$

=1.202056903159594285399738161511449990764986292340498881...

1/1^3+1/2^3+1/3^3+1/4^3+1/5^3+1/6^3+1/7^3+1/8^3+1/9^3+1/10^3+1/11^3+1/12^3
……= ζ (3)

and, For example,
{sin((2 π /3) - sin(4 π /3) + sin(6 π /3)} / {sqrt(3)*1^3} +
{sin(8 π /3) - sin(10 π /3) + sin(12 π /3} / {sqrt(3)*2^3} +
{sin((14 π /3) - sin(16 π /3) + sin(18 π /3)} / {sqrt(3)*3^3} +
{sin(20 π /3) - sin(22 π /3) + sin(24 π /3} / {sqrt(3)*4^3} +
………
………= ζ (3)
and,
{-cos((2 π /3) - cos(4 π /3) + cos(6 π /3)} / (2*1^3) +
{-cos(8 π /3) - cos(10 π /3) + cos(12 π /3} / (2*2^3) +
{-cos(14 π /3) - cos(16 π /3) + cos(18 π /3)} / (2*3^3) +
{-cos(20 π /3) - cos(22 π /3) + cos(24 π /3} / (2*4^3) +
………
………= ζ (3)
While turning circles, factorial increases.

and, ζ(5) considered.

$$\zeta(5)=\sum_{n=1}^{\infty}\frac{\sin\frac{(10n-8)\pi}{5}-\sin\frac{(10n-6)\pi}{5}+\sin\frac{(10n-4)\pi}{5}-\sin\frac{(10n-2)\pi}{5}}{n^5\sqrt{5+2\sqrt{5}}} \quad (1)$$

be equivalent to

\zeta(5)= sum_(n=1)^infty [sin((10n-8)*pi/5)+sin((10n-6)*pi/5)- sin((10n-4)*pi/5) - sin((10n-2)*pi/5) + sin(10n*pi/5)] / [\sqrt(5 + 2 *\sqrt(5))*n^5]

= sum_(n=1)^infty [2sin((10n-8)*pi/5)+2sin((10n-6)*pi/5)] / [\sqrt(5 + 2 *\sqrt(5))*n^5]

=1.0369277551433699263313654864570341……= ζ(5)

$$\zeta(5)=\sum_{n=1}^{\infty}\frac{2\sin\frac{(10n-8)\pi}{5}+2\sin\frac{(10n-6)\pi}{5}}{n^5\sqrt{5+2\sqrt{5}}}$$

be equivalent to

sum_(n=1)^infty [cos((10n-8)*pi/5) -cos((10n-6)*pi/5) -cos((10n-4)*pi/5) +cos((10n-2)*pi/5) + cos(10n*pi/5)] / [{1 + \sqrt(5)}*n^5]

= sum_(n=1)^infty [2cos((10n-8)*pi/5) -2cos((10n-6)*pi/5) + 1] / [{1 + \sqrt(5)}*n^5]

= 1.036927755143369926331365486457…..

$$\zeta(5)=\sum_{n=1}^{\infty}\frac{\cos\frac{(10n-8)\pi}{5}-\cos\frac{(10n-6)\pi}{5}-\cos\frac{(10n-4)\pi}{5}+\cos\frac{(10n-2)\pi}{5}+1}{n^5(1+\sqrt{5})} \quad (1)$$

be equivalent to

$$\zeta(5)=\sum_{n=1}^{\infty}\frac{2\cos\frac{(10n-8)\pi}{5}-2\cos\frac{(10n-6)\pi}{5}+1}{n^5(1+\sqrt{5})}$$

and, ζ(7) considered.

$$\zeta(7)=\sum_{n=1}^{\infty}\frac{\sin\frac{(14n-12)\pi}{7}+\sin\frac{(14n-10)\pi}{7}+\sin\frac{(14n-8)\pi}{7}-\sin\frac{(14n-6)\pi}{7}-\sin\frac{(14n-4)\pi}{7}-\sin\frac{(14n-2)\pi}{7}}{n^7(2\sin\frac{\pi}{7}+2\cos\frac{\pi}{14}+2\cos\frac{3\pi}{14})} \quad (1)$$

