Research and report

ζ(3), ζ(5), ζ(7), ζ(9), ζ(11), ζ(13) are irrational numbers

Toshiro Takami
mmm82889@yahoo.co.jp

Abstract

Since ζ(3) could be represented by sin, cos and π, we report here. I spelled
In wolframAlpha,
and, ζ(5), ζ(7), ζ(9), ζ (11), ζ(13) considered.

From these equations, it can be said that ζ(3), ζ (5), ζ (7), ζ (9), ζ (11), ζ(13) are irrational numbers.
ζ (15), ζ (17) etc. can also be expressed by these equations.

discussion

$$\zeta(3) = \frac{\zeta(2)\pi^2}{15\zeta(4)} \sum_{n=1}^{\infty} \frac{\sin\frac{(6n-4)\pi}{3} - \sin\frac{(6n-2)\pi}{3}}{\sqrt{3}n^3} \tag{1}$$

=1.2020569031595942853997381615114499907649862923404988817...

1/1^3+1/2^3+1/3^3+1/4^3+1/5^3+1/6^3+1/7^3+1/8^3+1/9^3+1/10^3+1/11^3+1/12^3
……= ξ (3)

and, For example,
{sin((2 π /3) - sin(4 π /3) + sin(6 π /3)} / {sqrt(3)*1^3} +
{sin(8 π /3) - sin(10 π /3) + sin(12 π /3} / {sqrt(3)*2^3} +
{sin((14 π /3) - sin(16 π /3) + sin(18 π /3)} / {sqrt(3)*3^3} +

{sin(20 π /3) - sin(22 π /3) + sin(24 π /3} / {sqrt(3)*4^3} +
………
………= ζ (3)
and,
{-cos((2 π /3) - cos(4 π /3) + cos(6 π /3)} / (2*1^3) +
{-cos(8 π /3) - cos(10 π /3) + cos(12 π /3} / (2*2^3) +
{-cos(14 π /3) - cos(16 π /3) + cos(18 π /3)} / (2*3^3) +
{-cos(20 π /3) - cos(22 π /3) + cos(24 π /3} / (2*4^3) +
………
………= ζ (3)
While turning circles, factorial increases.

and, ζ(5) considered.

$$\zeta(5) = \frac{21\zeta(6)}{2\zeta(4)\pi^2} \sum_{n=1}^{\infty} \frac{2\sin\frac{(10n-8)\pi}{5} + 2\sin\frac{(10n-6)\pi}{5}}{n^5\sqrt{5+2\sqrt{5}}} \tag{1}$$

= 1.0369277551433699263313654864570…..

and, ζ(7) considered.

$$\zeta(7) = \frac{10\zeta(8)}{\pi^2\zeta(6)} \sum_{n=1}^{\infty} \frac{2\sin\frac{(14n-12)\pi}{7} + 2\sin\frac{(14n-10)\pi}{7} + 2\sin\frac{(14n-8)\pi}{7}}{n^7(\sin\frac{\pi}{7} + \cos\frac{\pi}{14} + \cos\frac{3\pi}{14})} \tag{1}$$

= 1.0083492773819228268397 9….. = ζ (7)

and, ζ(9) considered.

$$\zeta(9) = \frac{99\zeta(10)}{10\pi^2\zeta(8)} \sum_{n=1}^{\infty} \frac{\sin\frac{(18n-16)\pi}{9} + \sin\frac{(18n-14)\pi}{9} + \sin\frac{(18n-12)\pi}{9} + \sin\frac{(18n-10)\pi}{9}}{n^9(\frac{\sqrt{3}}{2} + \sin\frac{\pi}{9} + \sin\frac{2\pi}{9} + \cos\frac{\pi}{18})} \tag{1}$$

zeta(9) = 1.0020083928260822144178527692324120604856058513948887 56...
….

and, ζ(11) considered.

$$\zeta(11) = \frac{6825\zeta(12)}{691\pi^2\zeta(10)} \sum_{n=1}^{\infty} \frac{\sin\frac{(22n-20)\pi}{11} + \sin\frac{(22n-18)\pi}{11} + \sin\frac{(22n-16)\pi}{11} + \sin\frac{(22n-14)\pi}{11} + \sin\frac{(22n-12)\pi}{11}}{n^{11}(\sin\frac{\pi}{11} + \sin\frac{2\pi}{11} + \cos\frac{\pi}{22} + \cos\frac{3\pi}{22} + \cos\frac{5\pi}{22})} \tag{1}$$

= 1.00049418860411946455870228252646993646……
zeta(11)= 1.00049418860411946455870228252646993646….…

and, ζ(13) considered.

$$\zeta(13) = \frac{691\zeta(14)}{70\pi^2\zeta(12)} \sum_{n=1}^{\infty} \frac{\sin\frac{(26n-20)\pi}{13} + \sin\frac{(26n-18)\pi}{13} + \sin\frac{(26n-16)\pi}{13} + \sin\frac{(26n-14)\pi}{13} + \sin\frac{(26n-12)\pi}{13} + \sin\frac{(26n-1}{13}}{n^{13}(\sin\frac{\pi}{13} + \sin\frac{2\pi}{13} + \sin\frac{3\pi}{13} + \cos\frac{\pi}{26} + \cos\frac{3\pi}{26} + \cos\frac{5\pi}{26})} \quad (1)$$

