

Refutation of the lonely runner conjecture with three runners

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Abstract: We evaluate the conjecture of the lonely runner with three runners. We do *not* assume a runner may be stationary as a no-go contestant. The result is that the conjecture diverges from tautology by one logical value and hence is refuted. We also assume a runner can be stationary; those *non* tautologous results also refute the conjecture.

We assume the method and apparatus of Meth8/VL4 with Tautology as the designated *proof* value, **F** as contradiction, **N** as truthity (non-contingency), and **C** as falsity (contingency). The 16-valued truth table is row-major and horizontal. The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables for more variables. (See ersatz-systems.com.)

LET p, q, r, s : runner-1, runner-2, runner-3, number of runners;
 \sim Not; $+$ Or ; $\&$ And; \setminus Not And; $>$ Imply; $=$ Equivalent; $@$ Not Equivalent;
 $(p=p)$ tautology; $(p@p)$ contradiction, zero 0.

From: en.wikipedia.org/wiki/Lonely_runner_conjecture

Remark 0: Other implementations of the conjecture assume a runner may *not* run but remain stationary, and name that the lonely runner. However this initial implementation makes no such assumption because a non-runner is *not* a runner and hence removed from consideration.

No runner is stationary. (1.1.1)

$$(((p+q)+r)@(p@p)) = (p=p) ; \quad \mathbf{FTTT} \quad \mathbf{TTTT} \quad \mathbf{FTTT} \quad \mathbf{TTTT} \quad (1.1.2)$$

No runner as equivalent to another runner implies the number of runners. (1.2.1)

$$(((p@q)\&(q@r))\&(p@r))>s = (p=p) ; \quad \mathbf{TTTT} \quad \mathbf{TTTT} \quad \mathbf{TTTT} \quad \mathbf{TTTT} \quad (1.2.2)$$

No runner is stationary, and no runner as equivalent to another runner implies the number of runners. (1.3.1)

$$(((p+q)+r)@(p@p))\&(((p@q)\&(q@r))\&(p@r))>s = (p=p) ; \quad \mathbf{FTTT} \quad \mathbf{TTTT} \quad \mathbf{FTTT} \quad \mathbf{TTTT} \quad (1.3.2)$$

Remark 1.1/2/3: While the truth table results for Eqs. 1.1.2 and 1.3.2 as rendered are equivalent, Eq. 1.2.2 is needed to establish that the unique runners establish the number of runners. Eqs. 1 as cast with model operators weaken the result.

One runner implies the fraction of a runner divided by the number of runners. (2.1.1)

$$(((p>((q+r)\s))+q>((p+r)\s)))+(r>((p+q)\s))) = (p=p) ; \quad \mathbf{TTTT} \quad \mathbf{TTTT} \quad \mathbf{TTTT} \quad \mathbf{TTTF} \quad (2.1.2)$$

One runner is equivalent to the fraction of a runner divided by the number of runners. (2.2.1)

$$(((p=((q+r)\s))+ (q=((p+r)\s)))+(r=((p+q)\s))) = (p=p) ;$$

F T T T T T T T T F (2.2.2)

Remark 2: Eqs. 2.1 and 2.2 show respectively one runner implying a fraction is closer to tautology than is one runner equivalent to a fraction.

We evaluate two arguments with the antecedent of Eqs. 1 and consequent of 2.1 / 2.2.

No runner is stationary, and no runner as equivalent to another runner implies the number of runners which implies one runner implies the state of 1 divided by the number of runners. (3.1.1)

$$(((p+q)+r)@(p@p))&(((p@q)&(q@r))&(p@r)>s) >$$

$$(((p>((q+r)\s))+ (q>((p+r)\s)))+(r>((p+q)\s))) ;$$

T T T T T T T T T F (3.1.2)

No runner is stationary, and no runner as equivalent to another runner implies the number of runners which implies one runner is equivalent to the state of 1 divided by the number of runners. (3.2.1)

$$(((p+q)+r)@(p@p))&(((p@q)&(q@r))&(p@r)>s) >$$

$$(((p=((q+r)\s))+ (q=((p+r)\s)))+(r=((p+q)\s))) ;$$

T T T T T T T T T F (3.2.2)

Remark 3: Eqs. 3.1 and 3.2 produce the same truth table result as *close* to tautology but divergent by one F value. This is due to T>F=F.

If we ignore Eq. 1.1 to establish that a runner can be permitted as stationary, to adopt the common assumption, the truth table result analogs for Eqs. 3 become:

$$(((p@q)&(q@r))&(p@r)>s) >$$

$$(((p>((q+r)\s))+ (q>((p+r)\s)))+(r>((p+q)\s))) ;$$

F T T T T T T T T F (4.1.2)

$$(((p@q)&(q@r))&(p@r)>s) >$$

$$(((p=((q+r)\s))+ (q=((p+r)\s)))+(r=((p+q)\s))) ;$$

T T T T T T T T T F (4.2.2)

Remark 4: By admitting a stationary runner, the conjecture is weakened for the implication of a runner as a fraction, but remains the same result as Eq. 3.2 for the equivalence of a runner as a fraction.

Excepting Eq. 1.2, the other Eqs. are *not* tautologous. This means that with or without assuming a runner can be stationary as a no-go, the lonely runner conjecture is refuted.