When calculating

{sin((14n-12)*pi/7)+sin((14n-10)*pi/7)+ sin((14n-8)*pi/7)}, {n=1}

= sin(2*pi/7)+sin(4*pi/7)+ sin(6*pi/7)= sin(π /7) + cos(π /14) + cos((3 π)/14)

sum_(n=1)^infty [sin((14n-12)*pi/7)+sin((14n-10)*pi/7)+ sin((14n-8)*pi/7)] / [{sin(π/7) + cos(π/14) + cos((3 π)/14)}*n^7]

= 1.00834927738192282683979….. = ζ (7)

be equivalent to

$$\zeta(7)=\sum_{n=1}^{\infty}\frac{2\sin\frac{(14n-12)\pi}{7}+2\sin\frac{(14n-10)\pi}{7}+2\sin\frac{(14n-8)\pi}{7}}{n^7(\sin\frac{\pi}{7}+\cos\frac{\pi}{14}+\cos\frac{3\pi}{14})} \qquad (1)$$

be equivalent to

{ 2cos((14n-12)*pi/7)-2cos((14n-10)*pi/7)-2cos((14n-8)*pi/7) +1 },{n=1}
=2cos(2*pi/7)-2cos(4*pi/7)-2cos(6*pi/7) +1
=1 + 2 sin(π /14) + 2 sin((3 π)/14) + 2 cos(π /7)

sum_(n=1)^infty [2cos((14n-12)*pi/7)-2cos((14n-10)*pi/7)-2cos((14n-8)*pi/7) +1]/[{1+2sin(π /14)+2 sin((3 π)/14)+2cos(π /7)}*n^7]
= 1.0083492773819228268397975498497967595998635605652387 06...
zeta(7)= 1.008349277381922826839797549849796759599863560565238706...

$$\zeta(7)=\sum_{n=1}^{\infty}\frac{2\cos\frac{(14n-12)\pi}{7}-2\cos\frac{(14n-10)\pi}{7}-2\cos\frac{(14n-8)\pi}{7}+1}{n^7(1+2\sin\frac{\pi}{14}+2\sin\frac{3\pi}{14}+2\cos\frac{\pi}{7})} \qquad (1)$$

and, ζ(9) considered.
{sin((18n-16)*pi/9)+sin((18n-14)*pi/9)+ sin((18n-12)*pi/9) + sin((18n-10)*pi/9)}, {n=1}
=sin(2*pi/9)+sin(4*pi/9)+ sin(6*pi/9) + sin(8*pi/9)]= sqrt(3)/2 + sin(π /9) + sin((2 π)/9) + cos(π /18)

sum_(n=1)^infty [sin((18n-16)*pi/9)+sin((18n-14)*pi/9)+ sin((18n-12)*pi/9) + sin((18n-10)*pi/9)] / [{sqrt(3)/2 + sin(π /9) + sin((2 π)/9) + cos(π /18)}*n^9]
= 1.0020083928260822144178527692324120604856058513948887 56.......
zeta(9) = 1.002008392826082214417852769232412060485605851394888756...
....

$$\zeta(9)=\sum_{n=1}^{\infty}\frac{\sin\frac{(18n-16)\pi}{9}+\sin\frac{(18n-14)\pi}{9}+\sin\frac{(18n-12)\pi}{9}+\sin\frac{(18n-10)\pi}{9}}{n^9(\frac{\sqrt{3}}{2}+\sin\frac{\pi}{9}+\sin\frac{2\pi}{9}+\cos\frac{\pi}{18})} \qquad (1)$$

and, ζ(11) considered.
{sin((22n-20)*pi/11)+sin((22n-18)*pi/11)+sin((22n-16)*pi/11)+ sin((22n-14)*pi/11)+sin((22n-12)*pi/11)}, {n=1}

