

Confirmation of the Collatz conjecture

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Abstract: Using the standard wiki definition of the Collatz conjecture, we map a positive number to imply that a divisor of two implies either an even numbered result (unchanged) or an odd numbered result (changed to the number multiplied by three plus one) to imply the final result of one. This is the shortest known confirmation of the conjecture, and in mathematical logic.

The Collatz conjecture is described at wikipedia.org/wiki/Collatz_conjecture, for which we decompose farther below:

"[A] sequence defined as follows: start with any positive number n . Then each term is obtained from the previous term as follows: if the previous term is even, the next term is one half the previous term. If the previous term is odd, the next term is 3 times the previous term plus 1. The conjecture is that no matter what value of n , the sequence will always reach 1." (0.0)

We assume the method and apparatus of Meth8/VL4 with Tautology as the designated *proof* value, **F** as contradiction, **N** as truthity (non-contingency), and **C** as falsity (contingency). The 16-valued truth table is row-major and horizontal.

LET p, q, r, s : positive integer, quotient, remainder, divisor;
 + Or, add; & And, multiply; \ Not And, divide;
 > Imply, greater than; < Not Imply, lesser than; = Equivalent; @ Not Equivalent;
 % possibility, possibly, for one or some; # necessity, necessarily, for all or every.
 (%s>#s) ordinal one; (%s<#s) ordinal two; (s=s) ordinal three, ; (s@s) ordinal zero.
 $\sim(y > x) (x \geq y)$

"[A] sequence defined as follows: start with any positive integer n ." (0.1)

$\sim((\%p>\#p)>p) = (p=p) ;$ **NFNF NFNF NFNF NFNF** (0.2)

Remark 0.0.1: Previously we used zero as the fiducial point for positive integers, when in fact ordinal 1 is the fiducial point. Hence "p is greater than or equal to one" is captured by "not one greater than p" as above.

We divide p by the divisor s to produce a quotient q and remainder r as a fraction of the divisor s . (1.1)

$(p\backslash s)=(q+(r\backslash s)) ;$ **TTTT TTTT TFTF FTTF** (1.2)

We define an even number as having the fractional part of remainder r as zero in the numerator and divisor s in the denominator, for a remainder of zero, to imply p is p divided by s . (2.1)

$(r=(r@r))>(p=(p\backslash s)) ;$ **FTFT TTTT FFFF TTTT** (2.2)

We define an odd number as having the fractional part of remainder r as one in the numerator and divisor s in the denominator, for a remainder of one divided by s, to imply p is p multiplied by three plus one. (3.1)

$$(r=(\%r>\#r))>(p=((p\&(p=p))+(\%p>\#p))) ; \quad \text{TTTT CTCT TTTT CTCT} \quad (3.2)$$

We build the argument that Eq. 0.1 implies the following: divisor s as two implies (Eq. 1) the form of p/2 as quotient plus fraction as remainder/2 which implies either (Eq. 2) the form of an even p or (Eq. 3) the form of an odd p, to imply the final result of one. (4.1)

$$\begin{aligned} &(\sim((\%p>\#p)>p)>(((s=(\%s<\#s))>(((p\backslash s)=(q+(r\backslash s))) > (((r=(r@r))>(p=(p\backslash s)))+ \\ &((r=(\%r>\#r))>(p=((p\&(p=p))+(\%p>\#p))))))) = (\%p>\#p) ; \\ & \qquad \qquad \qquad \text{TTTT TTTT TTTT TTTT} \end{aligned} \quad (4.2)$$

Eq. 4.2 as rendered is tautologous, hence confirming the Collatz conjecture. We note this is the shortest known such proof, and in mathematical logic.