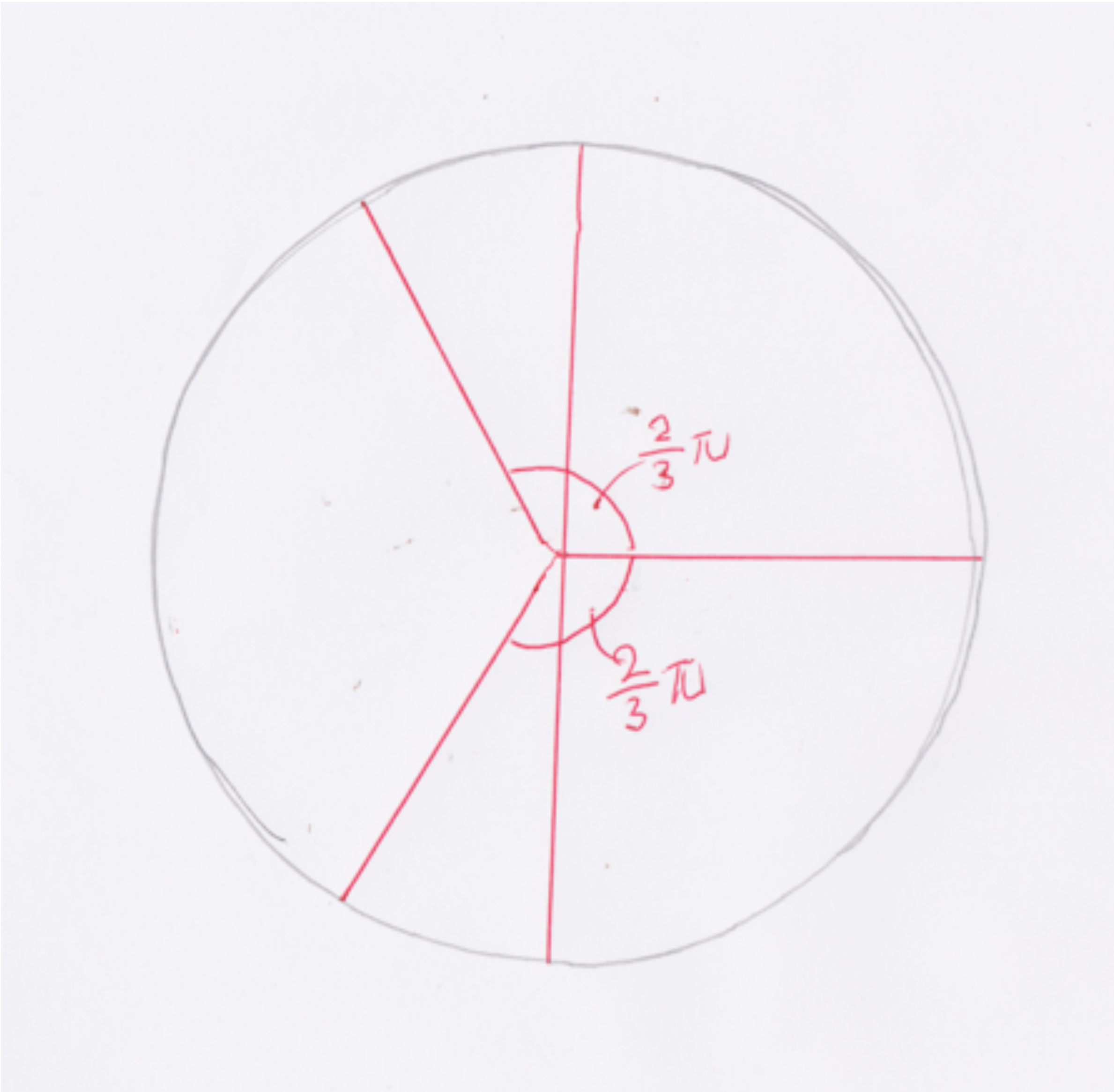
=1.0001227133475784891467518365263573957142751...
zeta(13)= 1.000122713347578489146751836526357395714275105895509845...

ζ (15), ζ (19) etc. can also be expressed by these equations.

conclusion

From these equations, it can be said that ζ(3), ζ (5), ζ (7), ζ(9), ζ(11), ζ(13) are irrational numbers.

The figure below is that of ζ (3).



References

1) https://en.wikipedia.org/wiki/Riemann_hypothesis

postscript

Writing the calculations on the way is very heavy for latex beginners, I want to forgive that I did not write.

☃～☃～☃～☃～☃～
☃～☃～☃～☃～☃～

I am a psychiatrist now and also a doctor of brain surgery before.

(home)
〒854-0067
47-8　kuyamadai, Isahaya City, Nagasaki Prefecture, Japan

mmm82889@yahoo.co.jp
I would like to receive an email. I will not answer the phone.
Currently 56 years old
Born on November 26, 1961
（I am very poor of English. Almost all document are google-translation.）
When converted to English by Google translation, it becomes cryptic to me.
But, I use google-translation.