=sin(2*pi/11)+sin(4*pi/11)+sin(6*pi/11)+ sin(8*pi/11)+sin(10*pi/11)]= sin(π /11) + sin((2 π)/11) + cos(π /22) + cos((3 π)/22) + cos((5 π)/22)

sum_(n=1)^infty [sin((22n-20)*pi/11)+sin((22n-18)*pi/11)+sin((22n-16)*pi/11)+ sin((22n-14)*pi/11)+sin((22n-12)*pi/11)]/[{sin(π /11)+sin((2 π)/11)+cos(π /22)+cos((3 π)/22)+cos((5 π)/22)}*n^11]
= 1.00049418860411946455870228252646993646……
zeta(11)= 1.00049418860411946455870228252646993646…….

$$\zeta(11)=\sum_{n=1}^{\infty}\frac{\sin\frac{(22n-20)\pi}{11}+\sin\frac{(22n-18)\pi}{11}+\sin\frac{(22n-16)\pi}{11}+\sin\frac{(22n-14)\pi}{11}+\sin\frac{(22n-12)\pi}{11}}{n^{11}(\sin\frac{\pi}{11}+\sin\frac{2\pi}{11}+\cos\frac{\pi}{22}+\cos\frac{3\pi}{22}+\cos\frac{5\pi}{22})} \quad (1)$$

and, ζ(13) considered.
{sin((26n-24)*pi/13)+sin((26n-22)*pi/13)+sin((26n-20)*pi/13)+ sin((26n-18)*pi/13)}, {n=1}
= [sin(2*pi/13)+sin(4*pi/13)+sin(6*pi/13)+sin(8*pi/13)+sin(10*pi/13)+sin(12*pi/13)]
=[sin(π /13) + sin((2 π)/13) + sin((3 π)/13) + cos(π /26) + cos((3 π)/26) + cos((5 π)/26)]

sum_(n=1)^infty[sin((26n-24)*pi/13)+sin((26n-22)*pi/13)+sin((26n-20)*pi/13)+ sin((26n-18)*pi/13))+sin((26n-16)*pi/13)+sin((26n-14)*pi/13)]/[{sin(π /13)+sin((2 π)/13)+sin((3 π)/13)+cos(π /26)+cos((3 π)/26)+cos((5 π)/26)}*n^13]
=1.0001227133475784891467518365263573957142751...
zeta(13)= 1.000122713347578489146751836526357395714275105895509845...

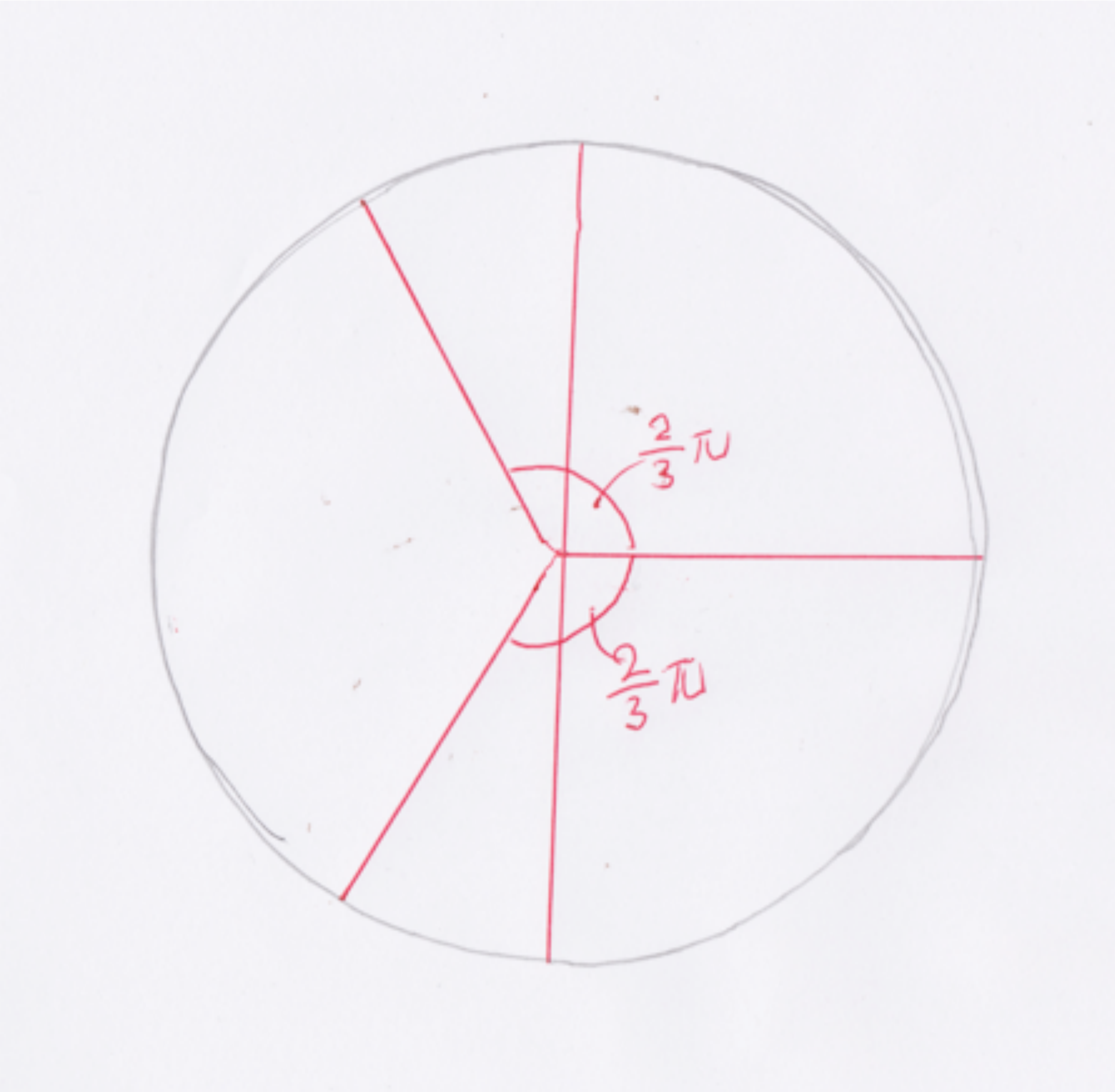
$$\zeta(13)=\sum_{n=1}^{\infty}\frac{\sin\frac{(26n-20)\pi}{13}+\sin\frac{(26n-18)\pi}{13}+\sin\frac{(26n-16)\pi}{13}+\sin\frac{(26n-14)\pi}{13}+\sin\frac{(26n-12)\pi}{13}+\sin\frac{(26n-10)\pi}{13}}{n^{13}(\sin\frac{\pi}{13}+\sin\frac{2\pi}{13}+\sin\frac{3\pi}{13}+\cos\frac{\pi}{26}+\cos\frac{3\pi}{26}+\cos\frac{5\pi}{26})} \quad (1)$$

ζ (15), ζ (19) etc. can also be expressed by these equations.

conclusion

From these equations, it can be said that ζ(3), ζ (5), ζ (7), ζ(9), ζ(11), ζ(13) are irrational numbers.

The figure below is that of ζ (3).



References

1) https://en.wikipedia.org/wiki/Riemann_hypothesis

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I am a psychiatrist now and also a doctor of brain surgery before.

(home)
〒854-0067
47-8 kuyamadai, Isahaya City, Nagasaki Prefecture, Japan

mmm82889@yahoo.co.jp
I would like to receive an email. I will not answer the phone.
Currently 56 years old
Born on November 26, 1961
（I am very poor of English. Almost all document are google-translation.）
When converted to English by Google translation, it becomes cryptic to me.
But, I use google-translation.